[PREPARED IN THE ORDNANCE COLLEGE.]

TEXT BOOK

OF

GUNNERY.

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PART I.

CONTENTS.

ABBREVIATIONS.

B.L.	Breech-Loading (applied to new type ordnance).
C.	Centigrade.
C.G.	Centre of Gravity.
cm.	Centimètre.
D of A.	Director of Artillery.
F.	Fahrenheit.
f/s.	Feet per second.
g	Gramme.
G.D.	Gravimetric density.
G.V.	Gravimetric volume.
Inst. C.E.	Institution of Civil Engineers.
kg.	Kilogram.
L.G.	Large-grain powder.
m.	Mètre.
mm.	Millimètre.
M.H.	Martini-Henry rifle.
L.M.	Lee-Metford Magazine rifle.
M.V.	Muzzle Velocity.
P.	Pebble powder.
Phil. Trans.	Philosophical Transactions of the Royal Society.
Proc. R.A.I.	Proceedings of the Royal Artillery Institution.
Proc. R.S.	Proceeding Royal Society.
Q.D.	Quadrant Depression.
Q.E.	Quadrant elevation.
R.B.L.	Rifled Breech-Loading (applied to old type guns).
R.C.D.	Royal Carriage Department.
R.G.F.	Royal Gun Factory.
R.L.	Royal Laboratory.
R.L.G.	Rifled large-grain powder.
R.M.L.	Rifled Muzzle-Loading.
R.U.S.I.	Royal United Service Institution.
R.V.	Remaining Velocity.
T.E.	Tangent elevation.

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ERRATA.

On page 6, last line, *for* "60 tons" *read* "34 to 44 tons."

„ 122, in line 14, *for* $\frac{2W+w}{W+w+w}\frac{V}{l}$ *read* $\frac{2W+w_1}{W+w+w_1}\frac{V}{l}$.

„ 122, in line 20, *for* Q*y* *read* Q*x*

„ 122, line 21, *read* V^2

„ 123, lines 18, 23, and 26, *for* C*w* *read* Cw_1

„ 164, last line, *read* 460 + 62

„ 171, line 7, *read* Table III

„ 281, 3rd para. from bottom, for " curves *read* curve "

„	310,	opposite 43	in column	9,	*read*	6563·6
„	311	„	81	„	5	„	6816·5
„	311	„	86	„	0	„	7672·4
„	311	„	86	„	6	„	7779·9
„	311	„	100	„	7	„	9935·6
„	312	„	132	„	4	„	2307·7
„	312	„	137	„	5	„	2569·4
„	312	„	162	„	3	„	3706·2
„	312	„	172	„	9	„	4512·9
„	313	„	235	„	0	„	4 6319·1
„	314	„	285	„	4	„	7704·5
„	319	„	56	„	1	„	·21723
„	319	„	82	„	6	„	·67552
„	323	„	218	„	3	„	·889443
„	323	„	224	„	4	„	·890667
„	323	„	225	„	9	„	·890950
„	325	„	53	„	0	„	56·48
„	325	„	53	„	1	„	60·11
„	325	„	53	„	2	„	63·84
„	325	„	53	„	3	„	67·67
„	325	„	53	„	7	„	84·03
„	325	„	64	„	3	„	943·06
„	325	„	72	„	7	„	959·79
„	326	„	103	„	5	„	690·84
„	326	„	111	„	9	„	296·65
„	327	„	134	„	6	„	451·76
„	327	„	143	„	6	„	830·32
„	327	„	149	„	2	„	053·95
„	328	„	197	„	2	„	783·67
„	328	„	203	„	6	„	988·83
„	328	„	212	„	2	„	248·81
„	328	„	215	„	1	„	334·46
„	329	„	236	„	1	„	891·43
„	329	„	246	„	5	„	162·75
„	329	„	251	„	0	„	11 281·20
„	330	„	260	„	1	„	516·99
„	330	„	266	„	1	„	665·56
„	330	„	268	„	1	„	713·76

In Table X, at head of 1st column, *for* v. ft. *read* V f/s

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TEXT BOOK OF GUNNERY.

PART I.

In the following pages of Part I (after the definitions have been given), the subject of Gunnery is considered in the order which naturally suggests itself, in the two parts in which it may be divided.

First, Exterior Ballistics, in which the motion of the projectile is considered after it has received its initial velocity of projection, when the projectile is moving freely under the influence of gravity and the resistance of the air, and it is required to determine the conditions so as to hit a certain object.

Secondly, Interior Ballistics, in which the pressure is analysed of the powder gas in the bore of the gun, and the investigation carried out of the requisite charge of powder to secure the initial velocity of the projectile; and in which the calculations are made of the strength of the various parts of the gun required to withstand the pressure at all parts of the bore.

The more mathematical parts of gunnery are resumed and completed in Part II.

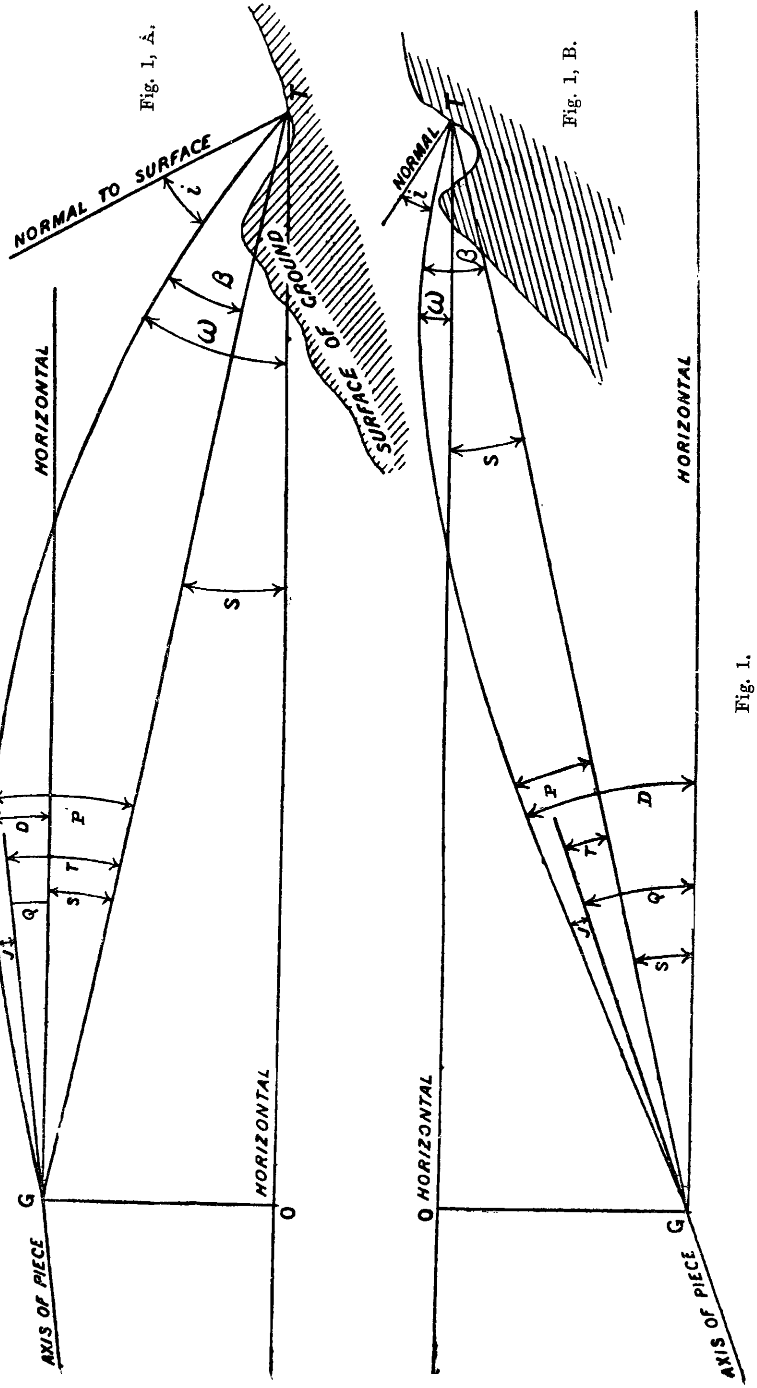

Fig. 1, A.

Fig. 1, B.

Fig. 1.

CHAPTER I.—DEFINITIONS AND UNITS.

The **axis of the piece** is the straight line passing down the centre line of the bore.

The **axis of the trunnions** is the straight line passing through the centre of the trunnions, at right angles to the axis of the piece.

The **calibre** is the diameter of the bore in inches measured across the lands.

The **line of sight** is the straight line passing through the sights of the piece and the point aimed at.

The **angle of sight** is the angle which the line of sight makes with the horizontal plane (S, fig. 1).

When the line of sight slopes downwards, as in fig. 1, A, for instance, in firing at a sea-target, the angle of sight is usually called the **Angle of Depression.**

The **line of departure** is the direction in which the shot is moving on leaving the piece; or, in other words, a tangent to the trajectory at the muzzle.

The **angle of departure** is the angle which the line of departure makes with a horizontal plane (D, fig. 1).

The **trajectory** is the curve described by the C.G. (centre of gravity) of the shot in flight (*i.e.*, the curved line GT in fig. 1, A, B).

Range is the distance GT from the muzzle of the gun G to the (second) intersection T of the trajectory with the line of sight.

The **planes of sight and departure** are vertical planes passing through the lines of sight and departure respectively.

Drift is the deflection of the projectile from the vertical plane of departure due to the rotation imparted by the rifling of the piece. It is sometimes termed Deviation.

The **quadrant angle** (French *niveau*) is the angle (Q, fig. 1) which the axis of the piece, when laid, makes with the horizontal plane.

It is termed quadrant *elevation* or *depression* (Q.E. or Q.D.), according as the piece is laid above or below the horizontal plane; the term *depressed fire* means that a piece is fired at a quadrant angle of depression (Q.D.).

The **angle of tangent elevation** (T.E., French *hausse*) is the angle between the axis of the gun and the line of sight (T, fig. 1).

The angle (S) made by the line of sight with the horizon must be added or subtracted to obtain the *quadrant angle* of elevation or depression from the *tangent angle*; subtracted in fig. 1, A, added in fig. 1, B.

The **angle of projection** is the angle between the line of departure and the line of sight (P, fig. 1).

Jump is the angle between the line of departure and the axis of the gun before firing (J, fig. 1).

The **angle of descent** is the angle which a tangent to the trajectory at the first point of impact makes with the line of sight (β, fig. 1).

The **angle of arrival** is the angle which a tangent to the trajectory makes with the horizontal plane (α, fig. 1).

The **angle of incidence** is the angle which a tangent to the trajectory at the point of impact makes with the normal to the surface struck (i, fig. 1).

Angles are measured in *degrees* and *minutes*, the circumference being divided into 360° (degrees), and each degree into 60′ (minutes).

A watch face will serve as a protractor, for measuring angles, each minute of time on the face being 6° of angle, and each hour is 30° of angle.

Angular velocity is measured always in *radians per second;* the *radian* is the name now given to the unit of circular measure, an angle subtended by an arc equal to the radius; the radian is thus

$$180 \div \pi = 57{\cdot}3^\circ, \text{ or } 3438'.$$

Thus to turn degrees or minutes of angle into radians of circular measure, divide by 57·3 or 3438.

If the line of sight GT in fig. 1 A slopes down at an angle of sight of S minutes at a range GT of R yards, in consequence of the gun G being h feet above the horizontal line OT through the target T, then

$$\text{Sin S} = \frac{h}{3\text{R}}:$$

When the angle of sight is small, sin S can be replaced by the circular measure of S′ or by S ÷ 3438; and then

$$\frac{\text{S}}{3438} = \frac{h}{3\text{R}}, \text{ or S} = 1146\frac{h}{\text{R}},$$

the depression-range-finding (D.R.F.) formula, of frequent use in the sequel.

So also for a line of sight sloping upwards as in fig. 1 B.

The gunner's rule that an "inch at 100 yards subtends a minute," and so on in proportion, is equivalent to replacing the number 1146 by 1200, which makes the radian 60° instead of 57°·3, and makes the circumference of a circle three times the diameter.

To face p. 4.

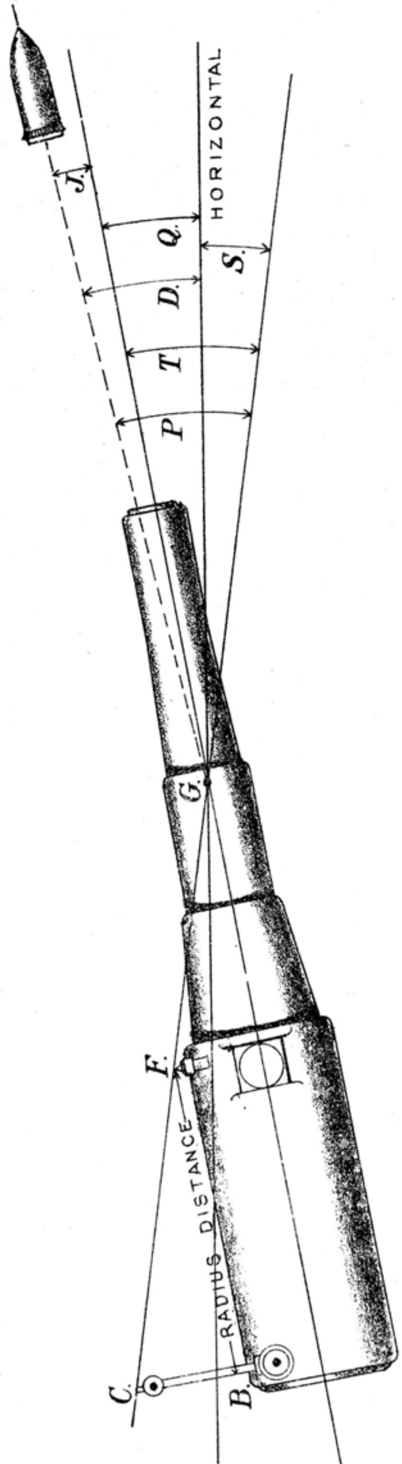

FIG. 2.

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The following example may help to define these angles :—

Example 1.—Firing out to sea at a range of 3,000 yards from a battery 300 feet high, the Angle of Depression

$$S = 114'\cdot6 = 1° 55'.$$

In the Range Table the angle of elevation is given as 2° 20′, with a jump of + 7′; then (fig. 1, A)

$$\begin{aligned} &T \text{ (angle of tangent elevation)} &&= 2° 20'; \\ &P \text{ (angle of projection)} &&= T + J = 2° 27' \\ &D \text{ (angle of departure)} &&= P - S = 0° 32' \\ &Q \text{ (quadrant elevation)} &&= D - J = 0° 25'. \end{aligned}$$

If the angle of descent is given in the Range Table as 2° 50′, then

$$\begin{aligned} &\beta \text{ (angle of descent)} &&= 2° 50', \\ &\omega \text{ (angle of arrival)} &&= \beta + S = 4° 45'. \end{aligned}$$

Example 2.—If the angle of tangent elevation is 2° 20′, quadrant angle 4° 15′, and jump 7′, find the angles of departure, of sight, and of projection.

From the definitions and from fig. 1, B,

$$\begin{aligned} &D \text{ (Angle of departure)} &&= \text{quadrant angle} + \text{jump} \\ & &&= 4° 15' + 7' \\ & &&= 4° 22' \\ &S \text{ (Angle of sight)} &&= \text{quadrant angle} - \text{angle of elevation} \\ & &&= 4° 15' - 2° 20' \\ & &&= 1° 55' \\ &P \text{ (Angle of projection)} &&= \text{angle of elevation} + \text{jump} \\ & &&= 2° 20' + 7' \\ & &&= 2° 27'. \end{aligned}$$

Dangerous space is the horizontal distance in which the trajectory would catch a given vertical target; *e.g.*, a shell with a slope of descent of 1 in 10 would catch the broadside of a ship 20 feet out of the water over a distance of 200 feet, which is thus the dangerous space.

Muzzle velocity is the velocity of a projectile on leaving the muzzle, in feet per second; abbreviated in writing to f/s.

Remaining velocity is the velocity at any point of the trajectory.

Striking velocity is the velocity at the point of impact

PHYSICAL DEFINITIONS.

Force is that which produces, tends to produce, or prevents motion in a body.

The unit of force employed in gunnery is the attraction of the earth on one pound or one ton.

Strictly speaking, it is the tension of a thread or rope, supporting one pound or one ton, thus allowing for a slight discount in the attraction of the earth due to its rotation. This unit of force is a statical or gravitational unit, and it changes slightly but quite inappreciably for our purposes, for different places on the earth's surface.

Stress is the action of balancing forces; it is estimated in units of force per unit area, generally in pounds or tons per square inch, abbreviated in writing to $lb/in.^2$ or $tons/in.^2$

Pressure is a stress tending to prevent the approach of two bodies together.

Tension is a stress tending to prevent the separation of two bodies.

Total thrust or pull P is the product of the stress p and the area A in square inches over which it acts, or

$$P = pA, \qquad p = P/A,$$

Strain is the deformation produced by stress.

Compression is the strain produced in a body by pressure.

Extension is the strain produced in a body by tension

The **limit of elasticity** of a substance is the least stress producing permanent strain.

For any stress less than the elastic limit, the ratio of stress to strain is found practically to be constant; this ratio is called the **modulus of elasticity**, and denoted by the letter E or M.

$$\text{Thus } E = \frac{\text{stress}}{\text{strain}} = \frac{\text{pressure}}{\text{compression}}, \text{ or } \frac{\text{tension}}{\text{extension}},$$

according as the stress is a pressure or a tension.

Under these circumstances, when the stress is removed the strain disappears, and the body returns to its original dimensions.

The **tenacity** of a substance is the least breaking tension.

Elasticity and tenacity are expressed in tons on the square inch in practical work.

For gun-steel we may take the modulus of elasticity

$$M = 12{,}500 \text{ tons/in.}^2,$$

and the tenacity as about 60 $tons/in.^2$

UNITS.

The **units** employed in gunnery are numerous; this is apt to lead to mistakes in calculations.

The **units of length** are—

Yards for ranges at practice, and in Hadcock's Table X.
Feet in Bashforth's tables, and in expressing the height of any point of the trajectory above plane, as, for instance, the position of the burst of a shell in the air.
Inches are used for calibres of ordnance, diameters of projectiles, and for the distances apart of the marks on the chronometer of Boulengé's instrument.
Thousandths of an inch are employed in denoting the shrinkages given to the parts of a gun.

The *units of weight* are—

Tons, cwts., and lbs., for ordnance.
Lbs. and oz., for projectiles, powder charges, and bursting charges of shells.
Grains, for bullets and powder charges of machine guns and small arms. 7000 grains = 1 lb. avoirdupois.

The metric units of the mètre and kilogramme are universally employed in Continental works on gunnery.

Weight is given in kilogrammes; or in grammes for small weights, such as rifle bullets and charges of powder.

Range is given in mètres (m.), and velocity in mètres per second (m/s).

Work and energy are measured in kilogramme-mètres (kg.-m), taking $g = 9{\cdot}81$, in metre-second units.

The calibre of foreign guns is expressed in centimètres (cm.) or millimètres (mm.); and centimètres are converted into inches by multiplying by 0·4, or more accurately by 0·3937, while inches are converted into millimètres by multiplying by 25 or 25·4 (Table XIII).

Thus a calibre of 12 centimètres is 4·7 inches, and the 75-mm. field gun and 7·5 mm. rifle have calibres of about 3 and 0·3 inches.

The conversion of other metric measures of length, weight, pressure, energy, velocity, &c., will be found in Table XIII.

CHAPTER II.—EXTERIOR BALLISTICS. THE RESISTANCE OF THE AIR, AND THE USE OF THE BALLISTIC TABLES.

SECTION 1.—CONSTRUCTION OF THE BALLISTIC TABLES.

THE first careful experiments on the resistance of the air were carried out by Sir Isaac Newton (1687) on spheres of glass, filled with air, water, or mercury, of various weights and diameters, let fall from the dome of St. Paul's Cathedral.

It was assumed that the resistance was proportional to the square of the velocity and the square of the diameter, and then from an observation of the time occupied in falling a given height (220 feet in Newton's experiments in St. Paul's Cathedral) it is possible to infer the resistance of the air from a mathematical formula.

The experiments by Benjamin Robins, in 1743, with his Ballistic Pendulum (Chapter IV), confirmed Newton's results for slow motion, for instance, at velocity below 800 or 900 feet per second (f/s); but for swift motion and velocity above 1200 f/s, Robins found that the resistance of the air was about three times the amount given by Newton's experiments, if calculated on the assumption of the *quadratic* law of resistance, that is, taking the resistance as varying as the square of the velocity.

Later experiments by Hutton in 1775, and at Metz in 1820, carried out with cannon balls against ballistic pendulums of large size, have also confirmed the law that the resistance of the air to a shot is proportional to the square of the diameter; but no simple mathematical law could be deduced from the experiments which would give the resistance at all velocities, both high and low.

The ballistic pendulum is nowadays completely superseded by electric screens and the chronograph, described in Chapter IV; the passage of the shot through the screens is recorded by an electric signal in the chronograph; the time occupied between the screens is read off, and the velocity and retardation of the shot can then be calculated, and thence the resistance of the air can be inferred.

Elaborate experiments of this nature were carried out by the Rev. F. Bashforth, B.D., in 1865—1870, and again in 1878—1879, and it is on these experiments that the value of the resistance of the air adopted in the construction of the Ballistic Tables has been obtained.

As in Newton's experiments, Mr. Bashforth found that the resistance of the air was proportional to the cross section or to the square of the diameter; so that if p denotes the resistance of the air to a 1-inch projectile, then the resistance to a d inch projectile, is given by

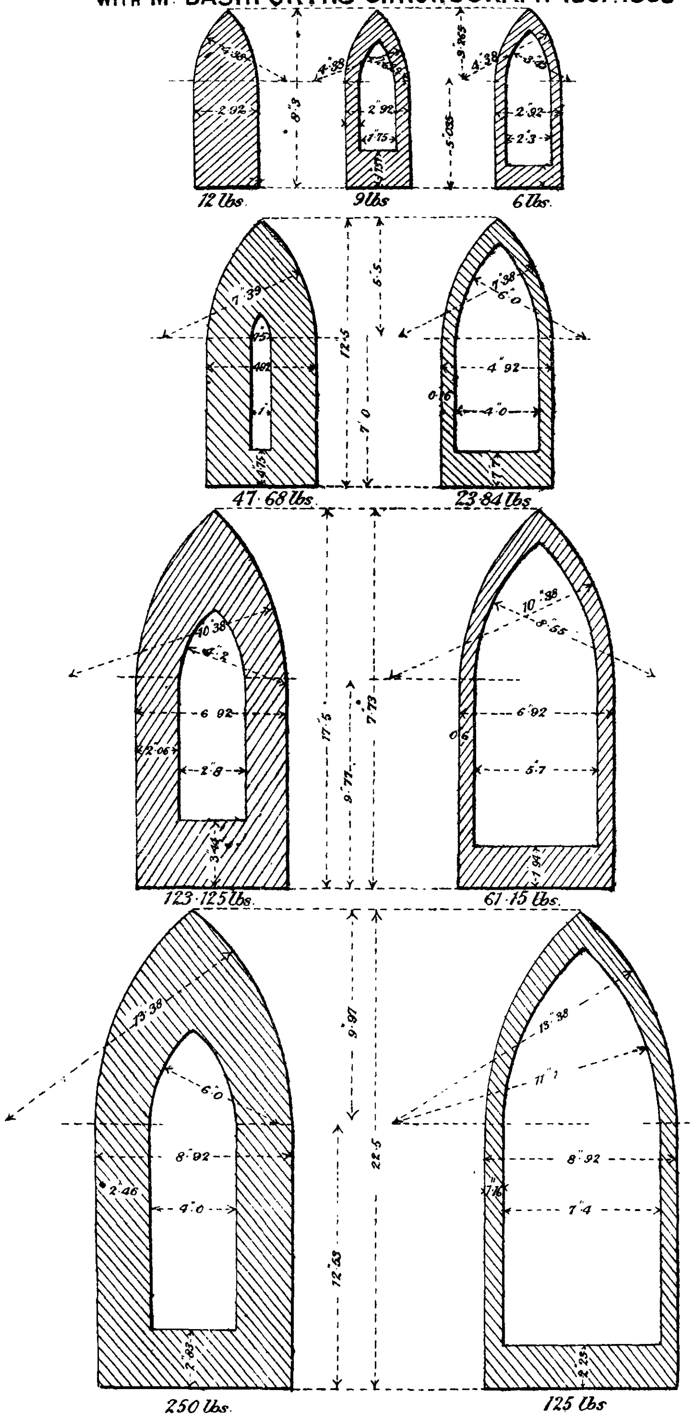
$$d^2p, \text{ pounds.}$$

The value of p in pounds, shown graphically in fig. 3, for projectiles of a standard shape fired under standard conditions, is given in Table II for velocities ranging from 100 f/s to 2800 f/s, as deduced from Bashforth's experiments, embodied in the coefficient K given in Table I, the relation between p and K being

$$p = \frac{K}{g}\left(\frac{v}{1000}\right)^3.$$

FIG I

DRAWINGS OF SECTIONS OF SHOT USED, IN EXPERIMENTS WITH MR BASHFORTHS CHRONOGRAPH 1867.1868



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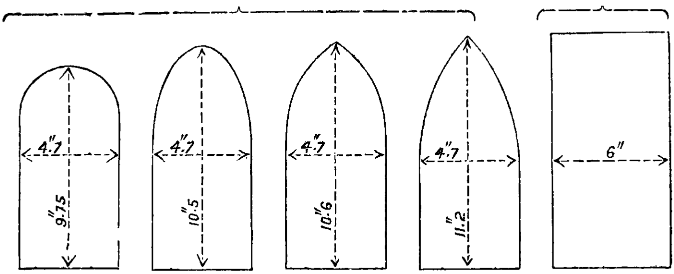
The standard shape adopted in the experiments was that of the service elongated projectile of that period (1865–79), having a cylindrical body and a flat base, and an ogival head struck with a radius of one and a half calibres or diameters, as shown in fig. 1.

To allow for difference in shape, projectiles of the annexed form, shown in fig. 2, were experimented with, and it was found that the

Fig. 2.

Forms of Shot experimented with

in 1886. in 1879.



(1) Hemispherical head. (2) Hemispheroidal head. (3) Ogival head (1 diam.) (4) Ogival head (2 diams.) (5) Flat head.

resistance of the air could be represented by

$$\kappa d^2 p, \text{ pounds,}$$

where κ is a factor depending on the shape of the head and smoothness of the surface, and κ is called the **coefficient of shape.**

Under ordinary conditions the value of κ may vary from 2 in flat headed proof projectiles, and 1·7 for spherical shot, to 0·95 for modern projectiles, and to 0·8 for the magazine rifle bullet.

With the improved steadiness in flight obtained with breech-loading, it is found that the resistance is reduced by a factor, σ, called the **coefficient of steadiness,** so that the resistance becomes

$$\kappa \sigma d^2 p, \text{ pounds.}$$

On the other hand, the coefficient σ may become greater than unity when the gun is worn, or the projectile imperfectly centered or rotated unsteadily.

Finally it was found that the resistance is proportional to the density of the air, so that if τ denotes the density referred to a certain standard density (534·22 grains per cubic foot in Bashforth's experiments), then the resistance R of the air is given, in pounds, by

$$R = \kappa \sigma \tau d^2 p.$$

and τ is called the **coefficient of tenuity.**

The allowance for tenuity becomes very important with high angle fire at long ranges, when the shot reaches elevated strata of the air where the density may be halved, or $\tau = \frac{1}{2}$.

But in all accurate records of practice it is important that the barometer and thermometer should be read, upon which the value of τ depends, as given in Table XI.

The product $\kappa \sigma \tau$ of the three coefficients κ, σ, τ is replaced by the letter n, and called the **coefficient of reduction,** so that we write

$$R = n d^2 p.$$

Fig. 3.

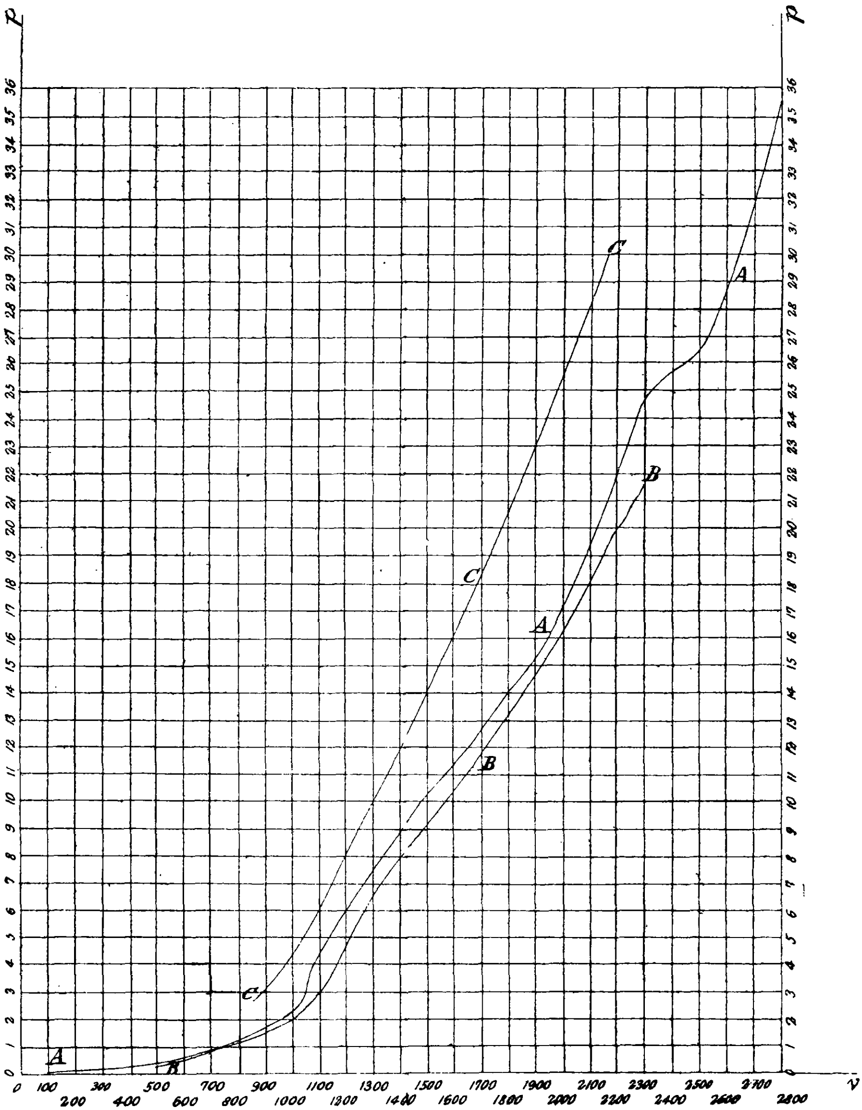


Diagram connecting v and p

To face p. 11.

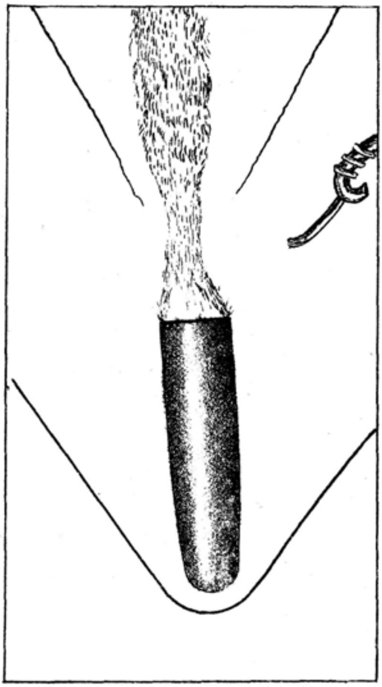


FIG. 4

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The curve connecting p and v, for the standard projectiles experimented with by Mr. Bashforth, is shown by the curve AA of fig. 3; the curve BB is drawn from the results of Krupp's experiments, where a shot with sharper point was employed; while the curve CC is drawn from the experiments with spherical shot.

Fig. 4 is drawn from a photograph of a bullet in flight, taken by Mr. C. V. Boys; the air waves diverging from the head and the base, and the trail of eddies in the wake of the bullet are very clearly shown, and resemble very closely the waves set up on the surface of water by a swift steamer. (*Nature*, March, 1893.)

EXPLANATION OF BALLISTIC TABLE III.

Suppose the projectile is flying horizontally, the effect of gravity being left out of account; the motion is retarded continually by the resistance of the air; and if Δv denotes the loss of velocity, in f/s, in a short interval of time of Δt seconds, then the *average* retardation r is given by

$$r = \frac{\Delta v}{\Delta t}.$$

But by Newton's Second Law of Motion, if the shot weighs w lbs. and the resistance of the air is R pounds, then

$$\frac{r}{g} = \frac{\mathrm{R}}{w}.$$

Putting—

$$\mathrm{R} = nd^2p.$$

where p refers to the *average* value in the interval of time Δt, then

$$\frac{\Delta v}{\Delta t} = \frac{\mathrm{R}}{w}g = \frac{nd^2pg}{w}.$$

or

$$\Delta t = \frac{w}{nd^2}\ \frac{\Delta v}{pg}.$$

The quantity $\frac{w}{nd^2}$ is very important in the Theory of Gunnery; it is called the *ballistic coefficient* of the projectile, and denoted by the letter C, so that

$$\mathrm{C} = \frac{w}{nd^2}$$

and then

$$\frac{\Delta t}{\mathrm{C}} = \frac{\Delta v}{pg}.$$

Since p is the same for all projectiles, and g is a constant, then if we take a constant drop in velocity (say, of 10 f/s, or $\Delta v = 10$),

$$\frac{\Delta v}{pg}$$

is a number *which is the same for all projectiles;* it is denoted by $\Delta\mathrm{T}$, and

$$\Delta\mathrm{T} = \frac{\Delta v}{pg},$$

and this is the time in seconds it takes the unit projectile under standard conditions, for which C = 1, to drop from velocity

$$v + \tfrac{1}{2}\Delta v \text{ to } v - \tfrac{1}{2}\Delta v \text{ f/s},$$

if v denotes the mean velocity in the interval.

The number ΔT is calculated numerically once for all, and entered in a column; afterwards the sum of the values of ΔT is made, and entered in a column denoted by T_v, as in Table III; so that if the shot takes t seconds for the velocity to fall from V to v f/s,

$$t = C(T_V - T_v). \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Thus, for example, while the velocity of the shot falls from 1510 to 1500 f/s, the mean velocity in the interval

$$v = 1505;$$

and the corresponding mean value of p from Table II is

$$p = 10{\cdot}3235.$$

Then, with $\Delta v = 10$, $g = 32{\cdot}19$, $\log g = 1{\cdot}5077$, $\log \Delta v/g = \bar{1}{\cdot}4923$.

Interval.	1490—1500.	1500—1510.	1510—1520.	1520—1530.
v	1495	1505	1515	1525
p		10·3235		
$\log p$		1·0138		
$\log \frac{\Delta v}{g}$		$\bar{1}$·4923		
$\log \frac{\Delta v}{pg} = \log \Delta T$		$\bar{2}$·4785		
ΔT	0·0305	0·0301	0·0298	0·0294
(In Table III) T	232·2818	232·3123	232·3424	232·3722

The vacant columns which precede and follow can be filled in as an exercise by similar numerical calculations.

Proportional parts must be employed for unit increments of velocity; and to save this trouble the Table III for T_v has these values interpolated for units in the velocity.

It will be noticed that Table III begins with the tabular number 75·399 against the velocity $v = 100$; this number has no particular signification, but it is made sufficiently large so as not to become negative, supposing the provisional values of the resistance of the air adopted for low velocities should be modified, leaving the numbers for higher velocities unchanged.

Thus it is probable that this number for T against the velocity $v = 100$ was originally put down by Mr. Bashforth as 75, but became modified to 75·399 on a subsequent revision of the table.

EXPLANATION OF BALLISTIC TABLE IV.

Next let Δs denote the number of feet described in the interval of time Δt seconds; then

$$\Delta s = v\Delta t$$

if v denotes the *mean* velocity in the interval; and dividing by C, the ballistic coefficient

$$\frac{\Delta s}{C} = v\frac{\Delta t}{C} = v\Delta T.$$

Denoting this number by ΔS, then

$$\Delta S = v\Delta T;$$

a number which can be tabulated in a Ballistic Table, as it is the same for all projectiles; it is the number of feet which the unit projectile will advance, under standard conditions, while its velocity drops from $v + \frac{1}{2}\Delta v$ to $v - \frac{1}{2}\Delta v$ f/s.

Thus in the interval during which the velocity drops from 1510 to 1500 f/s, we can put

$$v = 1505,$$

and, continuing the numerical calculations,

Interval	1490—1500.	1500—1510.	1510—1520.	1520—1530.
v	1495	1505	1515	1520
$\log v$		3·1775		
$\log v\Delta T = \log \Delta S$		1·6560		
ΔS		45·29		
ΔS	45·60	45·30	45·20	45·10
(In Table IV) S	43116·4	43162·0	43207·2	43252·3

Any slight discrepancies which may be encountered in the last figure in these calculations may be explained as due to small variations in the density of the air, or to the use of four figure logarithms; the Slide Rule may be used to perform these calculations with sufficient accuracy.

The calculated values of ΔS are summed, and entered in the first column of Table IV; and the values of S for unit increments of velocity are interpolated afterwards.

Now if a shot goes s feet while the velocity drops V to v f/s,

$$s = C(S_V - S_v). \qquad \text{.................. (2)}$$

Here again, to avoid negative numbers in case of a revision of Table IV, an arbitrary number $S = 1066$ is entered against $v = 100$; but as the Table is used for differences $S_V - S_v$, this number does not affect the results; this number 1066 was probably 1000 before a revision of the Table at low velocities.

These two Tables III and IV for T_v and S_v are called Bashforth's Ballistic Tables for Time and Distance (or Space); the numbers T_v and S_v are called the *reduced time* and *reduced* distance; and

$$T_V - T_v,\ S_V - S_v,$$

represent the time taken in seconds and the distance gone in feet, between any initial velocity V and final velocity v, by a shot for which the *ballistic coefficient*

$$C = \frac{w}{nd^2}.$$

is unity; for instance a 1-inch 1-pr., or 3-inch 9-pr., of standard shape, under standard conditions, so that $n = 1$.

EXPLANATION ON BALLISTIC TABLE V.

A third table, Table V, due to Mr. W. D. Niven, called the *Degree Table*, gives the *deviation* in direction in the vertical plane of a projectile in a flat trajectory, between any initial velocity V and final velocity v, f/s.

For if Δi denotes in *radians* (or circular measure) the change of direction, or of slope i, in the trajectory in the time Δt seconds, then resolving normally (as proved in Part II),

$$v\frac{\Delta i}{\Delta t} = g \cos i.$$

When i is small, $\cos i$ may be replaced by unity; and

$$\Delta i = \frac{g}{v}\Delta t.$$

But if $\Delta\delta$ denotes the number of degrees in Δi radians,

$$\Delta\delta = \frac{180}{\pi}\Delta i = \frac{180g}{\pi}\frac{\Delta t}{v}.$$

Dividing by the ballistic coefficient C, and denoting $\frac{\Delta\delta}{C}$ by ΔD, then

$$\Delta D = \frac{180g}{\pi}\frac{\Delta T}{v};$$

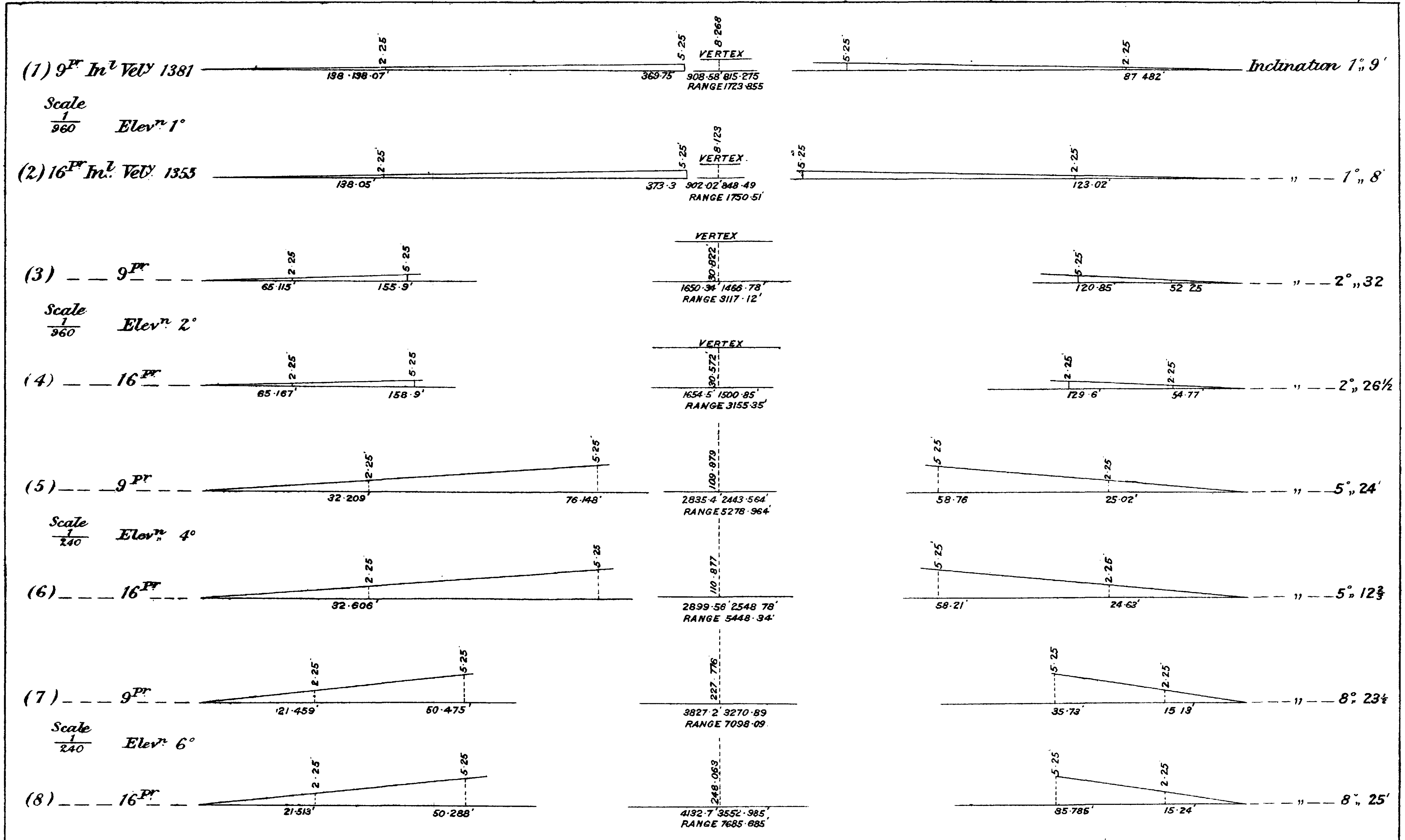
and ΔD is a number which is the same for all projectiles, being the number of degrees of deviation in direction of motion of the unit projectile, while its velocity drops from $v + \frac{1}{2}\Delta v$ to $v - \frac{1}{2}\Delta v$ f/s, the projectile flying in a nearly horizontal direction.

In continuation of the preceding numerical calculations—

Interval	1490—1500.	1500—1510.	1510—1520.	1520—1530.
v	1495	1505	1515	1525
$\log\frac{\Delta T}{v}$		$\bar{5}\cdot3010$		
$\log\frac{180g}{\pi}$		3·2658		
$\log\frac{180g}{\pi}\frac{\Delta T}{v} = \log\Delta D$		$\bar{2}\cdot5668$		
ΔD		0·03688		
(In Table V) D	83·8983	83·9359	83·9727	84·0090

To face p. 15.

FIG. 5.



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E. Weller & Grahams. Ltd Litho. London.

The summation of the calculated ΔD, and the interpolation of the values of D for unit increments of velocity in Table V, is carried out in the same manner as before in Tables III and IV.

These calculations will serve as a type of those required in a revision of the present Ballistic Tables, consequent on a redetermination by experiment of the value of p with modern projectiles.

Now if the direction of motion changes through δ degrees, while the velocity drops from any initial velocity V to any final velocity v, then

$$\delta = C(D_V - D_v). \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This Table V is useful in finding angles of departure and descent in direct fire for a given range of X feet.

For if V denotes the initial velocity, and v the final velocity at the end of the range of X feet, then

$$\frac{X}{C} = S_V - S_v$$

or

$$S_v - S_V - \frac{X}{C},$$

whence v is found from Table IV; and then the timo of flight T seconds is given by Table III from the formula

$$T = C(T_V - Tv).$$

In a flat trajectory the vertical component of the resistance of the air is insensible, so that we may assume that the shot takes equal time in going up to the vertex of the trajectory, and in coming down again; in other words the vertex is at the point of *half time;* so that if v_0 denotes the vertex velocity, then

$$\frac{\frac{1}{2}T}{C} = T_V - Tv_0,$$

or

$$T_V - Tv_0 = \tfrac{1}{2}(T_V - T_v)$$

$$Tv_0 = \tfrac{1}{2}(T_V + T_v),$$

whence the corresponding v_0 is found from Table III.

Now if the angles of departure and descent are denoted in degrees by ϕ and β, employing Table V,

$$\phi = C(D_V - Dv_0).$$

$$\beta = C(Dv_0 - D_v).$$

On this assumption of the vertical component of the resistance as insensible, the vertical height y at any time t is the same as for a body projected vertically upwards into the air for T seconds; and, therefore, according to Elementary Dynamics,

$$y = \tfrac{1}{2}gTt - \tfrac{1}{2}gt^2 = \tfrac{1}{2}gt(T - t) = \tfrac{1}{2}\, gtt',$$

where t' denotes the time down to the ground again; this is Colonel Sladen's formula, and it is very useful in plotting out ordinates of a flat trajectory.

Taking $g = 32$, makes $y = 16tt'$;

and at the vertex of the trajectory, where $t = t' = \frac{1}{2}$ T,

$$H = \tfrac{1}{8}gT^2 = 4T^2 = (2T)^2;$$

hence the practical rule—the square of double the time of flight in seconds is the height in feet of the vertex of the trajectory.

It will be noticed, however, that the application of Sladen's formula sometimes will give an appearance to the first part of the trajectory of an upward curvature, as is observable with a golf ball.

The approximation is then unsuitable; and the method must be replaced by one in which gravity is first left out of account, and the projectile is supposed to move in the initial direction of projection against the resistance of the air; and afterwards the effect of gravity is supposed to be restored by depressing the projectile a depth $\frac{1}{2}gt^2$ feet, where t denotes the time of flight to any point; this is equivalent to neglecting the vertical component of the resistance compared with its component parallel to the original direction of projection.

To find when Sladen's formula becomes unsuitable we must find when

$$\frac{R}{w} = \cot\phi,$$

$$\text{or } \frac{cd^2p}{w} = \cot\phi,$$

$$\text{or } \tan\phi = \frac{C}{p},$$

the value of p corresponding to the initial velocity V, and ϕ denoting the angle of departure; for greater values of ϕ the method is unsuitable, and the second method must be adopted at the outset.

The trajectories in fig. 5 have been drawn to scale by Colonel Kensington, late Professor of Artillery, Royal Military Academy; they are useful in showing to the eye the flatness of the trajectories and the closeness of approximation of the formulas.

Explanation of Tables VI and VII.

It is convenient to have a table giving the change in the tangent of the slope of the trajectory, and this is given by the function I_v in Table VI, such that, if the slope changes from ϕ to θ in fig. 6, while the velocity changes from V to v,

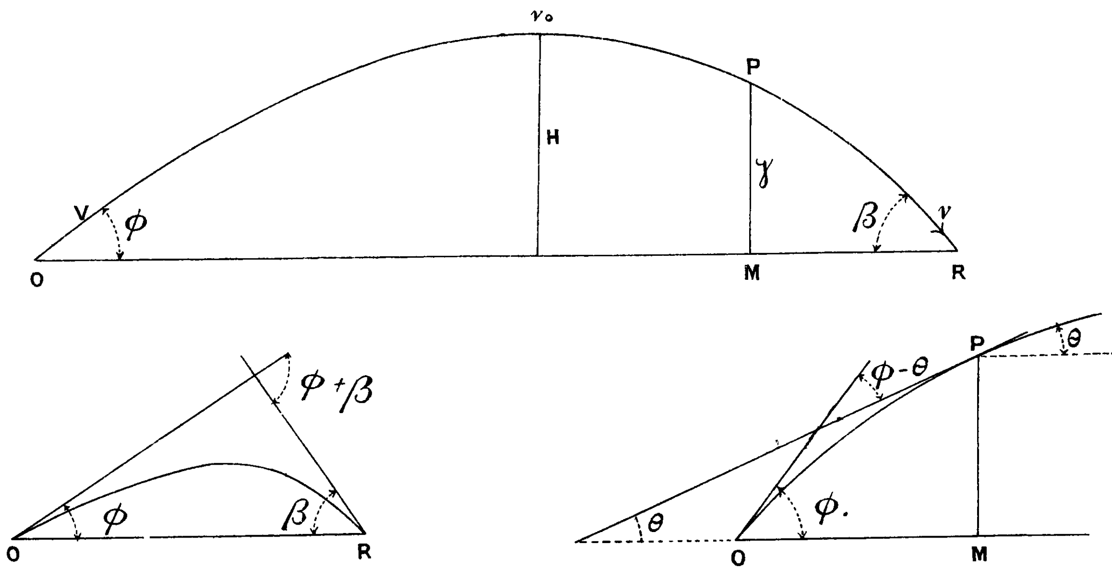
$$\tan\phi - \tan\theta = C(I_V - I_v).$$

In Direct Fire the angles ϕ and θ are small enough for $\tan\phi$ and $\tan\theta$, and the circular measure of ϕ and θ to be practically the same, so that the function I can be derived from D by multiplying by the factor

$$\frac{\pi}{180} = 0{\cdot}01745;$$

or if calculated independently, by omitting the factor $\frac{180}{\pi}$ employed in the computation of ΔD, so that

$$\Delta I = g\frac{\Delta T}{v}.$$



ν_0
P
H
γ
V
ϕ
β
ν
O
M
R
$\phi + \beta$
ϕ
β
O
R
P
θ
$\phi - \theta$
$\phi.$
θ
O
M

FIG 6

The *altitude function* A, invented by Colonel Siacci, has been tabulated by Mr. A. G. Hadcock in Table VII; the theory of this function will be given in Part II; it is required for determining the height y in feet of the shot at any intermediate range of x feet, in High Angle as well as in Direct Fire.

In Direct Fire, the formula required is

$$\frac{y}{x} = \tan\phi - C\left(I_V - \frac{A_V - A_v}{S_V - S_v}\right),$$

where v is the remaining velocity determined by Table IV, from

$$x = C(S_V - S_v).$$

If v denotes the striking velocity at the end of the range of X feet where $y = 0$ again,

$$\tan\phi = C\left(I_V - \frac{A_V - A_v}{S_V - S_v}\right),$$

which when the angle of departure ϕ is small, may be replaced by

$$\sin 2\phi = Ca,$$

where

$$a = 2\left(I_V - \frac{A_V - A_v}{S_V - S_v}\right),$$

thus determining the angle ϕ for a range X, when V and C are given.

To save the labour of calculating v and thence a, in terms of V and v, Mr. Hadcock has prepared Table X, a table of double entry, in which the value of a for any range of R yards can be read off corresponding to any initial velocity V, and reduced range $\frac{R}{C}$ yards.

To use Table X, look out on the line for the given velocity V, and in the column corresponding to $\frac{R}{C}$ yards, the number a; then

$$\sin 2\phi = Ca, \text{ or } \tan\phi = \tfrac{1}{2}Ca,$$

which determines ϕ, with sufficient accuracy.

Proportional parts must be used when the data, V and $\frac{R}{C}$, fall between two lines or columns of Table X.

In preparing in this way the column of elevations in a range table, it is convenient to calculate ϕ for given muzzle velocity V, for every 100 C yards of range, thus taking multiples of 100 in $\frac{R}{C}$; afterwards to interpolate the values of ϕ for the hundreds of yards of actual range; specimen calculations will be given in the sequel.

2. APPLICATION OF THE BALLISTIC TABLES.

In the application of the preceding Ballistic Tables, the notation employed is :—

w = Weight of projectile, in pounds.
d = Diameter of projectile, in inches.
$n = \kappa\sigma\tau$ = Coefficient of reduction.
κ = Coefficient for shape of projectile.
σ = Coefficient for steadiness of projectile.
τ = Coefficient for tenuity or density of air.
$C = \frac{w}{nd^2}$ = Ballistic coefficient.
V = Initial velocity, in feet per second, f/s.
v = Final velocity, in feet per second, f/s.
t = Time of flight, in seconds.

(1.) $$t = C\,(T_V - T_v).$$

s = Distance advanced, in feet.

(2.) $$s = C\,(S_V - S_v).$$

ϕ = Initial direction, in degrees, with horizon.
θ = Final direction.
δ = Deviation in direction $= \phi - \theta$.

(3.) $$\delta = C\,(D_V - D_v).$$

(4.) $$\tan\phi - \tan\theta = C\,(I_V - I_v).$$

In using Table X, in which the velocity v and the range in yards R are given, and it is required to determine ϕ, the initial direction, look out on the line for the given velocity V, and on the column of range corresponding to $\frac{R}{C}$ yards, the number, and denote it by a: then

(5.) $$\sin 2\phi = Ca,$$

which determines ϕ.

In High Angle Fire, V and v must be replaced in the formulas by U and u, called by Colonel Siacci the *pseudo-velocities*, where

(6.) $U = V\cos\phi\sec\eta$; $u = v\cos\theta\sec\eta$.
η = mean inclination in the arc ϕ to θ

(7.) $= \frac{1}{2}(\phi + \theta)$, approximately.

(8.) $\sec\eta = \frac{i(\phi) - i(\theta)}{\tan\phi - \tan\theta}$, if a closer approximation is desired at high angles of observation; $i(\theta)$ is tabulated in Table VIII.

Now, in High Angle Fire

(9.) $${}_\phi t_\theta = C\,(T_U - T_u).$$

(10.) $${}_\phi x_\theta = C\cos\eta\,(S_U - S_u).$$

(11.) $$\tan\phi - \tan\theta = C\sec\eta\,(I_U - I_u).$$

(12.) $${}_\phi\left(\frac{y}{x}\right)_\theta = \tan\phi - C\sec\eta\left(I_U - \frac{A_U - A_u}{S_U - S_u}\right).$$

In calculating the trajectory over an arc from inclination ϕ to θ, equations (7) and (8), determine η; and V being given, U is found from (6), and then (11) gives I_u and u.

Thence (9), (10), (12), give t, x, and y.

Other useful formulas are—

(13.)	If T is whole time of flight, and v_0 is vertex velocity	$\frac{1}{2}T = C(T_V - Tv_0)$ seconds. or $Tv_0 = \frac{1}{2}(T_V + Tv)$
(14.)	The angle of projection	$\phi = C(D_V - Dv_0)$.
	The angle of descent ..	$\beta = C(Dv_0 - Dv)$.
(15.)	Height y at any point where T is the whole time	$y = \frac{1}{2}gt\ (T - t)$ feet.
(16.)	Maximum height ..	$H = 4(T)^2 = (2T)^2$ feet.
(17.)	Muzzle energy (M.E.)	$= \dfrac{wV^2}{2g \times 2240}$ ft.-tons.
(18.)	Striking energy (S.E.)	$= \dfrac{wv^2}{2g \times 2240}$ ft.-tons.
(19.)	Penetration of wrought iron plate: Captain Orde Browne's rough rule	$p = \dfrac{vd}{1000}$ inches.
(20.)	Penetration of wrought iron plate: For moderate striking velocities	$p = \dfrac{v}{608 \cdot 3}\sqrt{\dfrac{w}{d}} - \cdot 14\ d$ inches.
(21.)	Penetration of wrought iron plate: For higher velocities. Tresidder's formula	$p^2 = \dfrac{wv^3}{d} \times \dfrac{1}{\log^{-1} 8 \cdot 8410}$.

In working the problems the coefficient of reduction n in the ballistic coefficient

$$C = \frac{w}{nd^2},$$

is first replaced by unity; other values of n, say 0·9, may then be taken to show the percentage of change in consequence.

The Slide Rule may replace, with ample accuracy, the four figure logarithms used in the calculations.

Problem 1.—Given C, V and v, to find t, from Table III and the formula

$$t = C(T_V - T_v).$$

1. Find how many seconds it will take for the velocity of the projectiles fired from the 13-pr. R.M.L. field gun, and from the 80-ton R.M.L. gun, to fall from 1595 f/s and 1540 f/s respectively, to 1000 f/s.

Here (1.) $d = 3$, $w = 13\frac{1}{4}$, $V = 1595$, $v = 1000$;

(2.) $d = 16$, $w = 1700$, $V = 1540$, $v = 1000$;

to find t in each case:—

	(1.)		(2.)
$\log w =$	1·1222	....	3·2304
$\log d^2 =$	0·9542	....	2·4082
$\log C =$	0·1680	....	0·8222
$T_V =$	232·5858	....	232·4308
$T_v =$	229·5207	....	229·5207
$\frac{t}{C} =$	3·0651	....	2·9101
$\log \frac{t}{C} =$	0·4864	....	0·4639
$\log t =$	0·6544	....	1·2861
$t =$	4·51 secs.	...	19·32 secs.

Problem 2.—Given V and t, to find v, from the formula

$$T_v = T_V - \frac{t}{C},$$

Find the remaining velocity, after 6 seconds, of the 10-in. B.L. projectile weighing 500 lbs., with muzzle velocity 2040 f/s, using Tables III.

Here $d = 10$, $w = 500$, $V = 2040$, and $t = 6$, to find v.

$$\log w = 2·6990$$
$$\log d^2 = 2·0000$$
$$\log C = 0·6990$$
$$\log t = 0·7782$$
$$\log \frac{t}{C} = 0·0792$$
$$\frac{t}{C} = 1·2000$$
$$T_V = 233·5666$$
$$T_v = 232·3666$$
$$v = 1518 \text{ f/s}.$$

It will be observed that in the last step, when looking out in the tables a value for v, the nearest tabular value to that obtained by calculation is taken. This is sufficiently accurate for all ordinary work, it being unnecessary to take into account fractional parts of a foot per second in Direct Fire.

Problem 3.—Given v and t, to find V, from the formula

$$T_V = T_v + \frac{t}{C}.$$

Find what must be the muzzle velocity of a shell, weighing 15 lbs., fired from a 12-pr. B.L. gun, in order that its remaining velocity after 7·5 seconds may be 900 f/s.

Here $d = 3$, $w = 15$, $v = 900$, to find V.

$$\begin{aligned}
\log w &= 1{\cdot}1761\\
\log d^2 &= 0{\cdot}9542\\
\log C &= 0{\cdot}2219\\
\log t &= 0{\cdot}8751\\
\log \frac{t}{C} &= 0{\cdot}6532\\
\frac{t}{C} &= 4{\cdot}5000\\
T_v &= 227{\cdot}9544\\
T_V &= 232{\cdot}4544\\
V &= 1548 \text{ f/s.}
\end{aligned}$$

Problem 4.—Given V and v, to find s, from Table IV and the formula

$$s = C(S_V - S_v).$$

Find the two ranges at which the remaining velocities will be 1500 f/s in the case of

(1.) The 6-in. B.L. projectile of 100 lbs. M.V. 1960 f/s;
(2.) The 12-in. B.L. projectile of 714 lbs. M.V. 1914 f/s.

Here,

	(1.)	(2.)
$w =$	100	714
$d =$	6	12
$\log w =$	2·0000	2·8537
$\log d^2 =$	1·5563	2·1584
$\log C =$	0·4437	0·6953
$V =$	1960	1914
$v =$	1500	1500
$S_V =$	45059·6	44885·8
$S_v =$	43162·0	43162·0
$S_V - S_v =$	1897·6	1723·8
$\log (S_V - S_v) =$	3·2782	3·2365
$\log C =$	0·4437	0·6953
$\log s =$	3·7219	3·9318
$s =$	5271 feet	8547 feet
$=$	1757 yards	2849 yards.

Problem 5.—Given V and s, to find v, from the formula

$$S_v = S_V - \frac{s}{C}.$$

Find the remaining velocity at 1,800 yards range of the projectile fired from the Martini-Henry rifle; also from the 2·5-in. and 12·5-in. R.M.L. guns.

Here, (1.) $w = 480$ grs. $= 0·06857$ lb., since 7000 grs. = 1 lb.

$d = 0·45$ inch, $V = 1315$.

(2.) $w = 7·625$, $d = 2·5$, $V = 1440$.

(3.) $w = 818$, $d = 12·5$ $V = 1442$,

and $s = 5400$, to find v in each case.

	(1.)	(2.)	(3.)
$\log w =$	$\bar{2}·8361$	0·8823	2·9128
$\log d^2 =$	$\bar{1}·3064$	0·7958	2·1938
$\log C =$	$\bar{1}·5297$	0·0865	0·7190
$\log s =$	3·7324	3·7324	3·7324
$\log \frac{s}{C} =$	4·2027	3·6459	3·0134
$\frac{s}{C} =$	15950·0	4425·0	1031·0
$S_V =$	42259·8	42884·4	42893·8
$S_v =$	26309·8	38459·4	41862·8
$v =$	432 f/s.	906 f/s.	1244 f/s.

Also, find t in each case from the formula

$$t = C(T_V - T_v).$$

	(1.)	(2.)	(3.)
$T_V =$	231·6690	232·1234	232·1299
$T_v =$	208·1291	228·0632	231·3572
$\frac{t}{C} =$	23·5399	4·0602	0·7727
$\log \frac{t}{C} =$	1·3718	0·6085	$\bar{1}·8880$
$\log C =$	$\bar{1}·5297$	0·0865	0·7190
$\log t =$	0·9015	0·6950	0·6070
$t =$	7·97 secs.	4·95 secs.	4·05 secs.

Problem 6.—Find the remaining velocity and time of flight of the 2·5-in. and 12·5-in. R.M.L. projectile for 2,000, 2,500, and 3,000 yards range.

Working by the same method as in the preceding example and putting $s = 6{,}000$, 7,500, and 9,000 successively, the following results are obtained :—

Range in yards.	2·5-inch.		12·5-inch.	
	v.	*t.*	*v.*	*t.*
2000	877	5·62	1225	4·53
2500	811	7·42	1180	5·77
3000	753	9·34	1138	7·06

Problem 7.—Find the striking velocity, and energy, also the number of inches of wrought-iron plate (unbacked) that can be penetrated by the 9·2-in. and 13·5-in. B.L. projectile at 1,000 yards range.

Here (1.) $d = 9{\cdot}2,\ w = 380,\ V = 2065,$

(2.) $d = 13{\cdot}5,\ w = 1250,\ V = 2016.$

and $s = 3{,}000$, to find v, E, and p (penetration in inches) in each case.

	(1.)	(2.)
$\log w =$	2·5798	3·0969
$\log d^2 =$	1·9276	2·2606
$\log C =$	0·6522	0·8363
$\log s =$	3·4771	3·4771
$\log \frac{s}{C} =$	2·8249	2·6408
$\frac{s}{C} =$	668·2	437·3
$S_V =$	45437·5	45264·9
$S_v =$	44769·3	44827·6
$v =$	1884 f/s.	1899 f/s.

The striking energy, in ft.-tons, $E = \dfrac{wv^2}{2g \times 2240}$,

	(1.)	(2.)
$\log v^2 =$	6·5502	6·5572
$\log wv^2 =$	9·1300	9·6541
$\log 2g \times 2240 =$	5·1591	5·1591
$\log E =$	3·9709	4·4950
$E =$	9,353 foot-tons	31,260 foot-tons.

The penetration p, in inches, may be obtained from the empirical formula (Krupp's)

$$v = 855 \frac{d^{\frac{3}{4}}}{w} p^{0\cdot7}$$

	(1.)	(2.)
$\log w^{\frac{1}{2}} =$	1·2899	1·5484
$\log v =$	3·2751	3·2786
$\log w^{\frac{1}{2}} v =$	4·5650	4·8270
$\log 855 =$	2·9320	2·9320
$\log d^{\frac{3}{4}} =$	0·7229	0·8477
$\log 855\, d^{\frac{3}{4}} =$	3·6549	3·7797
$\log p^{0\cdot7} =$	0·9101	1·0473
$\log p =$	1·3001	1·4961
$p =$	19·95 inches	31·34 inches.

Problem 8.—Given v and s, to find V, from

$$S_V = S_v + \frac{s}{C};$$

the problem required at proof when the remaining velocity v f/s of the projectile, at a distance s feet from the muzzle is ascertained by means of the chronograph.

Find the muzzle velocity, when the time taken by a shot fired from a 12-pr. B.L. gun in passing between two screens placed 180 feet apart, is found by the chronograph to be 0·1077 second.

The first screen is 50 yards from the muzzle of the gun.

The *mean* velocity is taken as the *actual* velocity at the mid-point between the screens, and then

$$v = \frac{180}{0\cdot1077} = 1669 \text{ f/s.}$$

The distance from the muzzle to the mid-point is

$$s = 150 + \frac{180}{2} = 240 \text{ feet.}$$

Now $w = 12\cdot5$, and $d = 3$, $s = 240$, $v = 1669$, to find V.

$$\log w = 1\cdot0969$$
$$\log d^2 = 0\cdot9542$$
$$\log C = 0\cdot1427$$
$$\log s = 2\cdot3802$$
$$\log \frac{s}{C} = 2\cdot2375$$
$$\frac{s}{C} = 172\cdot8$$
$$S_v = 43902\cdot3$$
$$S_V = 44075\cdot3$$
$$V = 1710 \text{ f/s.}$$

Now suppose the gun to have been fired with the same charge at the Woolwich proof butts, the time between screens to have been 0·1106 second (or about 0·003 of a second longer than the time of the service projectile in the previous example), the projectile being flat-headed; to allow for this, the coefficient of reduction, κ, referred to on p. 19, must be employed.

Experiment has shown that for a flat head, $\kappa = 1·817$ on the average, but in practical work it is generally assumed that $\kappa = 2$.

We have,

$$C = \frac{w}{\kappa d^2},$$

and

$$v = \frac{180}{0·1106} = 1628 \text{ f/s}.$$

Then

$\kappa =$	2	1·817
$\log \kappa =$	0·3010	
$\log d^2 =$	0·9542	
$\log \kappa d^2 =$	1·2552	
$\log w =$	1·0969	
$\log C =$	$\bar{1}$·8417	
$\log s =$	2·3802	
$\log \frac{s}{C} =$	2·5385	
$\frac{s}{C} =$	345·5	
$S_v =$	43728·8	
$S_V =$	44074·3	
$V =$	1710 f/s.	= 1738 f/s.

As another illustration, determine the M.V of the rifle which will have a striking velocity of 875 f/s at a range of 1,000 yards, given $d = 0·303$ inches, weight of bullet 215 grains and $n = 0·8$.

Problem 9.—Find the remaining velocity, time of flight, and height of the trajectory of the M.H. rifle bullet weighing 480 grains at intervals of 100 yards for a range of 1000 yards; muzzle velocity 1315 f/s, and $C = 0·3386$; calculated as in Problem 5 from

$$n = 1,\ d = 0·45,\ w = 480 \div 7000,\ \log C = \bar{1}·5297.$$

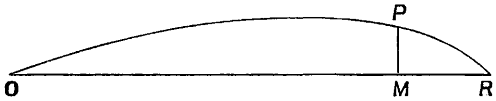
The annexed scheme of calculation on p. 26 shows a systematic procedure; the blank places are to be filled in.

SCHEME OF THE CALCULATION OF TRAJECTORY OF M.-H. RIFLE AT 1,000 YARDS RANGE.

Range in yards	100	200	300	400	500	600	700	800	900	1000
s	300	600	900	1200	1500	1800	2100	2400	2700	3000
$\log s$	2·4771									
$\log C$	$\bar{1}$·5297									
$\log \frac{s}{C}$	2·9474									
$\frac{s}{C}$	885·9	1771·8								
S_V	42259·8	42259·8								
S_v	41373·9	40488·0								
v	1166	1053	982	922	869	821	778	738	700	664
T_V	231·6690									
T_v	230·9490									
$\frac{t}{C}$	0·7200									
$\log \frac{t}{C}$	$\bar{1}$·8573									
$\log C$	$\bar{1}$·5297									
$\log t$	$\bar{1}$·3870									
t	0·2438	0·5137	0·811	1·127	1·461	1·878	2·196	2·590	3·009	3·449
T	3·4490	3·4490	3·449	3·449	3·449	3·449	3·449	3·449	3·449	3·449
t'	3·2050									
$\log t'$	0·5058									
$\log \frac{1}{2}g$	1·2067									
$\log y$	1·0995									
y	12·57	24·27	34·43	42·09	46·76	47·50	44·28	35·82	21·31	0·00

Problem 10.—Given V, x and y, to find v, t, and thence T and R.

Fig. 1.



At a range OM of 1500 yards, a 5-inch B.L. shell, having a muzzle velocity of 1750 f/s, grazes the top of a traverse PM, 8 feet high; how far beyond will it strike the ground, *i.e.*, to find MR.

Here, $w = 50$, $d = 5$, $V = 1750$,

$x = OM = 4500$, and $y = MP = 8$.

We will suppose the coefficient of reduction to be unity and atmospheric conditions to be normal; then $n = 1$, and,

$$C = \frac{w}{d^2} = 2{\cdot}0.$$

(1.) Employ the formula

$$S_v = S_V - \frac{S}{C},$$

to obtain the remaining velocity v at a range of 1500 yards at P; and y being so small compared with x we may put $x = s$; it will be found that at P

$$v = 1266 \text{ f/s}.$$

Had a coefficient of reduction $n = 0{\cdot}9$ been employed, this would have been 12 f/s more, a difference small enough not to affect appreciably the final result.

(2.) Employ the formula

$$t = C(T_V - T_v),$$

to obtain the time t to travel 1500 yards to P, it will be found that

$$t = 3{\cdot}04 \text{ seconds}.$$

(3.) Employ the formula

$$h \text{ or } y = \tfrac{1}{2}gt(T - t),$$

to find the time of flight T to end of trajectory R.

Then $$T = t + \frac{2y}{gt},$$

and it will be found that

$$T = 3{\cdot}201 \text{ seconds}.$$

(4.) Employ the formula

$$T_v = T_V - \frac{T}{C},$$

to find the velocity at the end of T seconds.
It will be found that at R

$$v = 1248 \text{ f/s}.$$

(5.) Employ the formula

$$s = C(S_V - S_v),$$

to find the distance MR, over which the velocity changes from 1266 to 1248 f/s.
It will be found that $s = 208{\cdot}4$ feet,
or MR = 69·5 yards,
and the whole range OR = 1569·5 yards.

The reason for these steps is as follows:—The distance required is given by (5) but the formula necessitates a knowledge of the final velocity at the end of the range, and this is obtained from (4). Again, the final velocity can only be found when T, the total time of flight, has been obtained from (3), and this, in its turn, depends on t, the time to the traverse, which can be found from (2) when the velocity at the traverse is known from (1).

The distance MR is called the *defiladed distance;* but if, instead of a traverse in the last example, a horseman (8 feet high) is supposed to be advancing towards the gun which continues firing at the same elevation, he on his horse may be struck by a direct hit whilst moving over the space from R to M; this is consequently called also the *dangerous* distance. Evidently the flatter the trajectory the greater the dangerous distance, and the greater the probability of hitting, if the range is not accurately estimated, and if, consequently, the correct elevation has not been given on the tangent scale.

The angle of descent β at various ranges is generally known, as it is recorded in Range Tables; the dangerous distance for a height h feet can then be readily found, for it is approximately $h \cot \beta$ feet.

In order to enable this calculation to be made still more rapidly, a column is frequently added to the range table, giving the *slope* of descent, "one in———," opposite each hundred yards of range, the number shown being the natural cotangent of the angle of descent β, then the dangerous distance is this number multiplied by the height h.

Another column sometimes also given is headed

"To hit an object 10 feet high, range must be known within ——— yards."

This is obtained by multiplying the permissible vertical error, in this case 5 feet or $1\frac{2}{3}$ yards, by the *slope* or $\cot \beta$ referred to above.

Further explanation is given in the Section on the Compilation of Range Tables.

Problem 11.—What is the greatest height H to which the projectile in the last question will rise above the ground?

The total time of flight T was found to be 3·20 seconds.

Therefore, by the formula,

$$H = (2T)^2 = 41 \text{ feet}.$$

Problem 12.—The 16-pr. R.M.L. has M.V. 1355 f/s and C = 1·43; the 20-pr. B.L. has M.V. 1677 f/s and C = 1·73.

Find the greatest height H of the trajectory in each case over a range of 1200 yards.

First find the remaining velocity v from

$$S_v = S_V - \frac{s}{C},$$

For the 16-pr. it is 1008 f/s; for the 20-pr. it is 1243 f/s.

Employ these values for v with

$$T = C(T_V - T_v),$$

to find T, the time of flight.

For the 16-pr. it is 3·145 seconds; for the 20-pr. it is 2·505 seconds.

Employing the formula

$$H = (2T)^2,$$

and substituting the values found above, we find the greatest height is 39·5 feet and 25·1 feet.

The details of the numerical calculation are given below.

	R.M.L. 16-pr.	B.L. 20-pr.
$\log s =$	3·5563	3·5563
$\log C =$	0·1553	0·2380
$\log \frac{s}{C} =$	3·4010	3·3183
$\frac{s}{C} =$	2518	2081
$S_V =$	42468·5	43936·3
$S_v =$	39950·5	41855·3
$v =$	1008	1243
$T_V =$	231·8255	232·8008
$T_v =$	229·6262	231·3524
$\frac{t}{C} =$	2·1993	1·4484
$\log \frac{t}{C} =$	0·3422	0·1608
$\log C =$	0·1553	0·2380
$\log t =$	0·4975	0·3988
$t =$	3·145	2·505
$\log 2 =$	0·3010	0·3010
$\log 2T =$	0·7985	0·6998
$\log H =$	1·5970	1·3996
$H =$	39·54	25·09
say,	39·5	25·1

Problem 13.—A 38-ton gun whose muzzle is 15 feet above the surface of the water, is aimed and fired at the middle of the side of a ship 30 feet high and 1000 yards distant. By mistake the range has been estimated to be 1100 yards, and elevation on the tangent scale is given accordingly. Will the ship's side be hit, and if so, where?

This is the same as finding the height of the ordinate at 1000 yards, when the total range is 1100 yards.

Take $C = 5{\cdot}181$, $MV = 1575$ f/s.

First, find the whole time of flight T for a range of 1100 yards.

$$S_v = S_V - \frac{3300}{5{\cdot}181}, \text{ with } V = 1575, \text{ whence } v = 1435 \text{ f/s.}$$

$$T = C\,(T_{1575} - T_{1435}), \text{ whence } T = 2{\cdot}195 \text{ secs.}$$

Next, find the time t to travel 1000 yards.

$$S_v = S_{1575} - \frac{3000}{5{\cdot}181}, \text{ whence } v = 1447 \text{ f/s.}$$

$$t = C\,(T_{1575} - T_{1447}),$$

whence $t = 1{\cdot}991$ secs.

Then from

$$\begin{aligned} h &= \tfrac{1}{2}gt\,(T - t), \\ &= \tfrac{1}{2} \times 32{\cdot}19 \times 1{\cdot}991 \times 0{\cdot}204, \\ &= 6{\cdot}536 \text{ feet.} \end{aligned}$$

or the side of the ship will be hit at a point 6·536 feet above the middle (which is level with the muzzle of the gun firing), or at 15 − 6·536 feet = 8·464 feet below the top.

This may also be found approximately by supposing $h = 300 \tan\beta$, if β is the angle of descent; but this will give rather too large a result, as it supposes the projectile to travel in a straight line for the last 100 yards of its course.

At 1100 yards the range table gives $\beta = 1°\ 24'$, whence $h = 7{\cdot}331$ feet instead of 6·536 feet as obtained by the more accurate calculation.

Problem 14.—The magazine rifle bullet strikes a vertical target at 500 yards at a certain spot when the M.V. is 2030 f/s, how much lower will the point of mean impact be, if the M.V. is only 1970 f/s, the elevation and other conditions being the same in both cases?

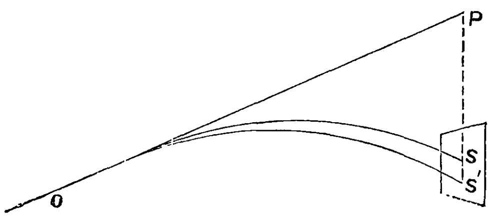
Here, $w = 215$ grains, $d = 0{\cdot}307$ inch, and take $n = 0{\cdot}7$, then

$$C = 0{\cdot}4656.$$

Find the striking velocity at the target from

$$s_v = S_V - \frac{1500}{C},$$

Fig. 2.



For the high velocity it is 1284 f/s, for the low velocity it is 1246 f/s.

Employ these values with

$$t = C(T_V - T_v)$$

to find the times of flight, t and t'; they are respectively

$$t = 0{\cdot}9358 \text{ sec.} \qquad t' = 0{\cdot}9651 \text{ sec.}$$

If gravity did not act in flight each bullet would reach the point P (fig. 2); but gravity will make them hit at points S and S′, such that PS and PS′ are the distances fallen in the times of flight; and PS, PS′ are found by the use of the formula

$$h = \tfrac{1}{2}gt^2,$$

and we have—

$$PS = 14{\cdot}99 \text{ feet.}$$
$$PS' = 14{\cdot}09 \quad ,,$$
$$\therefore \text{ the } \textit{difference } SS' = 0{\cdot}9 \quad ,,$$
$$= 10{\cdot}8 \text{ inches, the height of one point of mean impact above the other.}$$

Problem 15.—Given V and v, to find δ,

$$\delta = C(D_V - D_v),$$

Find the angle of projection ϕ, and descent β of the 12-inch B.L. 47-ton gun firing a projectile weighing 714 lbs., with M.V 1914 f/s to a range of 4000 yards; find also the striking velocity, time of flight, and height of vertex.

As this is a heavy B.L. gun, we take $n = 1$, then

$$C = \frac{w}{nd^2} = \frac{714}{144} = 4{\cdot}958,$$

$$\log C = 0{\cdot}6953$$

(1.) We have $V = 1914$ f/s, $s = 12{,}000$ feet, and $C = 4{\cdot}958$, to find v; from

$$S_v = S_V - \frac{s}{C},$$

and we obtain $\qquad v = 1354$ f/s.

(2.) Next find the whole time of flight T, from

$$T = C(T_V - T_v),$$

whence $\qquad T = 7{\cdot}487$ secs.

(3.) Suppose the vertex of the path of the projectile to be reached at half of the whole time of flight $= 3{\cdot}743$ secs.; then denoting by v_0 the velocity at the vertex, we have—

$$T_{v_0} = T_V - \frac{3{\cdot}743}{C},$$

and we find

$$v_0 = 1592 \text{ f/s.}$$

(4.) Given V and v_0, we can now find ϕ from

$$\phi = C(D_V - Dv_0),$$

whence we obtain

$$\phi = 3^\circ \cdot 961,$$

or the angle of projection is 3° 57′

From this must be deducted a certain number of minutes for jump, due to the particular nature of mounting.

5.) We can now calculate the angle of descent (β) from our knowledge of v_0 and v, by

$$\beta = C(D_{v_0} - D_V),$$

Thus with $v_0 = 1592$ f/s, and $v = 1354$ f/s,

we find $\beta = 4^\circ\ 43'$.

(6.) For the determination of the height of the vertex we have the whole time of flight T = 7·487.

Whence

$$H = (2T)^2 \text{ feet}$$
$$= (14{\cdot}974)^2$$
$$= 224{\cdot}2 \text{ feet.}$$

(7.) And collecting results, we have—

Angle of projection	ϕ	= 3° 57′
„ descent	β	= 4° 43′
Striking velocity	v	= 1354 f/s.
Time of flight	T	= 7·487 secs.
Height of vertex	H	= 224·2 feet.

EXAMPLES.

A.

1. The Range Table for a 10-inch R.M.L. gun shows angles of elevation as follows: 1000 yards, 1° 28′; 2000 yards, 3° 15′; 3000 yards, 5° 19′: what will be the Q.E. when the gun is mounted 100 feet above mean tide level?
2. Certain angles of descent are 1°, 1° 24′, 2° 17′, 4° 24′: find the corresponding slopes of descent.
3. At exactly half tide a gun is known to be 150 feet above the sea level; the quadrant angle of depression of a ship being 50′, find the range.
4. The width of a row of dummies put out is known to be 25 yards, their angular width is observed by the telescopic sight to be 45′: find the range.
5. A 9-feet target is observed from the gun to have an angular height of 5′, another observer makes it 6′: what difference would this make in the calculated range?
6. A ridge of ground, 100 yards distant from a gun, is observed to have an angular height above the horizontal plane of 3° 49′; what is the height of the top of the ridge in feet?

B.

1. With a 9-pr. R.M.L. gun, find the remaining velocity at 1000 yards range, muzzle velocity V = 1200 f/s. Here $w = 9$, $d = 3$, $s = 3000$, to find v.
2. Find the "time of flight" T of this 9-pr. shell over the same range.
3. Find the maximum height of the trajectory.
4. Compare the maximum height with that of the 12-pr. B.L. shell, having a time of flight of 2·07 secs. for a thousand yards range.
5. The first graze of a trial shot from a 5-inch B.L. gun is observed to occur after 6 secs. The muzzle velocity being 1750 f/s, calculate the range, considering atmospheric conditions normal, and taking $n = 1$, $w = 50$.
6. Calculate the height above the horizontal plane of the 9-pr. shell in problem (1) at 200 yards short of the 1000 yards target.
7. Plot the 9-pr. R.M.L. trajectory for 1000 yards approximately.
8. (*a.*) With the 12-pr. B.L. find the remaining velocity at a range of 2000 yards, V = 1710 f/s, $w = 12\frac{1}{2}$ lbs., $n = 0·9$.
   (*b.*) Find the time of flight.
   (*c.*) Find the angle of descent, the angle of elevation being 2° 38′, and jump 22′.
   (*d.*) Find the maximum height of the trajectory.
9. With the 10-inch B.L. gun, supposing the remaining velocity at 270 feet from the muzzle to be 2020 f/s, find the muzzle velocity. Take $n = 0·95$, $w = 500$.
10. A 12-inch B.L. gun is fired at Shoeburyness with a service projectile of 714 lbs. Screens are placed at 50 and 110 yards from the muzzle, and a velocity, at the middle point between them of 1901 f/s, is observed: find the M.V., $c = 5$.
11. The same gun is fired at the Woolwich Proof Butts, with the same charge and weight of projectile, but the latter is flat-headed, and the screens in this case are placed at 60 and 120 yards from the muzzle; a middle point velocity of 1886 f/s. being observed, find the M.V.
12. A 4-inch B.L. projectile is observed to strike an earthwork, the range of which is known to be 2300 yards, in 5 seconds: estimate the M.V., $n = 1$.
13. Calculate the time of flight of the 6-inch Q.F. projectile for a 2000 yards range, with a cordite charge giving M.V. 2250 f/s, $w = 100$.
14. Calculate the muzzle energies in ft.-tons of the 6-inch B.L. M.V. 1960 f/s, and 16·25-inch M.V 2087 f/s.
15. Compare the striking energy of the 6-inch B.L. M.V 1960 f/s, with that of the 6-inch Q.F. in example (13).
16. Suppose a shell weighing 15 lbs. to have been fired from the same gun as in example (8), and with the same (4 lbs. S.P.) charge, what will be the difference in :—

    (i.) Time of flight.
    (ii.) Maximum height of trajectory.

17. Calculate the angles of elevation and descent for the 12-pr.

    (i.) With a $12\frac{1}{2}$ lb. projectile.
    (ii.) With a 15 lb. projectile.
    Muzzle velocities as in example 8.
    The jump is 22′

18. Fill in the various columns of a Range Table for the 6-inch B.L. gun for the 2000 yards range.

Muzzle velocity = 1960 f/s.
Jump = 7 minutes.
Take n = 1.

19. With the same gun as in example 12, suppose a longer experimental shell weighing $30\frac{1}{4}$ lbs. to be fired with the same charge, how far will it range for the same time of flight, viz., 5 secs.? (take $n = 1$).
20. It is desired to fire a 12-pr. B.L. gun, layed indirectly by Q.E. for a range of 2400 yards from the position referred to in problem A (6): will the shell clear the ridge?
21. Using Captain Orde Browne's rough rule, what will be the penetration into wrought-iron armour plate of (*a*), the 9·2-inch, and (*b*) 10-inch B.L. projectiles at 2000 yards range?
M.V. 2065 and 2040 f/s respectively.
22. If in the preceding question, *compound* armour plate had been fired at, what penetration might be expected with each gun?
23. What are the longest ranges at which 16-inches thickness of compound armour could be perforated by a 12-inch B.L. projectile (*a*), weight 714 lbs., M.V. 1914 f/s;
(*b*), weight 850 lbs., M.V. 2350 f/s?
24. With a 6-inch B.L. howitzer, calculate the times of flight for a range of 2200 yards, of (*a*), a shell weighing 81 lbs., M.V 1200 f/s; (*b*), a longer shell weighing 100 lbs., fired with the same charge of powder (take $n = 1$).
25. A 3-pr. Hotchkiss Q.F. gun mounted on the upper top on the mast of a battleship 92 feet above the water line is fired horizontally: at what range will the shot strike the sea?
26. Determine the average velocity of a rifle bullet over a range of 100 yards, when the greatest rise of the bullet above the line of sight is 2, 1, or 0·5 inches.
27. Determine the ballistic properties, C and V, of a rifle which is to have a striking velocity of 800 f/s at a range of 1000 yards, without rising more than 32 feet. Calculate the weight of the bullet in grains for a calibre $d = 0{\cdot}303$ inch, taking the coefficient of reduction $n = 1$, 0·9, and 0·8.
28 Work out, as in Problem 9, p. 124, the trajectory for a range of 1000 metres of the French military rifle, in which $d = 8$ mm., $w = 14$ grams, and $V = 630$ m/s; and of the German rifle, in which $d = 7{\cdot}9$ mm., $w = 14{\cdot}5$ grams, and $V = 620$ m/s.
Spanish Mauser, 7 mm., $w = 11$ or 12 grams, $V = 670$ m/s.

ANSWERS TO BALLISTIC PROBLEMS.

A.

1. − 27, 2° 18′, 4° 41′.
2. 1 in 57, 41, 25 and 13.
3. 3438 yards.
4. 1910 „
5. 344 „ $\left\{ 2063 \; \dfrac{1719.}{344} \right.$
6. 20 feet.

B.

1. v = 914·3 f/s.
2. t = 2·93 secs.
3. 34·34 feet.
4. 34·4 : 17·14.
5. 2624 yards.
6. 23·45 feet.
7.
8.
9. V = 2034 f/s.
10. M.V. = 1914 f/s. Take C = 5.
11. „ = 1914 „
12. „ = 1892. Take n = 1, and velocity half way at $\dfrac{3 \times 2300.}{5}$
13. t = 3·12 secs.
14. 2665 ft.-tons and 54363 ft.-tons.
15. 100 : 132 32 per cent. higher.

	12½ lb.	15 lb.	
16. t =	4·66	4·85 secs.	c = 0·9 in both cases.
H =	86·8	94·5 feet	
17. α =	2° 37′	2° 56′	n = 0·9 in both cases.
β =	3° 52′	4° 4′	
18. V =	1443	3·58.	Col. (5) 2·91.
α =	1° 43′	2° 7′	„ (4) 81 (5) × (9).

Slope = 1 in 27.

19. V = 1720 f/s, v = 1065 f/s, s = 2211 yards, 89 yards less range.
20. *No.* See Range Table, page 619 of *Treatise on Service Ordnance,* 1893. Taking times for 100 and 2400 yards as 0·2 and 6 secs., gives h = 18·67 feet.
21. (*a.*) 1717 f/s, 15·8 ins. (*b.*) 1728 f/s, 17·28 ins.
22. 12·6 ins. 13·8 ins.
23. (*a.*) v. 1667 f/s. Range, 1652 yards. (*b.*) v. = 1667 f/s, take $p = \dfrac{16}{0{\cdot}8} = \dfrac{vd}{1000}$. Range, 4771 yards.
24. (*a.*) 6·44 secs. (*b.*) 6·74 secs.
25. 1178 yards. Take M.V. = 1873 f/s $\left\{ w = 3 \text{ lb., } 5 \text{ oz.}; \; d = 1{\cdot}85; \; \sigma = 0{\cdot}88. \right.$
26. 1470, 2078, 2940 f.s.
27. C = 0·371, V = 1850 f/s, w = 236, 212, and 189 grains.

SECTION 3.—COMPILATION OF RANGE TABLES.

The data for the compilation of a Range Table are obtained from practice carried out at Shoeburyness on as calm a day as possible.

This practice includes 5 rounds fired at a wood target 9 feet square, at 500 yards, for guns of small calibre or low velocity, or at 1000 yards for larger natures, the velocity of the projectile near the gun and the point of impact on the target being observed each round, and the muzzle velocity and jump are calculated from these.

Recently, remaining velocities have also been observed at the 500 or 1,000 yards target; from these and the velocities observed near the muzzle, the coefficient of reduction can be determined and employed in calculating the muzzle velocity and the jump, as explained in § 1 below.

Further series of about 5 rounds each are then fired at various elevations, say 1, 2, 4, 7, 10 and 14 degrees, according to the nature of the gun. The exact range of each round is noted, also its lateral deviation and the time of flight.

The average range, lateral deviation, and time of flight for each series are then tabulated; the mean errors in range and in direction are also computed. The height of the axis of the gun above the sands at the range is entered in the report.

The Shoeburyness report also gives the muzzle velocity and the mean jump obtained when firing the series at the target at 500 or 1,000 yards. For this series the gun is laid, by means of cross wires in the bore, on the bull's eye. It is then elevated through an angle which is judged to be sufficient to cause the shots to strike the vicinity of the bull's eye, say through an angle of 20 minutes for 500 yards range.

The mean point of impact of the series is then determined and marked on the target.

If this point is the same height on the target as the bull's eye, 20 minutes is the elevation (T.E.) for 500 yards.

If at 500 yards the mean point of impact is, say 2 feet, above or below the bull's eye, the T.E. would be 20′ ∓ D, where

$$\tan D = \frac{2}{500 \times 3}, \ \cot D = 750,$$

so that D is about 4′·6; otherwise obtainable from the formula (Chap. I, p. 4)

$$D' = \frac{1146h}{R} = \frac{1146 \times 2}{500} = 4{\cdot}584,$$

Jump.—Jump is worked out as follows:—The report will either state the quadrant elevation given to the gun, or the angle of elevation as given by the tangent sight (T.E.). In the former case the angle of depression S of the gun when laid on the bull's eye by cross-wires in the bore will also be given. From this information the height AC of the prolongation of the axis of the bore, before firing, above the centre of the target is obtained.

The muzzle velocity and range being known, the remaining velocity v and time of flight t are worked out by Bashforth's Tables, using the coefficient of reduction as found above, and making the necessary correction for tenuity. Then $s = \frac{1}{2}gt^2 = \text{AP}$ gives the vertical space through which the projectile will have fallen during the time of flight, and consequently a point P is obtained on the target at which the shot should have struck had there been no jump.

If the point of impact I coincides with P, the jump is nil.

If I is above or below P, the jump is positive or negative respectively, and given by

$$\text{Tan J} = \tan \text{IOP} = \frac{\text{PI}}{3\text{R}},$$

$$\text{or J} = \frac{1146}{\text{R}} \times \text{PI}.$$

The range to the target being R yards, and PI being measured in feet.

If the gun is laid on the bull's eye by tangent sight instead of by cross-wires in the bore, a correction must be made for the vertical distance between the line of sight and the axis of the piece.

2. *Angle of Elevation.*—The angle of elevation (T.E.) for the different ranges recorded at practice are obtained by adding the quadrant angle of elevation (Q.E.) to the angle of depression of the line of sight (S) from the gun to the mean point of impact.

Thus, suppose the range is R yards, and the height of the gun above the mean point of impact is h feet, then

$$\sin \text{S} = \frac{h}{3\text{R}}, \text{ or, in minutes, S} = 1146 \frac{h}{\text{R}},$$

and if Q is the quadrant angle, then Q + S = T, the angle of tangent elevation; and a curve connecting elevation and range can now be plotted.

A specimen of the Abstract from a practice report as forwarded from Shoeburyness is shown here.

REPORT OF EXPERIMENTAL PRACTICE WITH 5-INCH B.L. GUN.

Projectile, C.S., common shell, pointed, Mark I. Weight 50 lb.
Charge, $5\frac{1}{4}$ lb. cordite, size 10. 10.4.00.

Number of rounds.	Elevation (quadrant).	Time of flight.	Range.	Error in range.	Lateral deviation.	Error in deviation.	Muzzle velocity.	Jump.	Axis of gun above sands.	Remarks.
	° ′	secs.	yards.	yards.	yards.	yards.	f/s.	min.	feet.	Barometer, 29·84.
5	0 10	5 rounds at 2″ × 9′ × 9′ wood target at 500 yards. Point of impact 6 feet above sands					1890	± 0	12·37	Thermometer, 53°.
5	2 0	4·21	2072	15·4	4·5	0·78			13·2	↑ Wind force.
5	5 0	8·99	3835	18·2	21·8	1·08			15·2	
5	8 0	13·14	5192	16·6	44·7	1·90			16·27	
5	12 0	18·26	6604	43·4	91·3	5·26			17·4	
5	15 0	21·89	7544	68·4	123·1	4·06			18·1	

Using the formula

$$\tan S = \frac{h}{3R} \text{ or } S' = 1146 \frac{h}{R}.$$

Range	500	2072	3835	5192	6604	7544
Elevation	10′	2°	5°	8°	12°	15°
Height (ft.)	6·4	13·2	15·2	16·27	17·4	18·1
Correction (min.) ..	15′	7	5	3	3	3
Corrected elevation ..	25′	2° 7′	5° 5′	8° 3′	12° 3′	15° 3′

The corrected elevations for the various ranges are now plotted, taking the elevations as ordinates and the ranges as abscissæ, and a curve is drawn through the points thus obtained by means of a flexible ruler.

The time of flight shown on the report corresponding to the range is now plotted, and a time of flight curve drawn through the points obtained

Specimen Range Tables of the 6-inch and 15-pr. guns are printed here, extending up to 3,000 yards, which is about as far as it is permissible to use the formulas of Direct Fire in calculation.

RANGE TABLE FOR 6-INCH B.L. GUNS, MARKS IX AND X.

Based on Practice of 6.6.00.

Minute 49,594.

$\frac{40185}{9241}$

Charge ... { weight, 20 lb. gravimetric density, $\frac{75 \cdot 0}{0 \cdot 369}$ nature, Cordite, size 30.

Projectile { nature, cast steel common shell, Mark II, pointed. weight, 100 lb.

Muzzle velocity, 2,498 f/s.

Nature of mounting, C.P. Marks III (A) and III (B).

Jump, + 2½ minutes.

Remaining velocity (actual).	To strike an object 10 feet high, range must be known within	Angles of descent.	5 minutes' elevation or deflection alters point of impact		Elevation.	Range.	Fuze scale for fuze, time and percussion, middle, No. 54, Marks I*, II, and III.	50 per cent. of rounds should fall in			Time of flight.	Penetration into wrought iron.
			Range.	Vertically or laterally.				Length.	Breadth.	Height.		
f/s.	yards.	° ′	yards.	yards.	° ′	yards.		yards.	yards.	yards.	secs.	ins.
2462	1908	0 3	166	0·14	0 1	100	...	...	...	...	0·13	18·93
2426	818	0 7	164	0·29	0 4	200	¼	...	...	...	0·26	18·53
2391	573	0 10	162	0·43	0 7	300	½	...	...	...	0·39	18·14
2356	408	0 14	160	0·58	0 10	400	¾	...	...	...	0·52	17·76
2322	337	0 17	158	0·72	0 13	500	1	...	...	...	0·66	17·38
2288	278	0 21	156	0·87	0 16	600	1½	16·3	0·60	0·10	0·80	17·00
2255	238	0 24	154	1·01	0 19	700	1¾	16·4	0·61	0·12	0·94	16·62
2222	210	0 28	152	1·16	0 23	800	2	16·6	0·62	0·14	1·08	16·25
2190	185	0 31	150	1·31	0 26	900	2½	16·7	0·63	0·16	1·22	15·88
2158	165	0 35	148	1·45	0 30	1000	2¾	16·9	0·64	0·18	1·36	15·52
2126	150	0 38	146	1·60	0 33	1100	3	17·0	0·65	0·20	1·50	15·17
2095	138	0 42	144	1·74	0 37	1200	2¼	17·2	0·66	0·22	1·64	14·82
2064	127	0 45	142	1·89	0 40	1300	3¾	17·4	0·67	0·24	1·78	14·49
2033	117	0 49	140	2·03	0 44	1400	4	17·6	0·68	0·26	1·82	14·16
2003	108	0 53	138	2·18	0 48	1500	4¼	17·8	0·69	0·28	2·07	13·84
1974	100	0 57	136	2·32	0 52	1600	4¾	18·1	0·70	0·30	2·22	13·53
1945	94	1 1	134	2·47	0 56	1700	5	18·4	0·71	0·33	2·37	13·23
1917	88	1 5	132	2·61	1 0	1800	5½	18·7	0·73	0·36	2·53	12·93
1890	82	1 9	130	2·76	1 4	1900	5¾	19·0	0·75	0·39	2·68	12·64
1863	77	1 14	128	2·91	1 8	2000	6	19·3	0·78	0·42	2·84	12·36
1837	72	1 19	126	3·05	1 12	2100	6¼	19·6	0·81	0·45	2·99	12·09
1812	68	1 24	124	3·20	1 16	2200	6¾	19·9	0·84	0·49	3·15	11·83
1787	64	1 29	122	3·34	1 20	2300	7	20·2	0·87	0·53	[illegible]	11·58
1762	60	1 34	120	3·49	1 24	2400	7½	20·6	0·90	0·57	3·47	11·34
1738	57	1 40	118	3·63	1 28	2500	7¾	20·9	0·93	0·61	3·63	11·12
1714	54	1 46	116	3·78	1 32	2600	8¼	21·3	0·96	0·66	3·80	10·91
1691	51	1 52	114	3·92	1 36	2700	8½	21·7	0·99	0·71	3·97	10·70
1669	48	1 59	112	4·07	1 41	2800	9	22·1	1·02	0·77	4·14	10·50
1647	45	2 6	110	4·21	1 45	2900	9¼	22·5	1·05	0·83	4·32	10·31
1626	43	2 13	108	4·36	1 50	3000	9¾	23·0	1·08	0·90	4·50	10·13

RANGE TABLE FOR 15-PR. B.L. GUN, MARK I.

Based on Practice of 21 and 22.5.95.

50185 / 8045

Minute 39,421.

Charge { weight, 15¾ oz.; gravimetric density, $\frac{118\ 85}{0 \cdot 233}$.; nature, size 5/11, cordite.

Projectile { nature, 15-pr., shrapnel shell, Mark II.; weight, 14 lb. 1 oz.

Muzzle velocity, 1,574 f/s.
Nature of mounting, travelling, field, Mark I.
Jump, + 18 minutes.

Remaining velocity.	5 minutes' elevation or deflection alters point of impact: Range.	Laterally or vertically.	Deflection for drift (telescopic sight).	Slope of descent.	Elevation.	Range.	Fuze scale for time and percussion fuze, Mark IV.	50 per cent. of rounds should fall in: Length.	Breadth.	Height.	Time of flight.
f/s.	yards.	yards.	mins.	1 in	° ′	yards.		yards.	yards	yards.	secs.
1530	63	0·14		381	−0 9	100	¼				0·23
1488	63	0·29	1	214	−0 3	200	¾				0·46
1449	63	0·43	1	149	0 5	300	1				0·68
1409	63	0·58	1	110	0 14	400	1¼				0·91
1370	63	0·72	2	86	0 23	500	1½	17	0·36	0·20	1·13
1332	63	0·87	2	72	0 32	600	2	17	0·36	0·27	1·37
1298	62	1·01	2	61	0 41	700	2½	17	0·36	0·35	1·60
1264	61	1·16	2	54	0 50	800	2¾	18	0·36	0·43	1·84
1232	60	1·31	2	46	0 59	900	3	18	0·36	0·50	2·07
1201	59	1·45	3	40	1 8	1000	3½	19	0·36	0·58	2·31
1171	56	1·66	3	35	1 17	1100	4	19	0·37	0·66	2·55
1144	53	1·74	3	31	1 27	1200	4¼	20	0·37	0·74	2·79
1117	51	1·89	3	27	1 38	1300	4½	20	0·37	0·83	3·04
1093	49	2·03	3	23	1 48	1400	5	21	0·87	0·93	3·28
1071	47	2·18	3	20	1 56	1500	5¼	21	0·37	1·03	3·54
1053	45	2·32	4	18	2 9	1600	5¾	22	0·38	1·14	3·80
1035	43	2·47	4	17	2 20	1700	6	22	0·38	1·26	4·06
1021	41	2·61	4	16	2 32	1800	6½	23	0·38	1·39	4·33
1006	39	2·76	4	14	2 44	1900	6¾	23	0·38	1·53	4·80
992	38	2·91	4	13	2 56	2000	7¼	24	0·38	1·68	4·87
977	37	3·05	5	12	3 9	2100	7¾	24	0·39	1·82	5·16
964	37	3·20	5	11	3 22	2200	8	24	0·47	1·97	5·45
952	36	3·34	5	10	3 35	2300	8½	25	0·58	2·14	5·75
940	35	3·49	6	10	3 50	2400	9	25	0·70	2·34	6·05
928	35	3·63	6	9	4 4	2500	9½	26	0·85	2·53	6·37
916	34	3·78	6	9	4 19	2600	10	26	1·02	2·76	6·71
904	33	3·92	7	8	4 34	2700	10½	27	1·18	3·00	7·04
893	32	4·07	7	8	4 50	2800	11	27	1·36	3·27	7·38
882	32	4·21	8	7	5 7	2900	11½	28	1·55	3·68	7·74
871	31	4·36	8	7	5 24	3000	12	28	1·74	3·94	8·08

A specimen computation for a 6-inch gun is given on p. 42, showing how the figures may be filled in for ranges intermediate to those observed at practice, by the aid of Bashforth's Tables.

A first requirement is the determination of the special value of n, the coefficient of reduction, to employ in the calculations.

Starting with $n = 1$, calculate by Bashforth's Tables of S and T the time of flight over a range obtained at practice; if this time of flight is greater than the observed time, we know that n must be reduced; so that, taking $n = 0{\cdot}9$ say, and repeating the calculation, if the calculated time of flight is now less than the observed time, we know that n lies between 0·9 and 1. In this way the most appropriate value of n over moderate ranges may be obtained, and checked by the observed elevations, employing the formula

$$\sin 2\phi = Ca$$

and Table X.

In the present 6-inch Range Table it will be found that $n = 0{\cdot}99$ fits in with the printed numbers up to 3,000 yards; this value of n is so nearly unity that we may take $n = 1$ in our calculations, and thus examine the effect of 1 % increase in the density of the air; and now calculate Column I, of remaining velocity; and next Column XII, of time of flight (p. 39).

The calculations are shown worked out for ranges of 500, 1,000, 2,000 and 3,000 yards; the intermediate columns can be filled in as an exercise.

A check on the numbers is given by the average velocity over a range, obtained by dividing the range in feet by the time of flight in seconds; this average velocity should not differ much from the remaining velocity at half range.

Next calculate the vertex velocity v_0 at the point of half time over the range, and thence the angles ϕ and β by formulas (13) and (14), p. 19.

The angle of elevation (T.E.) is $\phi - J$, tabulated in Column VI; and the angle of descent β is tabulated in Column III.

Column V merely gives R tan 5′ for every 100 yards in the range R, and is the same for all guns; and then Column IV gives

$$R \tan 5' \cot \beta,$$

and is obtained by the multiplication of the figures in Column V by $\cot \beta$.

The difference of the elevation for every 100 yards in Column VI will serve as a check upon the figures in Column IV, for if ΔE is the change of elevation in minutes to make the range change 100 yards, as shown in Column VI, then we may employ proportional parts, and put

$$\frac{\Delta R}{100} = \frac{5}{\Delta E}.$$

Column III can be calculated independently from the formula

$$\cot \beta = \cot 5' \frac{\Delta R}{R} = 688 \frac{\Delta R}{R},$$

where ΔR is the change in the range R due to a change of 5′ in elevation.

Column II is obtained by multiplying $\cot \beta$ in Column III by $\frac{5}{3}$, or by multiplying by 10 and dividing by 6.

CALCULATION OF RANGE TABLE. 6-inch gun. $w = 100$, $d = 6$, $n = 1$, $V = 2498$.

Range in yards.	0		500		1000		2000		3000	
$\frac{s}{C}$	0 ·0		540 ·0		1080 ·0		2160 ·0		3240 ·0	
S_v	46752 ·0		46212 ·0		45672 ·0		44592 ·0		43512 ·0	
v	2498		2313		2135		1838		1578	
T_v	234 ·1886		233 ·9642		233 ·7207		233 ·1741		232 ·5389	
$\frac{t}{C}$	0		0 ·2244		0 ·4679		1 ·0145		1 ·6497	
t	0		0 ·624		1 ·299		2 ·855		4 ·582	
$\frac{1}{2}\frac{t}{C}$	0		0 ·1122		0 ·2340		0 ·5072		0 ·8248	
T_{v0}	234 ·1886		234 ·0764		233 ·9546		233 ·6814		233 ·3638	
v_0	2498		2404		2305		2110		1930	
D_{v_0}	85 ·7633		85 ·6909		85 ·6015		85 ·3740		85 ·0820	
$\frac{\phi}{C} = D_V - D_{v_0}$	0		0. 0724		0 ·1618		0 ·3893		0 ·6813	
ϕ	0		0° ·2		0° ·45		1° ·08		1° ·89	
ϕ'	0		12′		27′		1° 5½′		1° 53½′	
(T.E.) $\phi - J$			9½′		24½′		1° 3′		1° 51′	
D_v			85 ·6094		85 ·4076		84 ·8970		84 ·2074	
$\frac{\beta}{C} = D_{v_0} - D_v$			0 ·0815		0° ·1939		0 ·4756		0 ·8746	
β			0° ·23		0° ·54		1° ·32		2° ·43	
β			0° 14′		0° 32′		1° 19′		2° 26′	
cot β			245		107		44		23	

Columns IX, X, XI of 50% zones are obtained as the result of as much experimental practice as possible, in the manner explained in Probability of Fire; if the practice is carried out over the sands, Column XI is obtained from IX by multiplying by tan β.

Column VIII—Fuze Scale. To obtain data for fuze scales, 5 rounds are fired with fuzes set full, and 5 rounds with fuzes set for medium range. A correction has to be made for the times of burning thus obtained for height of barometer, in accordance with the rule given in the Treatise on Ammunition.

A curve is then drawn on squared paper, and the most suitable graduation of the fuze for any range can be read off.

Column XIII, of Penetration into Wrought Iron, is calculated from the remaining velocity by one of the formulas (19), (20), or (21), p. 19.

The Slide Rule will be found useful in performing the computations with accuracy quite sufficient for practical purposes.

To check these calculations of T.E. by Hadcock's Table IX, make $V = 2500$ f/s, an increase of 2 f/s only, and calculate ϕ, for hundreds of yards of $\frac{R}{C}$, as follows:—

$\frac{R}{C}$	200	400	800	1200
R	556	1111	2222	3333
a	0·00327	0·00680	0·01568	0·0269
sin 2ϕ	0·0091	0·019	0·0435	0·0775
2ϕ	31′	1° 5′	2° 30′	4° 27′
ϕ	15½′	32½′	1° 15′	2° 13½′
T.E.	13′	30′	1° 12½′	2° 11′

The agreement with the figures printed in the Range Table is not always very close; further practice alone can settle which figures are more correct.

High Angle Fire.

As a specimen of the use of the formulas on p. 18 for High Angle Fire, employ them to determine the range of this 6-inch gun when fired at an angle of 14°, the muzzle velocity being 2500 f/s.

According to the result of practice embodied in the subsequent omitted parts of the Range Table, the range should be about 10,600 yards, with a time of flight of 26 seconds.

This will make the height of the vertex about

$$H = (2T)^2 = (52)^2 = 2{,}704 \text{ feet};$$

and the average height of the shot, for the tenuity correction, may according to the rule laid down by Major Ingalls, U.S.A., be taken as two-thirds of this, or 1,800 feet.

At this height the barometer will have fallen about 1·8 inches, at the rate of 1 inch per 1,000 feet, implying a reduction of 6°/₀ in the density of the air, at the normal barometric height of 30 inches; so that we may take the tenuity factor

$$\tau = 0{\cdot}94.$$

The calculation is carried out in parallel columns for a tenuity factor

$$\tau = 0{\cdot}90,$$

so as to obtain an idea of the change in range and time of flight due to a change of atmospheric conditions.

Over the arc up to the vertex,

$$\phi = 14°,\ \theta = 0,\ \eta = 7°,\ V = 2500 \text{ f/s}.$$

$$\log V = 3{\cdot}3979$$
$$\log \cos\phi = \bar{1}{\cdot}9869$$
$$\log \sec\eta = 0{\cdot}0032$$
$$\log U = 3{\cdot}3880$$
$$U = 2443 \text{ f/s}.$$

Now with $w = 100$, $d = 6$,

$\tau =$	0·94	0·90
$\log C = \log \dfrac{w}{\tau d^2} =$	0·4706	0·4895
$\log \sec\eta =$	0·0032	0·0032
$\log C \sec\eta =$	0·4738	0·4927
$\log C \cos\eta =$	0·4674	0·4863
$\tan\phi - \tan\theta =$	tan 14°	
$\log(\tan\phi - \tan\theta) =$	$\bar{1}$·3968	$\bar{1}$·3968
$\log C \sec\eta =$	0·4738	0·4927
$\log(I_U - I_u) =$	$\bar{2}$·9230	$\bar{2}$·9041
$I_U - I_u =$	0·083753	0·080187
$I_U =$	0·894068	0·894068
$I_u =$	0·810315	0·813881
$u =$	1069	1084

$T_U =$	234·1231	234·1231
$T_u =$	230·2948	230·4143
$\frac{t}{C} =$	3·8283	3·7088
$\log \frac{t}{C} =$	0·5830	0·5692
$\log C =$	0·4706	0·4895
$\log t =$	1·0536	1·0587
${}_{14}t_0 =$	11·32	11·45 seconds

$S_U =$	46590·1	46590 1
$S_u =$	40641·6	40770·2
$\Delta S = \frac{x}{C \cos \eta} =$	5948·5	5819·9
$\log \frac{x}{C \cos \eta} =$	3·7744	3·7649
$\log C \cos \eta =$	0·4674	0·4863
$\log x =$	4·2418	4·2512
${}_{14}x_0 =$	17450	17830 feet

$A_U =$	11104·98	11104·98
$A_u =$	5971·91	6075·91
$\Delta A =$	5133·07	5029·07
$\log \Delta A =$	3·7104	3·7015
$\log \Delta S =$	3·7744	3·7649
$\log \frac{\Delta A}{\Delta S} =$	$\bar{1}$·9360	$\bar{1}$·9366
$\frac{\Delta A}{\Delta S} =$	0·8630	0·8642
$I_V =$	0·894068	0·894068
$\frac{1}{2}a =$	0·031068	·029868
$\log \frac{1}{2}a =$	$\bar{2}$·4924	$\bar{2}$·4752
$\log C \sec \eta =$	0·4738	0·4927
$\log C \sec \eta \cdot \frac{1}{2} a =$	$\bar{2}$·9662	$\bar{2}$·9679
$C \sec \eta \frac{1}{2} a =$	0·09251	0·09287
$\tan \phi =$	0·24933	0·24933
$\frac{y}{x} =$	0·15682	0·15646
$\log \frac{y}{x} =$	$\bar{1}$·1953	$\bar{1}$·1945
$\log x =$	4·2418	4·2512
$\log y =$	3·4371	3·4457
${}_{14}y_0 =$	2735	2790 feet

At the vertex

$$v_0 = u \cos \eta \sec \theta = u \cos \eta$$

$\log u =$	3·0290	3·0351
$\log \sec \eta =$	0·0032	0·0032
$\log v_0 =$	3·0258	3·0319
$v_0 =$	1061	1076 f/s

In the range table the slope of descent is given as 1 in 2, so that $\cot\beta = 2$, and β is about 26°.

In the descending branch, take an arc extending over 0° — 26°, so as to locate the shot when descending at an angle of 26°.

The data now are

$$\phi = 0,\ \theta = -\beta = -26°,\ \eta = -13°,$$

and

$V = v_0 =$	1067	1076
$\log C =$	0·4706	0·4895
$\log\sec\eta =$	0·0113	0·0113
$\log C\sec\eta =$	0·4819	0·5008
$\log C\cos\eta =$	0·4593	0·4782

to calculate u, x, y, and t.

Now, with $\phi = 0$,

$$U = V\cos\phi\sec\eta = V\sec\eta.$$

$\log V =$	3·0258	3·0319
$\log\sec\eta =$	0·0113	0·0113
$\log U =$	3·0371	3·0432
$U =$	1089	1105 f/s.

$\log\tan\beta =$	$\bar{1}$·6882	$\bar{1}$·6882
$\log C\sec\eta =$	0·4819	0·5008
$\log\dfrac{\tan\beta}{C\sec\eta} =$		
$\log(I_U - I_u) =$	$\bar{1}$·2063	$\bar{1}$·1874
$I_U - I_u =$	0·1608	0·1539
$I_U =$	0·81503	0·81836
$I_u =$	0·65423	0·66446
$u =$	804	814 f/s

$S_U =$	40811·0	40936·4
$S_u =$	36594·4	36796·5
$\Delta S = \dfrac{x}{C\cos\eta} =$	4216·6	4139·9
$\log\dfrac{x}{C\cos\eta} =$	3·6250	3·6170
$\log C\cos\eta =$	0·4593	0·4782
$\log x =$	4·0843	4·0952
${}_0x_{26} =$	12140	12460
${}_{14}x_0 =$	17450	17830
${}_{14}x_{26} =$	29590	30290 feet
	(9863)	(10097) yards

$A_U =$	6109·02	6211·19
$A_u =$	2981·15	3114·19
$\Delta A =$	3127·87	3097·00
$\log \Delta A =$	3·4953	3·4910
$\log \Delta S =$	3·6250	3·6170
$\log \frac{\Delta A}{\Delta S} =$	$\bar{1}$·8703	$\bar{1}$·8704
$\frac{\Delta A}{\Delta S} =$	0·7418	0·7482
$I_U =$	0·81503	0·81836
$\frac{1}{2}a =$	0·07323	0·07016
$\log \frac{1}{2}a =$	$\bar{2}$·8647	$\bar{2}$·8461
$\log C \sec \eta =$	0·4819	0·5008
$\log C \sec \eta \cdot \frac{1}{2}a =$	$\bar{1}$·3466	$\bar{1}$·3469
$C \sec \eta \cdot \frac{1}{2}a =$	0·2221	0·2223
$\log \frac{y}{x} =$	$\bar{1}$·3466	$\bar{1}$·3469
$\log x =$	4·0843	4·0952
$\log y =$	3·4309	3·4421
${}_0y_{26} =$	2698	2768 feet

But

${}_{14}y_0 =$	2735	2790

so that the shot is still Δy feet above the horizontal plane through the point of projection, where

$\Delta y =$	37	22 feet

For such a small height we may prolong the trajectory in a straight line to meet the ground, giving an extra range $\Delta x = \Delta y \cot \beta$, so that with $\cot \beta = 2$,

$\Delta x =$	74	44 feet
${}_{14}x_{26} + \Delta x =$	29664	30334 feet
	(9888)	(10111) yards

To obtain a range of 10,600 yards by calculation, a still smaller value of n would have to be taken, probably about 0·87 or 0·86, due to taking a correction for $\kappa\sigma$ of about 0·93 to 0·95.

For the time of flight in the descending branch

$T_U =$	230·4519	230·5662
$T_u =$	225·8706	226·1205
$\frac{t}{C} =$	4·5813	4·4457
$\log \frac{t}{C} =$	0·6611	0·6480
$\log C =$	0·4706	0·4895
$\log t =$	1·1317	1·1375
${}_0t_{26} =$	13·54	13·73 seconds
${}_{14}t_0 =$	11·32	11·45
${}_{14}t_{26} =$	24·86	25·18

A method will be given in Part II of calculating such a long trajectory as this by smaller steps in successive arcs.

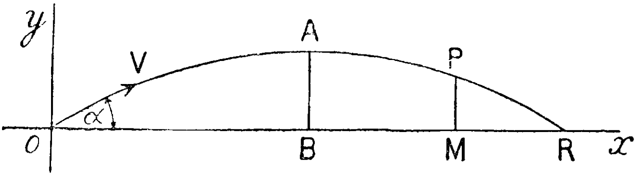
Consult Proc. R.A.I., July—Aug., 1901, p. 149. *Compilation of Range Tables*, by Major H. P. HICKMAN, R.A.

SECTION 4.—UNRESISTED MOTION OF A PROJECTILE.

Although the preceding problems and illustrations of artillery fire have shown that the force of gravity is usually a comparatively small effect in comparison with the resistance of the air, insomuch that it may be neglected in a calculation to a first approximation of the trajectory, occasions may still arise where it is the resistance of the air which is the negligable quantity, as in the case of high-angle howitzer fire, with heavy projectiles, and small charges and low initial velocities.

For this reason it is advisable to add a short account of the theory of the motion of an unresisted projectile in accordance with the principles discovered by Galileo in 1638.

FIG. 1.



The shot is supposed to be projected with velocity V f/s at an elevation α, and to have advanced a horizontal distance OM $= x$ feet, and to have ascended a vertical height MP $= y$ feet, in t seconds from leaving the muzzle (fig. 1).

Then
$$\text{OM} = x = \text{V}t \cos \alpha$$
$$\text{MP} = y = \text{V}t \sin \alpha - \tfrac{1}{2}gt^2;$$

in accordance with the laws of motion; the horizontal velocity V cos α of the shot remaining unaltered during the flight, while the vertical velocity diminishes from

$$\text{V} \sin \alpha \text{ to } \text{V} \sin \alpha - gt, \text{ in } t \text{ seconds.}$$

If **T** denotes the time of flight down to the ground again, taking the ground as horizontal, and if the range over the ground is X feet, then, putting $y = 0$, we find

$$\text{T} = \frac{2\text{V} \sin \alpha}{g},$$

and
$$\text{X} = \text{VT} \cos \alpha = \frac{\text{V}^2 \sin 2\alpha}{g},$$

so that
$$\sin 2\alpha = \frac{g\text{X}}{\text{V}^2},$$
$$\text{V} = \sqrt{(\text{X}g \operatorname{cosec} 2\alpha)},$$

giving the requisite elevation α to attain a range of X feet, with an initial velocity V f/s, or the velocity V required to attain a range X with elevation α; and then

$$\text{T} = \sqrt{\frac{2\text{X} \tan \alpha}{g}}.$$

Also

$$V \sin \alpha = \tfrac{1}{2}gT,$$

so that

$$y = \tfrac{1}{2}gTt - g\tfrac{1}{2}t^2$$
$$= \tfrac{1}{2}gt\,(T - t) = \tfrac{1}{2}gtt',$$

if

$$t' = T - t;$$

so that with $g = 32$,

$$y = 16tt',$$

Colonel Sladen's formula employed in the preceding examples for plotting ordinates of a trajectory.

At the highest point of the trajectory, where $y = H$, suppose,

$$t = t' = \tfrac{1}{2}T,$$

so that

$$H = \tfrac{1}{8}gT^2 = 4T^2 = (2T)^2;$$

hence the practical rule, "the square of twice the time of flight in seconds is the greatest height ascended in feet."

For instance, in the Jubilee rounds, fired in 1888, a time of flight of nearly 70 seconds was observed in a range of 12 miles; the rule would make the height ascended about 19,600 feet—more than the height of Mont Blanc.

Since

$$t = \frac{x}{V \cos \alpha},$$

therefore

$$y = x \tan \alpha - \frac{gx^2}{2V^2 \cos^2 \alpha},$$

the invariable relation connecting the co-ordinates x and y of a shot moving in the trajectory; and the form of this equation shows that the curve is a *parabola*, in accordance with the principles of co-ordinate geometry.

Putting $y = 0$ gives the range

$$X = \frac{2V^2 \sin \alpha \cos \alpha}{g},$$

as before; while putting $x = X$ gives the greatest height,

$$H = \tfrac{1}{4}X \tan \alpha.$$

We can prove that the trajectory of an unresisted projectile is a parabola by geometrical considerations (fig. 2), in the manner originally employed by Galileo.

Suppose the shot is projected from O in the direction OT with velocity V f/s, then, in the absence of gravity and resistance, the shot will be found, after t seconds, at T, where

$$OT = Vt \text{ (feet)}$$

But in the same time, t seconds, a body let fall from O would, if unresisted, have reached a point U vertically below O, at a depth

$$OU = \tfrac{1}{2}gt^2, \text{ feet.}$$

Galileo combined these two states of motion, and supposed them to take place simultaneously; so that the body, after t seconds, would be found at P, vertically below T, at a depth

$$TP = \tfrac{1}{2}gt^2.$$

The elimination of t leads to the invariable relation for all points on the trajectory OP,

$$\frac{OT^2}{TP} = \frac{V^2t^2}{\frac{1}{2}gt^2} = \frac{2V^2}{g} = 4OH,$$

if OH is measured vertically upwards from O to a height

$$OH = \frac{\frac{1}{2}V^2}{g};$$

this is the vertical height the body would reach if projected vertically upwards with velocity V; or it is the vertical depth the body would have to fall to acquire the velocity V; and OH is called the *impetus* or *head* of the velocity V.

The above relation for an unresisted trajectory,

$$OT^2 = 4OH \,.\, TP,$$

or

$$PU^2 = 4HO \,.\, OU,$$

defines a *parabola*, according to a fundamental property of a curve, from which the name *parabola* was originally derived; the curve exhibiting graphically the comparison (*parabola*) between a length PU and its square represented by OU, or between a length OU and its square root represented by PU.

But nowadays the parabola is defined as a curve described by a point which moves so that its distance from a fixed point, F, called the *focus*, is equal to its distance from a fixed straight line, HK, called the *directrix*.

We proceed then to translate the previous relation into this new geometrical property.

The focus F is determined by drawing HY perpendicular to OT, and producing it to F, making YF = HY, or the angle YOF = angle YOH; and the directrix will be the horizontal straight line HK through H.

Draw PN horizontally to meet OH in N, and let HF meet PU, parallel to OT, in Z.

Then, since

$$PU^2 = 4HO \,.\, OU,$$

and, by similar triangles,

$$\frac{PU}{PN} = \frac{HO}{HY} = \frac{OU}{YZ},$$

therefore,

$$\begin{aligned} PN^2 &= 4HY \,.\, YZ \\ &= (HY + YZ)^2 - (HY - YZ)^2 \\ &= HZ^2 - FZ^2 \\ &= HP^2 - FP^2 \end{aligned}$$

or

$$FP^2 = HP^2 - PN^2 = PK^2$$
$$FP = PK,$$

the fundamental property of the parabola.

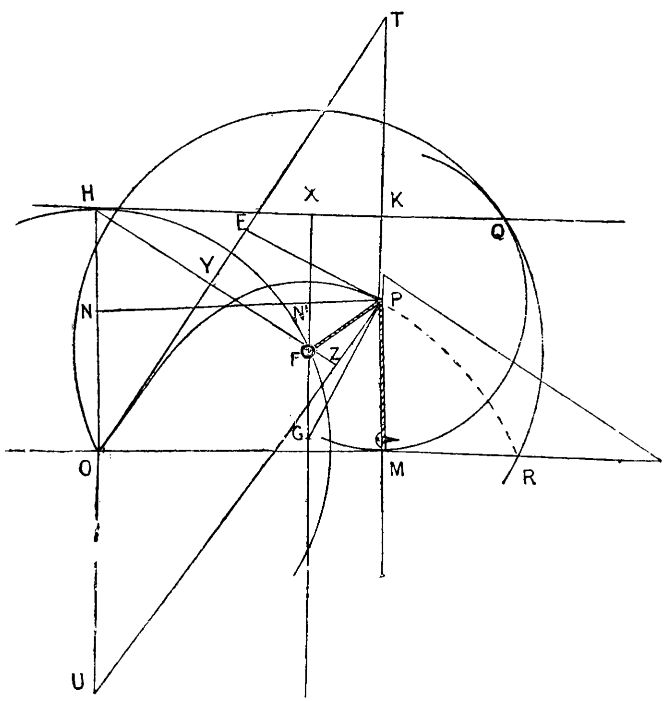
To describe the parabola mechanically, place a straight edge along OR, and a set square against it, KM being the edge at right angles to HK; take a thread of length KM, and fasten one end to M, and the other end to the focus F; then, if the thread is kept taut by a pencil at P, the parabola will be described by P as the set square, KM, slides along the straight edge OR, because FP = PK.

Suppose, then, that the direction of projection is required requisite for striking a given point, P, with the given velocity of projection V from O.

Describe the circle with centre P, and radius PK, cutting the circle with centre O and radius OH in F and F′ (fig. 6, p. 209).

Then the requisite directions of projection are the perpendiculars from O on HF and HF′; the upper direction corresponding to *high angle* or *mortar fire*, the lower direction to *direct fire*.

Fig. 2.



OPR a parabola, focus F; directrix HK.
HF a circle, centre O.
OQR a circle, centre F.
QM a circle, centre P.
E the middle point of OT, EP the tangent at P.
PG the *normal* at P.
PU parallel to OT, and perpendicular to FH.
PMR a set square, sliding on a straight edge OR.
FPM a thread, fastened by pins at F and M.

If the circles do not intersect, the point P is out of range; if the circles touch, then OP is the maximum range on the inclined plane OP; the direction of projection then bisects the angle POH.

If a circle is struck with centre F and radius FO, cutting the horizontal line through O in R, then OR is the range on this horizontal line.

Also, if FP produced cuts this circle in Q, then the length of the thread

$$FP + PM = HO = FO = FQ = FP + PQ,$$

or

$$PM = PQ,$$

so that a point P on the parabola is always equidistant from the horizontal line OMR and the circle OQR.

The velocity at P is the resultant of the original velocity V of projection, and of the velocity gt imparted by gravity; the direction of motion or tangent at P will therefore be EP, where E is the middle point of OT, and therefore equidistant from OH and PK.

For

$$\frac{ET}{TP} = \frac{\frac{1}{2}Vt}{\frac{1}{2}gt^2} = \frac{V}{gt},$$

and, therefore, by the triangle of velocities EP is the direction of motion at P.

The tangent EP bisects the angle FPK, because

$$FP = PK, \text{ and } FE = EH = EK, \text{ E being the mid-point of OT.}$$

If the *normal* at P, that is, the perpendicular through P to the tangent at P, cuts the axis XF of the parabola in G, then

$$\begin{aligned} FGP &= \text{complement of } GPN = NPE \\ &= \text{complement of } EPK \\ &= \text{complement of } EPF = FPG, \end{aligned}$$

so that

$$FG = FP = PK = N'X,$$

if PN cuts the axis of the parabola in N′; thence

$$N'G = FX, \text{ a constant.}$$

The length N′G is called the subnormal; thus the subnormal in a parabola is of constant length; this is the fundamental property of the parabola employed by Professor Hart in his discussion of the parabolic trajectory (*Messenger of Mathematics*, x, p. 64).

Further developments of parabolic motion will be found in Part II, Chapter III.

The parabolic theory is sometimes useful in assigning limits within which the real trajectory in a resisting medium must lie.

If V, v denote the initial and final velocities, and α, β denote the angles of departure and of descent in a real trajectory over a range of X feet, then this trajectory lies between two parabolic trajectories, having angles of departure α and β.

The height of the trajectory H lies between the heights of the parabolas, and therefore

$$\tfrac{1}{4}X \tan\beta > H > \tfrac{1}{4}R \tan\alpha.$$

The time of flight, T, lies between the parabolic times of flight, and therefore

$$\sqrt{\frac{2X \tan\beta}{g}} > T > \sqrt{\frac{2X \tan\alpha}{g}}.$$

and so on.

Also $\qquad V > \sqrt{(Xg \operatorname{cosec} 2\alpha)},\ v < \sqrt{(Xg \operatorname{cosec} 2\beta)}.$

Thus in the trajectory of the projectile weighing 380 lbs., fired at 40° elevation from the 9·2-inch wire gun, with velocity 2375 f/s. Lieutenant Wolley Dod, R.A., found by calculation ("Proc. R.A. Institution," vol. xvi) a range of 20,765 yards, a height of vertex, 17,110 feet; an angle of descent, 53° 50′; time of flight, 63·8 seconds; and final velocity, 1090 f/s.

Here X = 62,295 feet, H = 17,110:

$$\alpha = 40°,\ \beta = 53° 50',\ T = 63{\cdot}8 \text{ seconds}.$$

Working with these data,

$$\tfrac{1}{4}R \tan\alpha = 13{,}068,\ \tfrac{1}{4}R \tan\beta = 21{,}305,$$

the mean being 17,180 feet;

$$\sqrt{\frac{2R \tan\alpha}{g}} = 57, \qquad \sqrt{\frac{2R \tan\beta}{g}} = 73,$$

the mean being 65 seconds; thus exhibiting on the largest scale the limits of the approximation.

As numerical exercises on the parabolic theory, the range tables of howitzers for low initial velocities may be calculated to a first approximation; for instance, for the 8-inch howitzer firing a shot weighing 185 lbs., with charges of 10, 7, 6, 5, 4, 3, 2 lbs. of powder; calculating the muzzle energy and velocity due to the realised energy of the powder from Table XIV; and thence on the parabolic theory the requisite elevation and time of flight for ranges of 200, 500, and 1,000 yards.

CHAPTER III.—ACCURACY.

SECTION I.—LAYING.

All guns are mounted in such a way that two motions can be given to the axis of the piece, viz., motion in a vertical plane, usually termed *elevation*; and *motion in a horizontal* plane usually termed *training*. The former is always effected by mechanism, the latter is carried out sometimes by hand and sometimes by mechanism. When a gun is elevated and trained for the purpose of hitting some target, it is said to be "laid."

Causes affecting the Motion of a Projectile.

The motion of a projectile, referred to the vertical plane of Departure, is affected by the following causes:—

1. Resistance of the air.
2. The force of gravity.
3. Wind, blowing up or down the line of fire: this, in the case of artillery fire, is usually neglected.

The united effect of 1 and 2 has already been discussed in Chapter II, where it is shown that for given ballistics an angle of tangent elevation can be found for each range.

The motion of the projectile out of the Plane of Departure referred to the horizontal plane is affected by the following causes:—

1. Drift.

This is an effect observable with all rifled guns, by which the shot is deflected in its flight more or less from the vertical plane of fire; the deflection is to the right when the gun is rifled with a twist on a right-handed screw, to the left with a left-handed twist.

Thus it was found by Mr. Rigby, Superintendent R.S.A.F., Enfield, that with two barrels rifled respectively, with right and left-handed twists, and laid parallel, the bullets struck on a target at 1000 yards on an average 15 inches farther apart than the muzzles, showing that the *drift* of the rifle bullet at this range is about $7\frac{1}{2}$ inches.

In artillery the right-handed twist is always employed; but small arms are now rifled with a left-handed twist, to counteract the pull off.

The drift increases rapidly with the elevation and range of the gun; thus it was found that the 9·2-inch fired at Shoeburyness with an elevation of 40° and a muzzle velocity 2375 f/s, sent a shot weighing 380 lbs. to a range of over 20,000 yards, and that the drift was about 1000 yards to the right of the vertical plane of fire.

But the general effect may be attributed to the observed tendency of the projectile to move with its axis nearly tangential to the trajectory.

To keep the point of the projectile continually turning downwards into the tangent of the trajectory, the projectile must be acted upon, as in the case of a top, by a couple whose axis is directed towards the centre of curvature of the trajectory.

This couple will be called into existence if the projectile moves in a slightly sidelong position, with its nose turned a little to the right of the vertical plane of motion; and now the drift may be supposed due to the projectile following its nose to the right, and a deflection to the right of the vertical plane of fire thus accumulates in consequence.

For given ballistic conditions, drift can usually be measured and allowed for. This will be referred to later.

2. Wind.

This effect of wind is usually estimated, and corrected for, when possible, by observation of fire.

3. Want of level, *i.e.*, anything which causes the plane containing the sights to be rotated out of the vertical plane, and so causes deviation between the vertical planes of Sight and Departure.

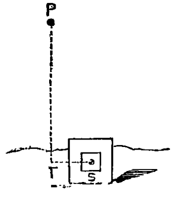
Thus with field, mountain, and certain siege guns, one wheel might, on account of the ground, be higher than the other: a screw gun, such as the 7-pr. of 400 lbs. might be "overscrewed," so that the trunnion ring would be rotated out of its proper position; a heavy gun, with or without trunnions, might, owing to some defect in the mounting or platform, be "down on one side."

The effect of this want of level is to deflect the projectile towards the lower side: its extent can usually be ascertained and allowed for. It is mathematically investigated later on.

Mountain guns that normally come into action on uneven ground are often provided with "reciprocating sights." The socket for the tangent bar is made capable of movement, and is provided with a spirit level, so that the plane containing the sights may be kept level.

From the above considerations it is apparent that if it is desired to hit a point S, the axis of the gun must be directed on some higher point P, say, in order to counteract gravity, &c., P being vertically over some point T, say, to the *left* of S, it being supposed that there are some disturbing causes whose united effect would be to deflect the projectile a distance ST to the right of the prolongation of the axis of the piece.

Fig 1.

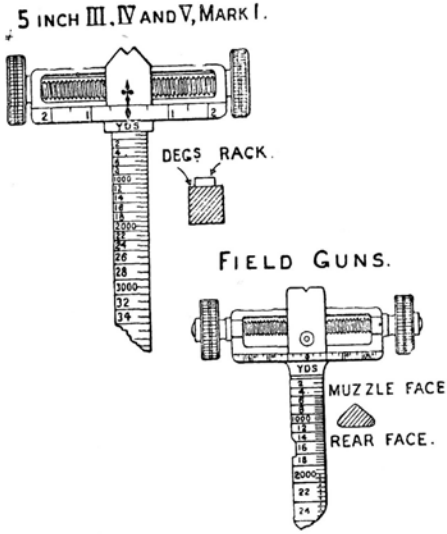


The virtual effect of travel of target is analagous to the effects that have just been discussed; as the target moves after the gun is laid, its motion between the completion of the laying and the moment the projectile reaches the end of the range must be allowed for in the laying; the component of this motion, referred to the vertical plane, is allowed for in the elevation, the component referred to the horizontal plane is allowed for in the training.

The Direction of the Line of Sight.

A consideration of the ordinary tangent sight and foresight will demonstrate how the line of sight is practically directed, and will lead up to the consideration of other appliances.

FIG. 2.



The tangent or hind sight consists of a steel bar with a cross head; the bar slides in a socket attached to the gun; it is of triangular or rectangular cross section, and is graduated usually on the rear face in yards and on the front in degrees. The cross head is provided with a sliding deflection leaf, in which is a central notch; the leaf can be moved along a scale right and left of a central zero. The foresight is usually hog backed or acorn shaped; as seen from the rear it has practically a triangular cross section.

When the gun is properly laid the line of sight passes through a point midway between the shoulders of the notch, the apex of the foresight, and the point aimed at.

When the steel bar is run down in the socket to its lowest position it is at zero. If the gun were laid with the tangent sight in this position, the line of sight would be parallel to the axis of the bore, and the gun would be laid "point blank."

When the gun is to be laid on an object at any range, the elevation due to the range must be given; this is effected by raising the tangent scale until the required graduation is level with the top of the socket; then, when the breech is lowered, so that the line of sight may be directed on the object, the angle between the axis of the gun and the line of sight will be the angle due to the range, *i.e.*, the angle of tangent elevation (*vide* fig. 2, Chapter I).

The graduations on the bar which register the elevation given, must first be calculated in degrees; they are determined from the relation

$$l = r \tan \theta,$$

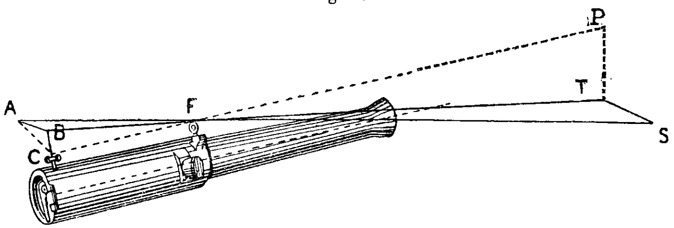
where l is the length in inches from the zero of the scale to the required graduation; r the radius distance in inches and θ the angle of elevation represented by the graduation in question; when the degree graduations are obtained, graduations in yards can be obtained from them by noting the ranges and their corresponding elevations in the range table of the gun.

The socket for the bar of the tangent sight is made so as to give the latter a set to the left; this is done to counteract drift, as will be explained later.

Mathematical Investigation.

The foregoing points with regard to drift and deflection may be considered mathematically as follows :—

Fig. 3.



Let BC in fig. 3 represent a tangent scale raised to the tangent elevation CB required for the distance FT, which may be called the range R; so that, if the plane FBC is vertical, with BF aligned on T, the shot would strike at T in the absence of drift or other lateral disturbance.

But if the shot strikes the vertical tangent through T to one side at S, the horizontal distance TS is called the *drift*, and denoted by D, and the angle TFS is called the *drift-angle*, and denoted by γ suppose.

To align the sights on S, the point struck, deflection BA must be given on a deflection scale, such that

$$\frac{AB}{BF} = \frac{TS}{FT} = \frac{D}{R} = \tan \gamma,$$

Through the fore-sight F draw FC parallel to the axis of the piece; then BFC is the angle of tangent elevation, denoted by E suppose.

If the angle ACB is denoted by θ,

$$\tan \theta = \frac{AB}{BC} = \frac{AB}{BF} \cdot \frac{BF}{BC} = \tan \gamma \frac{1}{\sin E}.$$

But R sin E is practically the same as R tan E or TP in fig. 1 or 3, and TP is practically $\frac{1}{2}gT^2$, the vertical distance fallen by the shot from its original direction of projection FP, in the time of flight T; and as the drift D is found in practice to vary very nearly as the square of the time of flight, the angle θ is very nearly the same for all ranges; it is a small angle, never exceeding 3°.

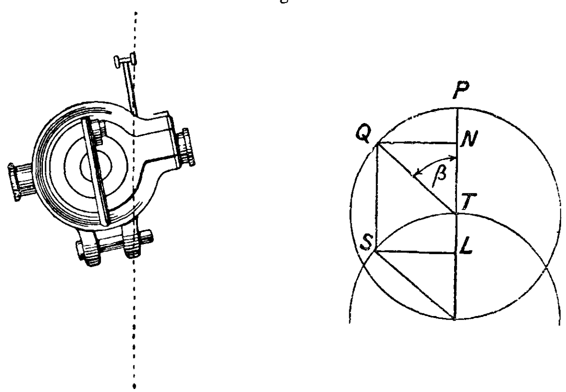
Advantage is taken of this circumstance by inclining the slot for the tangent scale laterally in the direction of AC at an average angle θ to the line BC, called the *permanent angle of deflection.*

With howitzers, however, the varying charges prevent the use of a permanent angle of deflection; their tangent scales are therefore perpendicular to the plane through the axis of the piece and the trunnion, and are provided with long deflection bars.

If the axletree of a gun is level and the sights are aligned on a point T at elevation E, then the axis of the bore, on opening the breech and looking through, will be aligned on P in fig. 1 vertically over T at a height

$$TP = R \tan E.$$

Fig. 4.



But if from inequalities or slope of the surface of the ground or platform, the axis of the trunnions, or the axletree, slopes at an angle β, and the sights are still aligned on T, the axis of the bore will point to Q on the circle PQ in fig. 4, where the angle $PTQ = \beta$. When the gun is fired the shot will strike at S, instead of T, on the circle TS; so that if E is the elevation for the range R, the shot will strike at a point S, at a distance LS towards the lower side, where

$$LS = R \tan E \sin \beta$$

being at the same time a distance equal to TL too low, where

$$TL = R \tan E \text{ vers } \beta.$$

Therefore to align the sights on S, the deflection AB must be given such that

$$\frac{AB}{BF} = \tan \gamma = \frac{D}{R} = \tan E \sin \beta,$$

the deflection being made in the direction of the higher wheel or trunnion.

When the angles E, β, and γ, are small, and we take $\pi = 3$, as is customary in these approximations in gunnery, and applying it to the last formula, we can put

$$\tan E = \frac{\pi E^\circ}{180} = \frac{E^\circ}{60}, \sin \beta = \frac{\beta^\circ}{60}$$

$$\tan \gamma = \frac{\pi \gamma'}{180 \times 60} = \frac{\gamma'}{3600},$$

so that for practical purposes,

$$\gamma' = E^\circ \beta^\circ,$$

or—the product of the slope of the trunnions in degrees, and of the elevation also in degrees, gives the minutes of deflection to be given towards the higher side.

A knot is a speed of one nautical mile (1,000 fathom) per hour, so that with a fathom of 6 feet a knot is 100 feet per minute, or

$$1 \text{ knot} = \tfrac{5}{3} \text{ f/s.}$$

Deflection for speed K (knots) of platform (ship) or target can thus be given on the deflection leaf AB in fig. 3 by means of the formula—

$$\frac{d}{r} = \frac{AB}{BF} = \frac{ST}{TF}.$$

$$\text{(i.)} \ \frac{d}{r} = \frac{\frac{5}{3} K}{V} \text{ (for platform),}$$

where V denotes the muzzle velocity.

$$\text{(ii.)} \ \frac{d}{r} = \frac{\frac{5}{3} K}{U} \text{ (for target),}$$

where U denotes the average velocity over the range,

which gives the deflection BA $= d$ (inches) at radius distance BF $= r$ (inches) for a speed of K (knots),

Thus if r is 3 feet $=$ 36 inches, and V $=$ 2,000, U $=$ 1,500 f/s., the length of a graduation d for 10 knots speed is given by

$$d = \frac{36 \times \frac{5}{3} \times 10}{2000 \text{ or } 1500} = 0{\cdot}3 \text{ or } 0{\cdot}4 \text{ inch,}$$

The deflection leaf is made use of to compensate for causes tending to divert the projectile to the left or right, it being a practical rule "never to lay off the target." The scale on the cross head is usually graduated in divisions, each division representing five minutes, on the same scale as the degrees on the tangent bar, and by its aid the notch on the leaf can be set to any required graduation; so that when the gun is laid, its axis will make an angle with the line it would have occupied had the notch remained at zero. Thus, suppose the notch were removed ten minutes to the left, then when the gun is laid its axis will make an angle of ten minutes with the position it would have occupied had the gun been laid with the notch at zero.

Methods of Sighting used in Practice.

It will be convenient to classify here the various methods employed in practice to cause the axis of the gun to assume the required direction: they may be grouped under four heads.—

A.—By the use of tangent elevation.

When tangent elevation is employed, not only must the *vertical plane* containing the line of sight be made to pass through the point aimed at, but the *line of sight* must itself pass through this point.

Examples.—The ordinary tangent sight and foresight. The bar and drum sight (Coast Artillery). Scott's sights (Telescopic) (Field Artillery).

B.—By the use of quadrant elevation, the gun being trained by eye.

Quadrant elevation is given by some mechanical means, which practically eleminates *personal error* as far as elevation is concerned, the layer being responsible only for the training. It is only necessary for him to cause the *vertical plane* containing the sights to pass through the object aimed at.

Examples.—1. Index plate and reader, or other range indicator, in conjunction with straight edged sights (Coast Artillery).

2. Clinometer and plane on gun, in conjunction with sights of various kinds (Siege Artillery).

C.—By the combined use of tangent and quadrant elevation.

This is the principle which underlies all auto-sights; here, as in method A, the *line of sight*, as well as the *vertical plane* containing the line of sight, must pass through the object aimed at.

Example.—Auto-sights for Q.F. guns (Coast Artillery).

D.—By using some adjunct away from the gun, so as to predict the correct quadrant elevation and training in sufficient time, so that the gun can be laid before the time comes to fire it.

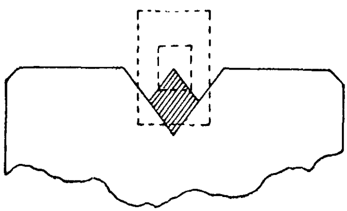
This method eliminates *personal error in laying:* its correctness depends on the accuracy of the instrument or adjunct that is used, and the fidelity with which the findings of the latter are transmitted to and given to the gun.

Examples.—All P.F. systems, especially the service method devised by Colonel Watkin, C.B., R.A.

Method A.

Of the above examples, the tangent scale and foresight, which has already been described, was, up to a few years ago, almost universally employed, and it still forms a portion of the equipment of field and siege artillery, but is seldom used with coast guns. As has been pointed out, when tangent elevation is employed, the line of sight must be directed on a point, and to ensure, as far as possible,

Fig. 5.



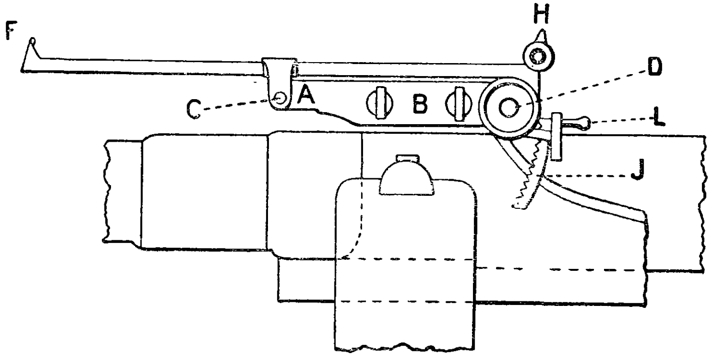
FULL SIGHT.

regularity in laying, the eye of the layer should be applied at a constant distance from the hind sight, a *full sight* being taken.

In other words, the line of sight should pass from the eye, through a point midway between the shoulders of the notch, through the apex of the foresight to the point aimed at. The usual position for the eye is about one foot in rear of the hind sight, though no hard and fast rule can be laid down; if it is brought too close it is impossible to correctly focus the three points to be aligned, the edges of the notch becoming blurred; if, on the other hand, the eye is too far away the apex of the foresight is indistinct, and a lower point is apt to be made use of, resulting in a gun being laid too low.

The bar and drum sight was originally only made use of in the Royal Navy, but lately its principle has been adopted in many land service mountings.

Fig. 6.

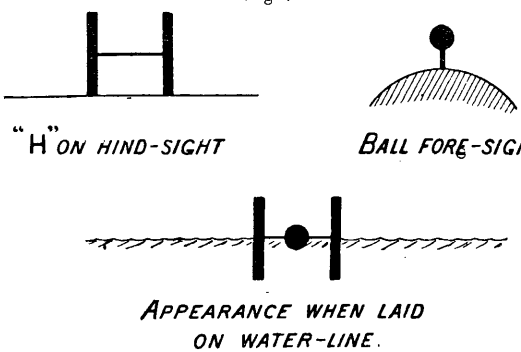


HF is a bar upon which the hind sight H and the foresight F are mounted; it is pivoted at C; when a handle L is rotated the elevating arc J raises the end H of the bar, so that the latter makes the desired angle of elevation with AB, which is attached to the mounting and is parallel to the axis of the bore. Ranges corresponding to the correct angles of elevation are engraved on the circumference of the drum D, which rotates round an axis perpendicular to that of L and is actuated by a wheel and worm.

Sights of this description possess the advantage of lending themselves to the use of a telescope, the optical axis of the latter taking the place of the imaginary line through the fore and hind sights.

For the sake of comparison with the full sight of fig. 5, the H sight, in use in the Navy, may be noted.

Fig. 7.



The hind sight is provided with an H, the foresight terminates in a small ball, and when the gun is properly laid the line of sight passes through the centre of the cross bar and the centre of the ball. This particular arrangement is well suited to the conditions of naval practice.

Scott's Sight.

This is a telescopic sight for use with field artillery, and is interesting as being the first employment of a telescope in the land service. The apparatus is fitted by means of a bracket to the right trunnion of the gun, and can be levelled so as to compensate for want of level in the axletree, owing to inequalities of ground, &c. The optical axis of the telescope takes the place of an imaginary line passing through the fore and hind sights.

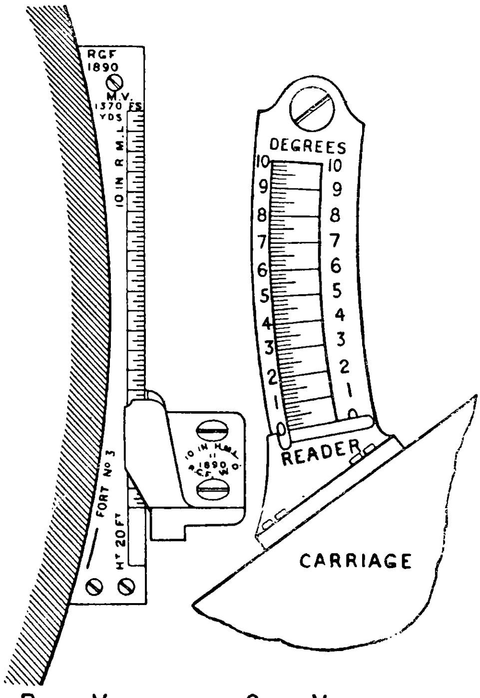
Many advantages are claimed for this sight, which is fully described in the Handbook for Scott's sights.

When tangent elevation, is employed it is immaterial whether the gun and the object aimed at are at the same level or not; the T.E. is not affected under any conditions which actually occur.

B Method.

An index plate is a graduated arc attached to a gun in connection with a reader attached to a gun mounting.

Fig. 8.



REAR VIEW. SIDE VIEW.

The index plate has usually been graduated in degrees on its side and in yards on its rear edge, the latter being in view of the elevating number. When the axis of the bore is horizontal the reader points to zero; when the gun is elevated the reader marks the corresponding degree above zero; when the gun is depressed the corresponding degree below zero; it thus registers *quadrant elevation.*

The degree graduations depend upon the radius of the sights, and are calculated from the relation

$$\text{angle} \times \text{arc} = \text{radius} \times 57{\cdot}3.$$

Guns that employ the index plate and reader are always placed at a certain height above the sea. In order therefore to determine the quadrant elevation due to any range, it is necessary to subtract from the tangent elevation, as given in the range table, the angle of depression due to the range; it sometimes happens that the latter angle is greater than the former, so that the quadrant elevation is *minus*, or really *quadrant depression.* When the quadrant elevation due to each 100 yards of range is determined, it is then possible to graduate the rear edge of the index plate in yards, by making the graduations thereon correspond with the degree graduations on the index plate.

Example

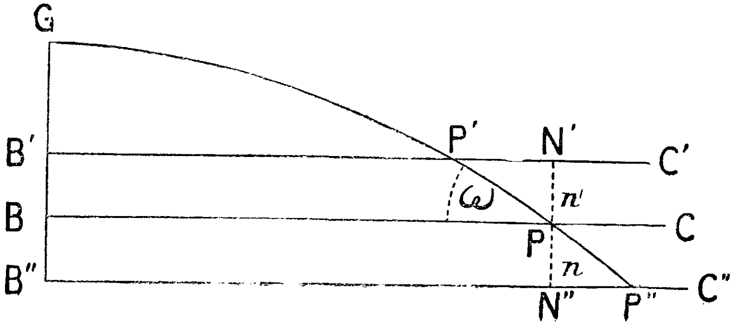
6" B.L. GUN, MARK VI.

Charge 48 lb. E.X.E. M.U. 1960 f/s.
Height above Mean Tide Level 100 feet.

Range in yards.	Tangent Elevation.	Angle of Depression.	Quadrant Angle.
1,000	0′·41	1″·54′	1°·13′ Depression
2,000	1′·38	·57	41′ Elevation
3,000	2′·45	·38	2°·7′ ,,
4,000	4′·80	·29	3°·39′ ,,
5,000	5′·47	·23	5°·24′ ,,

As owing to the tide the sea level alters from time to time, the *height of a gun above mean tide level* is usually taken as the height of site, and quadrant elevation calculated therefrom, and a "tide correction" applied where necessary, by adding or subtracting from the range given to the gun.

FIG. 9.



Thus let a gun be placed at G, at a height GB above *mean tide* level. Let B′ C′ be *high water* level and B″ C″ *low water* level. Then if a gun is correctly laid to strike P, with a quadant elevation due to a height G B, if it is "high water" it will strike "short" at P′, if it is "low water" it will strike at P″ and be "over."

Intermediate fluctuations of the tide would have corresponding effects.

The distance P′N′ or P″N″ evidently depends on the rise or fall of the tide and the angle of arrival at the range in question.

Thus, suppose the level of the water to be n feet above or below mean tide, then for a range R yards and an angle of arrival ω, the *range correction due to tide* would be

$$\tfrac{1}{3}n \cot \omega \text{ (yards).}$$

Tables of tide corrections are made out to suit local conditions, and in practice the amounts are recorded in multiples of 25 yards. The accurate method of arriving at them is as follows:—

Actual Correction.	Recorded Correction.
0 to $12\frac{1}{2}$ yards.	0 yard.
$12\frac{1}{2}$ „ $37\frac{1}{2}$ „	25 yards.
$37\frac{1}{2}$ „ $62\frac{1}{2}$ „	50 „

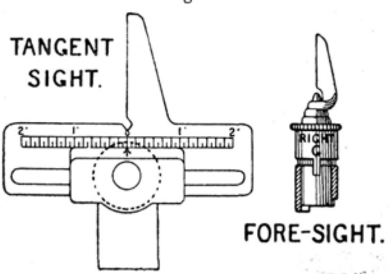
&c., &c. (*Vide* G.A. Drill, 1899, Vol. I., p. 195).

Quadrant elevation is sometimes put on the gun by means of a range indicator consisting of a dial graduated in yards, an index, actuated by a steel band, being constrained to move in accordance with the inclination assumed by the axis of the piece. The graduations of the dial are obtained on the principles already discussed. At other times, quadrant elevation is obtained by means of a clinometer placed on a prepared plane cut on a howitzer; its reading, of course, gives the inclinations of the axis of the latter. This method is usual in siege artillery.

It is thus evident that in Method B the *giving of the desired elevation* depends not on the skill of the layer but on the accuracy of the means employed.

The gun is trained by eye, straight edged sights being usually employed in connection with index plates or range indicators. Straight edged sights may be described as a tangent sight and foresight with blades attached to them as in the figures:—

Fig. 10.



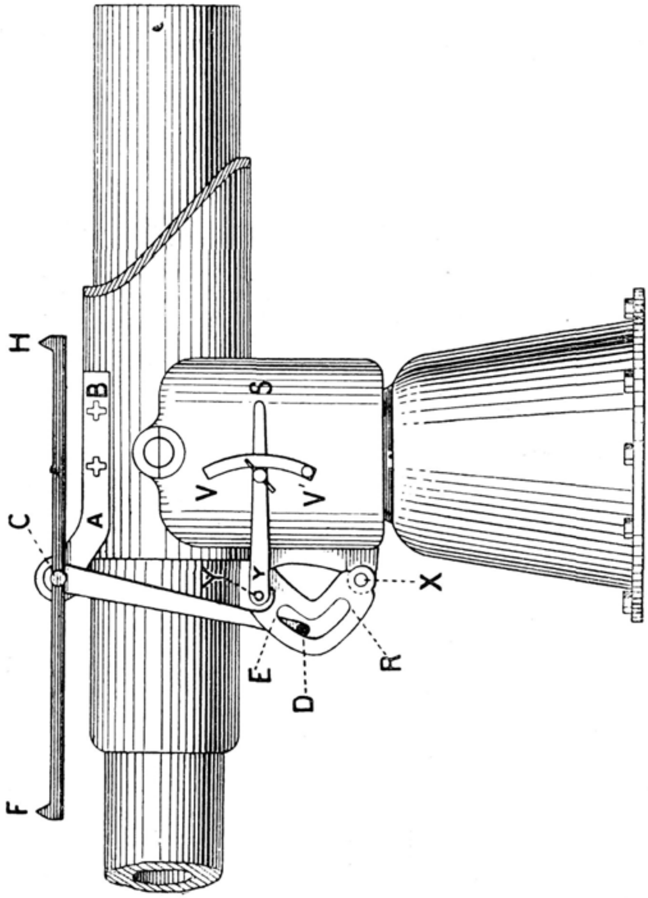
These blades serve two purposes: they are found to be of use in keeping a gun trained on a moving object, and they obviate the constant altering of the height of the hind sight consequent on change of range, as the blades cover some 1,000 yards of water.

In siege artillery the line is sometimes obtained by laying back on some auxiliary point, quadrant elevation being given to the howitzer by a clinometer.

Method C.

All automatic sights are based on the same principle, namely: that for a given range, there is a corresponding T.E. and Q.E., when the ballistic conditions and the height of site are constant.

FIG. 11.



The following is an example of how this principle may be applied:—FH in fig. 11 is a bar sight, similar to that of fig. 6, which carries an arm CD. C is a pivot carried by AB, a straight piece of metal which always remains parallel to the axis of the bore; the bar carrying the fore and hind sights, F and H, with its attached arm CD, is capable of movement about C, but this movement is governed by the pin D, which moves in the cam ER. The front surface of ER is cut so that when the line of sight HF is directed on an object, the angle between HF and AB will be the T.E. due to the range. This would be the case when the reading of a clinometer placed on FH (which would give the angle of sight or angle of depression), added to the reading of a clinometer placed on AB (which would give the Q.E.), were together equal to the range table angle of T.E., due to the range of the point upon which HF is directed.

From the above description it is seen that the line of sight and the axis of the piece have relative motion; one cannot be altered without interfering with the other.

Corrections for varying heights of tide are made as follows:—The plate in which the cam ER is cut is pivoted at X, and can be slightly rocked by the action of the handle SY, which can be clamped at any position along the arc VV′, which carries graduations in accordance with local conditions.

Method D.

As this method may practically be considered as entirely instrumental, it need not be further discussed here; full descriptions will be found in the Manual of Position Finding.

Theory of Range Finding.

The range R of a distant object C to the front of a measured base A B of length c, is taken as

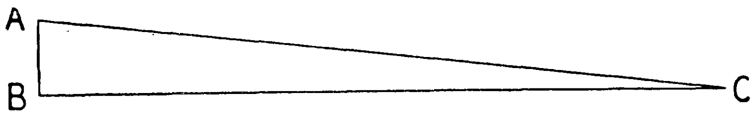
$$c \operatorname{cosec} C;$$

when the angle of convergence C of the lines of sight AC and BC, in other words the *parallax* of C, has been measured:

$$c \operatorname{cosec} C$$

is the diameter of the circle round ABC: the letters and notation employed in Trigonometry is employed here for the moment.

Fig. 12.



If the angle C is small, as in practice, and is expressed in degrees or minutes, we can put

$$\sin C = \frac{C^\circ}{57{\cdot}3}, \text{ or } \frac{C'}{3438},$$

and then

$$R = 57{\cdot}3 \frac{c}{C^\circ}, \text{ or } 3438 \frac{c}{C'},$$

giving the range R in yards, if c is measured in yards; but if c is measured in feet,

$$R = 1146 \frac{c}{C'};$$

this is the formula of p. 4 used with the D.R.F., in which case the base AB is vertical, and ABC is a right angle, and AC is the range R.

But with field range finders, such as the mekometer, or the Barr and Stroud range finder, the base AB is horizontal.

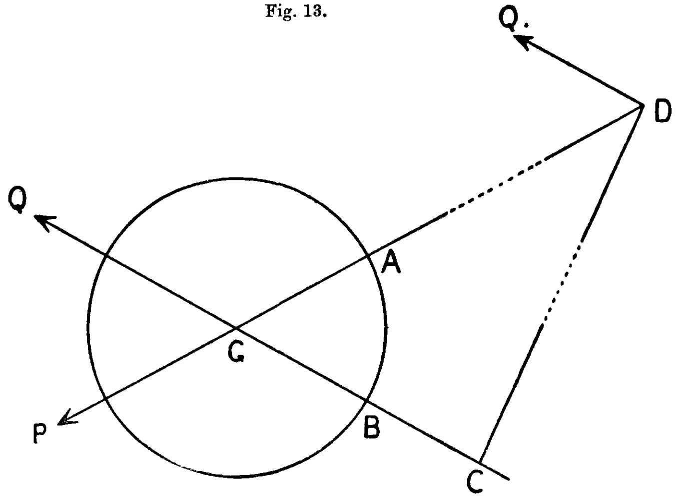
Range finders differ from position finders in that they can only measure the distance from the point they occupy to the object. Many range finders are movable and can be used in close proximity to the guns, but a depression range finder must be in a chosen position, and sometimes is at a considerable distance from the guns it serves, consequently ranges as measured by it will not be true for the guns, until a correction called a "group difference" is applied.

The group difference depends on "displacement" and the angle at which the guns are trained.

The guns of a fort are formed into groups, consisting of one or more guns: in the latter case a gun is selected to be the pivot gun. "Displacement" is the distance in yards between the pivot of the D.R.F. and the pivot of the group it serves. Sometimes two or more groups are served by the same instrument: the displacement must be measured in each instance.

A consideration of fig. 13 will make apparent the effect of the training on group difference.

Fig. 13.



Let G be the pivot gun and AB the circular training arc; let D be the position of the D.R.F. Then GD will be the displacement. Draw the line DAGP; then if the gun and range finder be directed on the line DP, it is evident that the range measured by the instrument will exceed the range from the gun by a distance GD, *i.e.*, the displacement.

Now let the gun be trained through 60° say, so that the pointer on the mounting moves from A to B. Draw the line QGBC, and let DC be a perpendicular on this line from D.

As the ranges measured are great compared with the displacement the lines of sight GQ and DQ, from gun and instrument respectively, may be assumed to be parallel, and the group difference is given by GC, when

$$GC = GD \cos AGB,$$

so that in this instance it is half the displacement.

In practice group differences are always tabulated in multiples of 25 yards (see G.A. Drill, 1899, Vol. I, p. 190), upon the same principle as that employed in Tables of Tide Correction.

SECTION II.—ACCURACY OF FIRE.

For good shooting there are two essential requirements: first, a good weapon and good ammunition; and, secondly, men who know how to use them.

But after all care has been taken, absolute certainty of hitting the same spot at each round is impossible, as several causes of error exist, which cannot be avoided.

Accuracy of fire is thus a comparative term; it is said to be good when a group of projectiles fall close together.

Causes of Inaccuracy.

The **chief causes of unavoidable inaccuracy**, which may exist on the experimental practice ground, where all the conditions are most favourable, are as follows:—

1. Want of accuracy in the gun, faulty ammunition, or unsuitable mounting.
2. Weather.

Range and Accuracy.

With the object of compiling Range Tables, a gun of each nature, when introduced into the Service, is sent to Shoeburyness, and series of rounds are fired at several different elevations for **range and accuracy**, with its service ammunition.

Five or more rounds are fired at each elevation.

Mean ranges and mean lateral deviation from the line of fire are then obtained for each elevation; the difference of each round from the *mean* gives the *error*, from which the 50 °/ₒ zones are worked out.

To take a practical example:—

EXAMPLE I.

No. of round.	Range.	Differences from mean, or errors.	Deviation right.	Differences from mean or errors.	Elevation.
	yds.	yds.	yds.	yds.	
1	4968	23	24·4	3·0	5°26′
2	4954	9	21·6	0·2	.
2	4962	17	22·8	1·4	..
4	4908	37	20·0	1·4	..
5	4934	11	18·4	3·0	..
Total ..	24726	97	107·2	9·0	
Mean ..	4945	19·4	21·4	1·8	

The second column in the above table gives the actual ranges. The mean range is obtained by adding all together and dividing by 5, since 5 rounds were fired.

The third column contains the *difference* of each round, irrespective of sign, from the mean range just found. The mean of these differences is then obtained, and called the mean error in range or mean longitudinal error.

The fourth column gives the lateral deviation from the line of fire. The mean deviation is at the bottom of this column.

And the fifth column gives the differences from this mean, with a mean at the bottom called the mean error in deviation or mean lateral error.

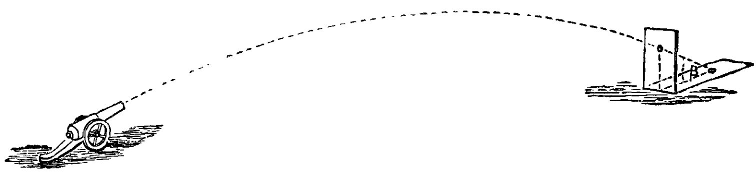
Collecting the results from the Table A we have—

Mean range	**4945**	yards.
Mean longitudinal error	**19·4**	„
Mean deviation right	**21·4**	„
Mean lateral error	**1·8**	„

When the position of the point of mean impact on the horizontal plane is known, fig. 14 shows how the magnitude of the angle of descent determines the position of the point of mean impact on a vertical target.

Thus if β be the angle of descent, and if the horizontal target is struck at a distance l from the vertical one, the latter will be struck at a height which equals $l \tan \beta$.

Fig. 14.



The angle of descent of the 8-inch projectile at 4,945 yards is known to be 7° 25′.

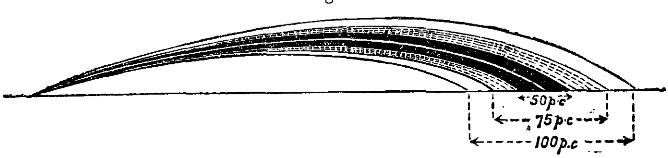
$\therefore$ **Mean vertical error** .... = 19·4 tan 7° 25′
= **2·5 yards.**

Vertical targets are employed at the shorter ranges, because they may then be of moderate size, and errors due to inequalities of the ground are eliminated, but at long ranges targets cannot generally be made large enough to catch all the rounds.

The **point of mean impact** on a horizontal target is the intersection of the lines of mean vertical and mean lateral deviation, and on a vertical target it is the intersection of the lines of mean vertical and lateral deviation.

The **mean trajectory** is that which strikes the point of mean impact: it is the central one of all the trajectories fired at the same elevation.

Fig. 15.



In fig. 15 the central white line represents the mean trajectory, the dark band is that in which 50 °/$_o$ of the trajectories lie; the shaded band is that which contains 75 °/$_o$, while the outer band contains the remainder. The width of these bands is exaggerated in fig. 15, for the sake of showing them clearly.

A practical illustration of dispersion or want of accuracy is given by a fire-hose, in which the stream of water is more separated at the end than at the beginning of its course through the air: the whole trajectory being a kind of bent cone, with its apex at the nozzle.

The Range Table 50°/$_o$ Zones.

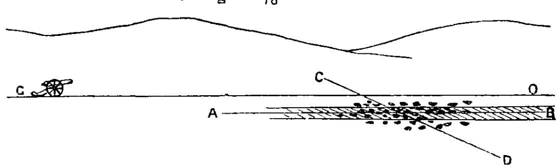
The mean longitudinal error × 1·69 is taken as the width of the 50 °/$_o$ **length zone**; the mean lateral error × 1·69 is taken as the width of the 50 °/$_o$ **breadth zone**; the mean vertical error × 1·69 is taken as the width of the 50 °/$_o$ **height zone.**

The factor 1·69 depends on the Theory of Probability explained in Part II, p. 243.

Thus, if GO, figs. 16 and 17, represents the direction of the gun, and AB is a straight line parallel to it, at a distance equal to the mean

Fig. 16.

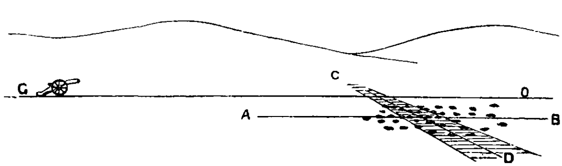
Showing 50 °/$_o$ breadth zone.



lateral deviation, and CD be a straight line at right angles to GO or AB, at a distance from the muzzle equal to the mean range; then if the zone in fig. 16, called the breadth zone, and that in fig. 17

Fig. 17.

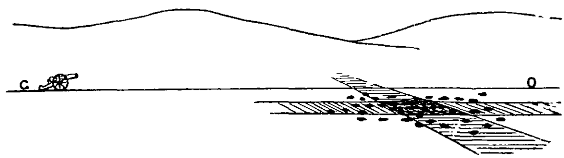
Showing 50 °/$_o$ length zone.



called the length zone, each contains 50 °/$_o$ of the hits on the surface of the ground, their widths must be 1·69 times the mean lateral error, and 1·69 times the mean longitudinal error respectively, and AB and CD are the central lines of these zones.

Fig. 18.

Showing 50 % length zone and 50 % breadth zone intersecting and forming 25 % rectangle.



If now we look at fig. 18, where these zones are superposed, we see a rectangle which must contain 50 % of 50 %, or 25 % of the total number of hits. In a similar manner a 25 % rectangle on a vertical target is made up of the intersection of the 50 % breadth and height zones.

At each range there is a horizontal and a vertical 25 % rectangle; the width of each is the same, as each has the same breadth zone, but the relation of the length of one to the height of the other depends on the angle of descent.

TABLE A.

PROBABILITY FACTORS.

Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.
1	0·02	21	0·40	41	0·80	61	1·27	81	1·94
2	0·04	22	0·41	42	0·82	62	1·30	82	1·98
3	0·06	23	0·43	43	0·84	63	1·33	83	2·03
4	0·07	24	0·45	44	0·86	64	1·36	84	2·08
5	0·09	25	0·47	45	0·89	65	1·39	85	2·13
6	0·11	26	0·49	46	0·91	66	1·42	86	2·18
7	0·13	27	0·51	47	0·93	67	1·45	87	2·24
8	0·15	28	0·53	48	0·95	68	1·48	88	2·30
9	0·17	29	0·55	49	0·98	69	1·51	89	2·37
10	0·18	30	0·57	50	1·00	70	1·54	90	2·44
11	0·20	31	0·59	51	1·02	71	1·57	91	2·52
12	0·22	32	0·61	52	1·04	72	1·60	92	2·60
13	0·24	33	0·63	53	1·07	73	1·64	93	2·69
14	0·26	34	0·65	54	1·09	74	1·67	94	2·78
15	0·28	35	0·67	55	1·12	75	1·71	95	2·91
16	0·30	36	0·70	56	1·14	76	1·74	96	3·04
17	0·32	37	0·72	57	1·17	77	1·78	97	3·22
18	0·34	38	0·74	58	1·19	78	1·82	98	3·45
19	0·36	39	0·76	59	1·22	79	1·86	99	3·82
20	0·38	40	0·78	60	1·25	80	1·90	100	

This Table A is calculated on the Theory of Probability explained in Part II, p. 243.

Taking the width of a 50 % zone as unity, the factor in the above table is the width of the zone containing the corresponding percentage: thus the 80 % and 20 % zone is respectively 1·90 and 0·38 times the width of the 50 % zone.

If the width of the 50 % zone is given in yards, the widths of other zones containing different percentages can be obtained by *multiplying* by their corresponding factors: thus, if the width of a 50 % zone is 3 yards, the widths of 25 % and 72 % zones are $0·47 \times 3 = 1·41$ yards and $1·60 \times 3 = 4·80$ yards, respectively.

Conversely, if it is required to find what percentages will fall in zones of given widths, the factors must be obtained by *dividing* each by the width of the 50 °/₀ zone.

Thus, with the same 50 °/₀ zone (3 yards wide) as before, what percentages will fall in zones 2 yards and 6 yards wide?

The factors are

$$\tfrac{2}{3} = 0{\cdot}67 \text{ and } \tfrac{6}{3} = 2{\cdot}00,$$

and they correspond to

35 °/₀ and 82·4 °/₀

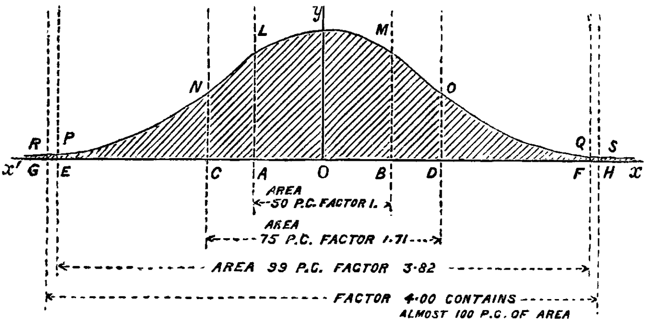
respectively.

The annexed fig. 19 represents the probability curve, the line GH being asymptotic to it; the total area contained between the curve and the line GH is proportional to the total number of rounds, or 100 °/₀.

The central area ABML represents half the total area, or 50 °/₀ of the rounds fired.

Areas of different widths contain percentages according to the table, the widths benig the same as the factors in the table: thus the 75 °/₀ area, CDON, is 1·71 times as wide as the 50 °/₀ area, and the 99 °/₀ area, EFQP, is 3·82 times as wide; it is a fair approximation to assume that an area four times as wide as the 50 °/₀ area contains all the area, though there is a small portion outside.

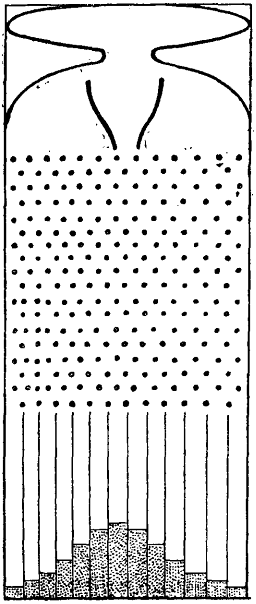
Fig. 19.



The curve of error in fig. 19 can be imitated experimentally in an instrument (fig. 20), invented by Mr. Francis Galton, and called by him the *Quicunx*, from the Latin word used to describe the arrangement in the planting of trees, which is imitated by the pins in this instrument.

A charge of small shot (or better, of spherical seeds, as not so heavy) is allowed to pour through the funnel at the top. The spherules knock against the pins and are scattered thereby in an arbitrary manner; but it is found that they group themselves in the stalls at the bottom in a manner which imitates closely the profile of the Probability Curve.

Fig. 20.



Examples on the use of Table A.

EXAMPLE 1.

If a zone of a certain width catches 20 % of the rounds fired, how much wider must another be to catch 80 %?

From the Table we find that the 20% zone is 0·38 of the width of the 50 % zone; and also that the 80 % zone is 1·9 times the width of the same standard.

Consequently the widths of the zones in question must be to each other as

0·38 : 1·9, or as 1 : 5,

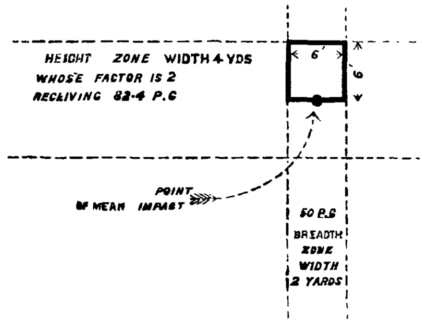
or the 80 % zone is five times as wide as the 20 % zone.

EXAMPLE 2.

If the breadth and height 50 % zones are each 2 yards wide, what percentage of hits may be expected on a target 6 feet square, if the point of mean impact is in the middle of the lower edge?

The 50 % breadth zone just includes the target (fig. 21).

Fig. 21.



The height zone to be employed must be one which is double the height of the target, for then the point of mean impact will be in the middle of the zone, and the whole of the target will be included. The factor for this zone is evidently 2, corresponding to a percentage of 82·4: but as the target only lies on one-half side, we must take half the percentage or 41·2 °/₀.

Consequently on the target we have

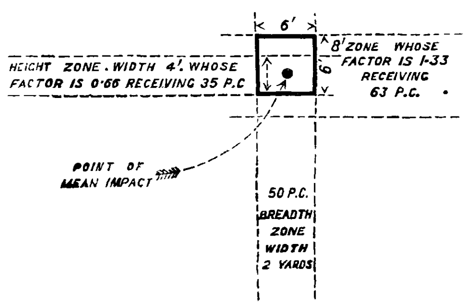
$$50\ {}^{\circ}/_{\circ} \text{ of } 41{\cdot}2\ {}^{\circ}/_{\circ} = 20{\cdot}6\ {}^{\circ}/_{\circ}.$$

Example 3.

If in the last example the point of mean impact is raised 2 feet, what improvement may be expected in the shooting?

As before, the 50 °/₀ breadth zone just includes the target (fig. 22).

Fig. 22.



For the height zones—take one 4 feet wide and another 8 feet wide. Then the target will be contained in the lower half of the 4-feet zone and in the upper half of the 8-feet height zone.

The 4-feet zone has a factor $\frac{4}{6} = 0{\cdot}67$, and it receives

35 °/₀ hits.

The 8-feet zone has a factor $\frac{8}{6} = 1{\cdot}33$, and it receives

63 °/₀ hits.

As the height band, which just contains the target, is composed of the halves of these two zones, it must receive $\frac{1}{2} \times 35 + \frac{1}{2} \times 63 = 49$ °/₀ hits;

and the whole target has 50 °/₀ of 49 °/₀ = 24·5 °/₀

—an improvement of 3·9 °/₀ of the total fired, or 19 °/₀ more hits on the target than in the last case, for

$$\frac{3{\cdot}9}{20{\cdot}6} = \frac{19}{100} \text{ nearly.}$$

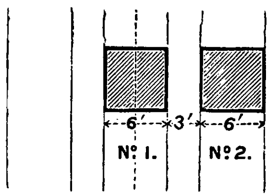
EXAMPLE 4.

Suppose there are two targets, 6 feet wide, and of the same height, 3 feet apart, fired at by a gun at a certain range; the width of the 50 °/₀ breadth zone being 6 feet.
Which plan will give the most hits on the target?

(1.) If the mean point of impact is at the middle of one?
(2.) If it is midway between the two?

Taking the first supposition (fig. 23)—
(Neglecting height errors, which bear the same proportion throughout.)
50 °/₀ must fall on the target (No. 1) aimed at.

Fig. 23.



To find out how many fall on the other (No. 2), take a zone just to include No. 2 target, the centre being the middle of No. 1. This zone must be 24 feet wide.

The factor for this zone is $\frac{24}{6} = 4$, corresponding to 100 °/₀.

Now take a zone, having the same centre, which will just *not* include the second target; this must be 12 feet wide. and the factor is $\frac{12}{6} = 2$, corresponding to 82·4 °/₀.

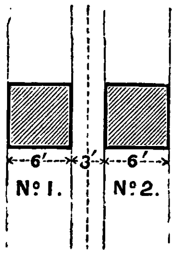
Hence (100 – 82·4) °/₀ fall in the spaces between the two zones, but since there is a target in only one of these spaces, we must divide by two to find out how many fall on the second target, and we thus obtain

$$\frac{100-82{\cdot}4}{2} = 8{\cdot}8\ \%.$$

In this case then we have—

On No. 1 target............	50 %
On No. 2 target............	8·8 %
Total	58·8 % on both.

Fig. 24.



On the second supposition (fig. 24)—

Take a zone to include both targets, this must be 15 feet wide.

And the factor is $\frac{15}{6} = 2{\cdot}5$ or 90·8 %.

We must subtract from this the numbers which fall in the zone between the targets 3 feet wide, and are lost.

Here the factor is—

$$\frac{3}{6} = 0{\cdot}5, \text{ corresponding to } 26{\cdot}5\ \%.$$

The difference of these two percentages, *i.e.*,

$$90{\cdot}8 - 26{\cdot}5 = 64{\cdot}3\ \%,$$

falls on the two targets in this case, which is more than on the first supposition.

The range tables 50 % zones must usually be considered as guides to the probable number of hits, for, in the first place, they sometimes depend on five rounds only, and, in the second place, the conditions of practice often differ materially from those obtaining during the trials at Shoeburyness. The best plan for determining the probable percentage of hits, is to work back step by step from the number of hits actually obtained at practice. An instance is given in the next example:—

EXAMPLE 5.

Firing at a "record target" 10 yards broad and 3 yards high, a battery made 10 °/₀ of hits and 10 °/₀ of lateral misses. What percentage of hits may be expected on a target 100 yards broad and 9 yards high, provided the practice is carried out by the same battery under the same conditions?

As there were 10 °/₀ of lateral misses, 90 °/₀ would have hit had the record target been of infinite height.

Now, the factor for 90 °/₀ is 2·44, and as $100 \times \frac{2{\cdot}44}{10}$ exceeds 4, the approximate factor for 100 °/₀; there will be no lateral misses when firing at the target 100 yards broad.

As there were 10 °/₀ of hits, and as 10 °/₀ missed laterally,

$$\frac{10 \times 100}{100 - 10} = 11{\cdot}1 \text{ °/₀}$$

would have hit the record target had it been infinitely broad.

Thus the question comes to be: if 11·1 °/₀ of hits may be expected on a target 3 yards high and of infinite breadth, what percentage may be expected on one 9 yards high and of infinite breadth?

The factor for 11·1 °/₀ = 0·202; the percentage corresponding to

$$\frac{9}{3} \times {\cdot}202 = 31{\cdot}6,$$

which is the answer required.

EXAMPLE 6.

Given that the 50°/₀ zones for length of the 6-inch B.L. and of the 9-inch R.M.L. guns, at a range of 2,000 yards, are 18 yards and 23 yards, determine the height of site which will put the 9-inch gun on equal terms with the 6-inch gun for accuracy of shooting at a sea-target, the 6-inch gun being at sea-level.

(The angle of descent at 2,000 yards of the 9-inch gun is given in the Range Table as 3° 20′.)

Small Arm Ammunition.

With **small arms** very large numbers of rounds are manufactured, and a certain proportion are fired from standard rifles to test the accuracy of the ammunition; the powder or cordite is first tested separately.

A different method to artillery practice is followed in this case.

Rifles in rests are laid on large vertical targets at 500 yards, and series of 20 rounds are fired from each under as nearly as possible the same conditions. The vertical targets are 24 feet square, and are divided into smaller squares of 3 feet side, and these again into smaller ones 6 inches square. The point of impact of each round is noted and plotted on a diagram, as shown opposite. The horizontal and vertical distance of each hit from some vertical and from some horizontal line is measured, the mean of each of these distances is then determined, and thus the point of mean impact; so far the plan resembles that previously described; but after this the plan adopted for small arms differs from the other: the *radial* distance of each hit from the point of mean impact is measured, and the mean of these radial distances gives the **figure of merit** furnished by the particular sample of ammunition employed; thus in Plate III the figures of merit of 20 shots from a Snider rifle on 22nd December, 1884, was 1 foot 1·35 inches, while the figures of merit of a sample of ammunition for the Martini rifle on the same day was 9·25 inches; as these were average samples of ammunition in each case, the figures may be taken as a fair comparison of the accuracy of the two rifles.

If a steady wind is blowing it makes but little difference, as though the point of mean impact is altered, the radial distances from this point remain unchanged, or nearly so. Gusts of wind, however, spoil the shooting.

Abnormal or Doubtful Rounds in Analysis.

Referring to Example 1, we see from its third column that the greatest difference of any round from the mean range is 37 yards, and the question arises, should this round be thrown out or not?

No rule can be laid down definitely, but Table B may, in many instances, be of some help. It may be used as follows:—

Multiply the mean longitudinal error by the factor in the table corresponding to the number of rounds fired. If the product thus obtained exceeds the error of the round in question, there is little doubt that the round should not be discarded. If the error exceeds the product, the round may be considered doubtful. The neighbouring means, that is to say, the mean longitudinal errors of the groups of rounds fired at the elevations immediately above and below, may in some cases remove the doubt; in other cases there may be some equally good extraneous evidence. It must be clearly understood that only one doubtful round at a time can ever be cast out by this method.

In Example 1 there are five rounds, the factor corresponding to which from Table B is 2·44.

The mean longitudinal error in the example is 19·4.

The product of these $= 2{\cdot}44 \times 19{\cdot}4 = 47$.

This product is greater than the error 37, that is, it is greater than the greatest error of any round, and consequently none of the five rounds can be considered doubtful.

If exactly four rounds are fired, and if they all fall in a zone of four times the mean difference, none of the rounds need be considered doubtful. In Example 1 all the rounds fell between 4,968 and 4,908, that is, in a zone of 60 yards. This being less than $4 \times 19{\cdot}4$, and four rounds being less than the number actually fired, no rounds are doubtful.

The fact that a doubtful round has been thrown out must be taken into consideration in the calculation for determining the 50 % zone, but this is a matter beyond the scope of this chapter.

To face p. 80.

Plate III.

Part I Chapter III.

PROOF OF AMMUNITION.

COMPARISON OF ACCURACY OF SNIDER & MARTINI HENRY RIFLES.

ROYAL LABORATORY, WOOLWICH.

22ND DECEMBER 1884.

No. 8. Diagram
L. C. TARGET C.

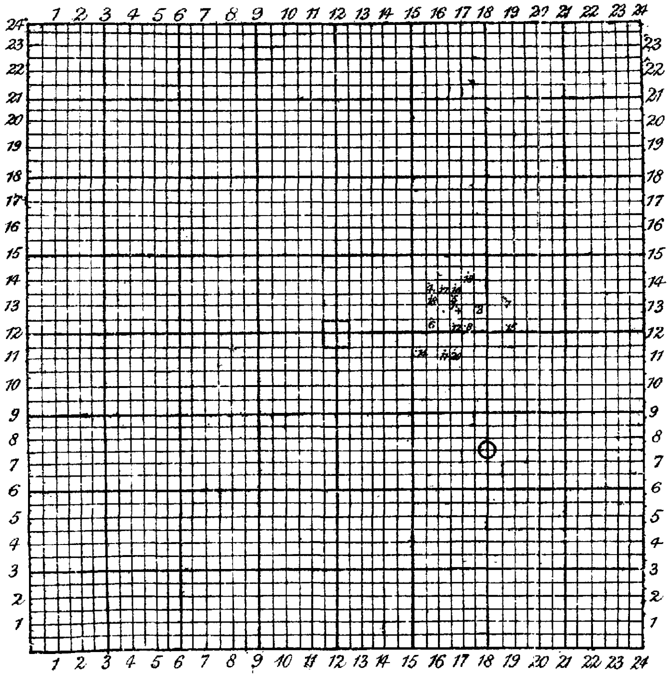
DATA.

Rifle 36th description of S. Enfield. Rifle
Powder 70 Grains R.F.G.
Bullets ·573 Clay Plug
Lubrication Wax
Cartridge Mark IX
Fired from Fixed Rest
Hits 20
Missed
Mean Absolute Deviation 1·335 Inches
Number of Shots 41 to 40
Stripped
Fouled
Range in Yards 500
Elevation 1°35′ 1°13′ 20
Point aimed at

Direction of Range N.N.E.
Wind { Direction of N.E. / Strength of 0 to 1 lb / Character of Steady
Thermometer 39°
Barometer 30·284
Degree of Humidity 92
Solar Heat

SNIDER RIFLE.

Record of 20 Shots from 33 Guager work of the 20th



Showing the Deviation of each Shot.

No. OF SHOT	HORIZONTAL MEASUREMENT		HORIZONTAL MEASUREMENT		ABSOLUTE DEVIATION. FROM POINT OF MEAN IMPACT	
1H	1	9	4	1	0	3
2H	2	3	4	3	0	8
3H	2	9	3	7	0	11
4H	0	9	4	7	.	3
5H	1	9	4	5	0	7
6H	0	9	3	7	1	1
7H	3	10	4	4	2	1
8H	2	2	3	5	0	3
9H	2	2	4	5	0	8
10H	1	9	4	8	0	11
11H	1	3	2	4	1	7
12H	1	9	3	3	0	5
13H	2	7	3	3	0	11
14H	0	1	2	1	2	5
15H	3	8	3	4	1	11
16H	1	3	3	11	0	7
17H	1	3	4	9	1	2
18H	2	3	5	3	1	7
19H	0	7	4	4	1	4
20H	1 .	9	2 .	5	1 .	5
	36	1	76 .	5	22 .	3
	1 .	9·8	3 .	9·85	1 .	135

ROYAL LABORATORY, WOOLWICH.

22ND DECEMBER 1884.

No. 10 Diagram.
Left TARGET G.

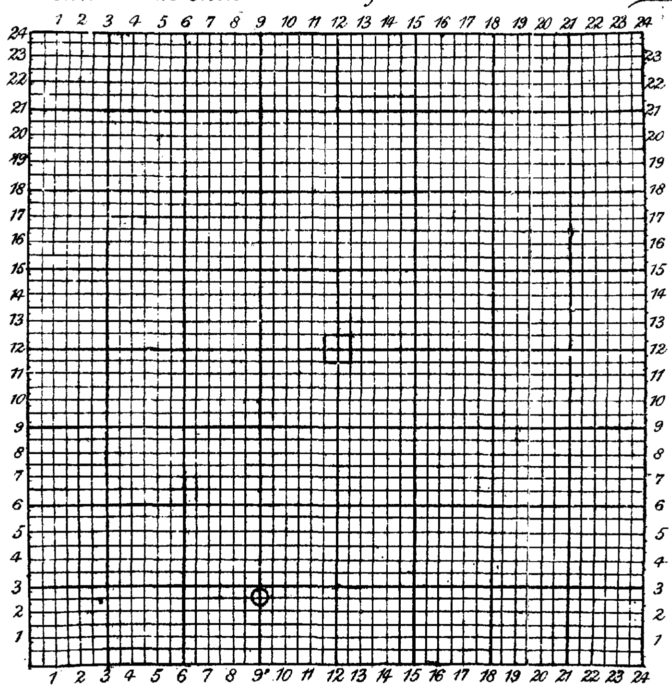
DATA.

Rifle 28 description of M. Henry Rifle
Powder 85 Grains R.E.G.2
Bullets 480 Grains
Lubrication Wax
Cartridge Mark III.
Fired from fixed Rest
Hits 20
Missed
Mean Absolute Deviation ·925 Inches
Number of Shots 61 to 80
Stripped
Fouled
Range in Yards 500
Elevation 1°22′ 1°20′40
Point aimed at

Direction of Range N.N.E.
Wind { Direction of N.E. / Strength of ½ to 1½ lbs. / Character of Steady
Thermometer 43°
Barometer 30·284
Degree of Humidity 78
Solar Heat

MARTINI-HENRY RIFLE

Record of 20 Shots from 53 Guager work of the 20·12·84.



Targets 24 × 24 feet
Squares 6 × 6 inches

Showing the Deviation of each Shot.

No. OF SHOT	HORIZONTAL MEASUREMENT		VERTICAL MEASUREMENT		ABSOLUTE DEVIATION FROM POINT OF MEAN IMPACT	
1H	0	10	2	11	0	1
2H	0	7	2	3	0	11
3H	0	2	1	5	1	7
4H	0	3	3	3	0	10
5H	1	2	3	3	0	6
6H	0	9	3	7	0	9
7H	1	3	[illegible]	10	0	3
8H	1	2	3	10	1	1
9H	0	9	2 .	5	0	5
10H	0	2	2 .	9	0	9
11H	0	3	2	11	0	7
12H	1	9	3	5	1	0
13H	2	0	3	4	1	3
14H	1	2	1	7	1	2
15H	1	4	3	10	1	2
16H	0	10	2	4	0	8
17H	0	5	2	5	0	8
18H	0	4	3	5	0	10
19H	1	5	2	4	0	8
20H	1	3	2	9	0	5
	17 .	4	56° .	10	15 .	5
	0 .	10·40	2 .	10·1		·925

1017.12.01.

To face p. 168

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TABLE B.

Rounds.	Factors.	Rounds.	Factors.
3	2·05	12	3·02
4	2·27	13	3·07
5	2·44	14	3·12
6	2·57	15	3·16
7	2·67	16	3·19
8	2·76	17	3·22
9	2·84	18	3·26
10	2·91	19	3·29
11	2·96	20	3·32

CHAPTER IV.—INTERNAL BALLISTICS.

THE investigation of the relations connecting the *pressure*, *volume*, and *temperature* of the powder gases inside the bore of a gun, and of the *work* done by the expansion of the powder, constitutes the branch of artillery science called *Internal Ballistics*.

Under the same head may be considered the Measurement of Velocity, at any point of the bore as well as outside near the muzzle; also the theory of Recoil.

SECTION I.—WORK REALISED BY THE EXPANSION OF POWDER GAS.

Definition of Work.

Work is performed when a force, P pounds or tons, pushes a body through a distance s feet; it is measured by the product Ps, called foot-pounds or foot-tons, according as P is in pounds or in tons.

The **unit of work** (called the **foot-pound**) is the amount of work which is performed in raising a weight of 1 lb. through a distance of 1 foot vertical against gravity; but for artillery purposes the **foot-ton** is the unit generally employed, *i.e.*, the amount required to raise 1 ton 1 foot high; a foot-ton contains 2,240 foot-pounds.

Work done by a uniform force can be *represented graphically* by a rectangular area in which the height is proportional to the force P, and the base to the distance s through which the force acts.

Thus, if AD is proportional to P, and AB to s (fig. 1), the area DB will represent graphically the work done.

Suppose at some point C on AB (fig. 2) the pressure suddenly alters; if at C we erect a perpendicular and mark off on it CF, proportional to the new pressure, and complete the rectangle FB, the work will be represented by the sum of the two areas DC and FB.

Fig. 1.

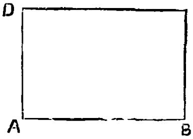


Fig. 2.

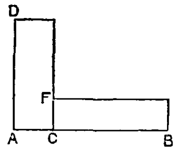
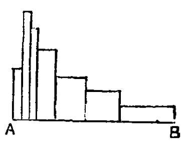


Fig. 3.



If the pressure changes more than once we must take a greater number of rectangles (fig. 3), and the sum of the areas then represents graphically the work done by a pressure P which has suddenly changed in magnitude several times in acting on the body, over the distance AB.

Now, suppose the pressure to change in magnitude, but to do so *gradually*, as in fig. 4, in this case the number of rectangles becomes indefinitely increased, and the work done is represented by the area enclosed by a curved line (the locus of the corners of an indefinite number of rectangles).

In fig. 4 we have such a diagram where the pressure (as in the bore of a gun) begins from a moderate pressure, say 10 tons per square inch, soon rises to a high maximum, say 20 tons per square inch; then falls off at the muzzle to about two or three tons per square inch, and ceases soon after the projectile leaves the muzzle.

If a force has pushed a weight through a given distance, and has caused it to move at a certain velocity, work is *stored up*.

Suppose the work stored up was produced by the weight falling from a certain height h feet, under the acceleration of gravity g, until it had attained the same velocity v f/s.

The relation between v and h under these circumstances is given by the elementary dynamical formula—

$$\tfrac{1}{2}v^2 = gh$$

or

$$h = \frac{v^2}{2g}.$$

If the body weighs w lb., the work done must by definition be wh foot-pounds; substituting the value of h just obtained, this work is equal to

$$\frac{wv^2}{2g} \text{ ft-lbs., or } \frac{wv^2}{2g \times 2240} \text{ ft.-tons.}$$

With foot-second units we take $g = 32$, or more accurately,

$$g = 32{\cdot}19, \log g = 1{\cdot}5077.$$

This expression is a measure of the work contained in a moving body in terms of its weight and velocity; in this form it is called kinetic energy, or shortly **energy**.

It should be noticed that the amount of work increases as the *square* of the velocity; thus if the weight of the projectile is unchanged, and its velocity is doubled, the energy becomes quadrupled.

The energy in ft.-lbs. due to the rotation of a rifled projectile is expressed by

$$\frac{w}{2g}(k\omega)^2,$$

in which k is the radius of gyration in feet, and ω is the angular velocity in radians per second.

The *radius of gyration* of the projectile about its axis is defined to be that radius at which the whole weight of the shot may be supposed concentrated in a ring, without altering its energy of rotation for given angular velocity ω.

If the pitch of the rifling is b feet, the shot will make a complete turn, an angle of 2π radians, when the shot advances b feet, the twist being uniform; so that if the shot is advancing with velocity v f/s, when the angular velocity is ω radians per second,

$$\frac{\omega}{2\pi} = \frac{v}{b};$$

and the energy of rotation is

$$\frac{4\pi^2k^2}{b^2}\,\frac{wv^2}{2g}\text{ ft.-lbs.,}$$

or $\frac{4\pi^2k^2}{b^2}$ of the energy of translation or striking energy.

This is always small in proportion to the energy due to translation, and may generally be neglected.

The work done by the powder pressures in the bore on the projectile must equal the energy contained in the projectile and the gun, less the work lost by friction.

Thus, if P is the mean total thrust in tons on the base of the projectile exerted over a length of bore s feet, we must have

$$Ps = \frac{wV^2}{2g \times 2240};$$

where V is the muzzle velocity of the projectile in feet per second, and w its weight in pounds, if the energy of rotation and of recoil and friction is neglected here.

The following examples will illustrate these definitions of work and energy:—

On firing the 9·2-inch B.L. gun, Mark V, the powder pressure acts on the base of the shell as it moves over a distance of 246 inches in the bore of the gun; the weight of the projectile is 380 lbs., and its M.V. 2065 f/s; what must be the mean propelling force on the base of the shell?

$$\text{Here } d = 9{\cdot}2,\ w = 380,\ V = 2065,\ s = 246 \div 12,\ g = 32{\cdot}19;$$

$$\text{whence } P = 548{\cdot}7 \text{ tons.}$$

Some additional thrust is also required to compress the driving-band into the grooves.

If the mean pressure p, in tons per square inch is required, we must divide the result just obtained by the area of the base of the shell in square inches.

$$\tfrac{1}{4}\pi d^2 = 0{\cdot}7854(9{\cdot}2)^2,$$

and we obtain $p = P/\frac{1}{4}\pi d^2 = 8{\cdot}25$ tons per square inch.

The *maximum* pressure, however, greatly exceeds this, as shown in fig. 4, A gun is a simple thermodynamic machine or heat engine, which does its work in a single stroke, and does not work in a series of cycles, as an ordinary steam engine (Anderson, *Conversion of Heat into Work*).

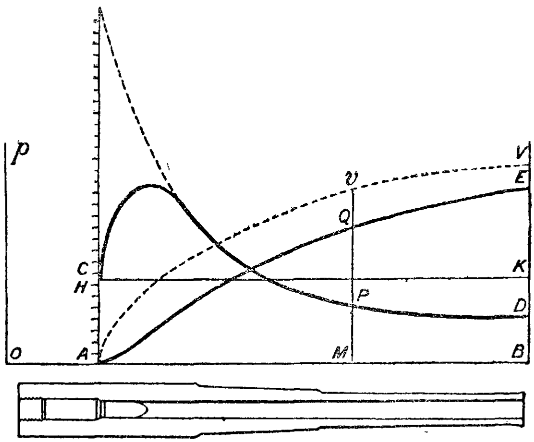
The Maxim gun, however, tends to a resemblance with a steam engine in its power of sustained repeated action.

When a gun is fired, the shot is expelled by the pressure of the powder gases, and the relation between p, the pressure, and x, the volume of the gas, is represented graphically on what may be called the *indicator diagram* of the gun by means of a curve, CPD, in which the ordinate MP represents the pressure p, and the abscissa OM represents the volume, x, each to an appropriate scale, when the base of the shot has advanced from A to M; the curve CPD starts from a point C, such that the ordinate AC represents the pressure when the shot begins to move (fig. 4).

Considering that the bore of the gun is cylindrical, the volume x grows at a uniform rate with the travel of the shot, so that we can take OM to represent volumes of expansion, provided the origin O is suitably placed to allow for the extra diameter usually given to the powder chamber OA.

The area AMPC then represents the work done by the powder (per unit area or square inch of cross section of the bore) when the base of the shot has advanced from A to M, the area ABDC

Fig. 4.



representing the total work per unit area done by the powder as the base of the shot is leaving the muzzle B.

If OM represents cubic inches of volume, and MP represents tons/in.^{2} (tons per square inch), then the areas represent inch-tons of work, reducible to foot-tons by dividing by 12.

The diagrams of figs. 11, 12, 13, p. 105, taken from a valuable paper by Sir Andrew Noble, communicated to the Royal Society on 21st June, 1894, represent the results of actual experiments carried out in a 6-inch gun, which could be lengthened abnormally to 100 calibres, or 50 feet of length.

Suppose the shot weighs w lb., and that it acquires velocity v f/s at M, then equating the kinetic energy and the work done in ft.-tons,

$$\frac{wv^2}{2240 \times 2g} = \frac{\text{area AMPC}}{12}.$$

This supposes the bore is smooth; but if it is rifled with a pitch of b feet, the angular velocity at M is $\frac{2\pi v}{b}$ radians per second; so that if the radius of gyration of the shot about its axis is k feet, the kinetic energy is replaced by

$$\frac{wv^2}{2240 \times 2g}\left(1 + \frac{4\pi^2 k^2}{b^2}\right) \text{ft.-tons};$$

and to allow for the friction of the bore, an empirical deduction is made from the pressure, represented in full by MP.

The curve of energy, AQE, is drawn such that its ordinate MQ represents to scale the work done by the powder, or the kinetic energy acquired by the shot, each proportional to the area AMPC; and thence the velocity curve AvV can be drawn, in which the ordinate Mv represents the velocity v, so that Mv is proportional to the square root of MQ.

Thus if, as in the pneumatic gun, we may take the pressure as uniform and represented by the line HK of average pressure, then the energy curve AQE will be a straight line, and the velocity curve AvV a parabola; in this case the gun may be made of uniform thickness, calculated by the formulas of Chapter V, and great economy of weight is secured.

The pressure in the bore is determined experimentally by *crusher gauges*, described below, which are screwed into the bore at regular intervals in its length, as shown in the figure opposite.

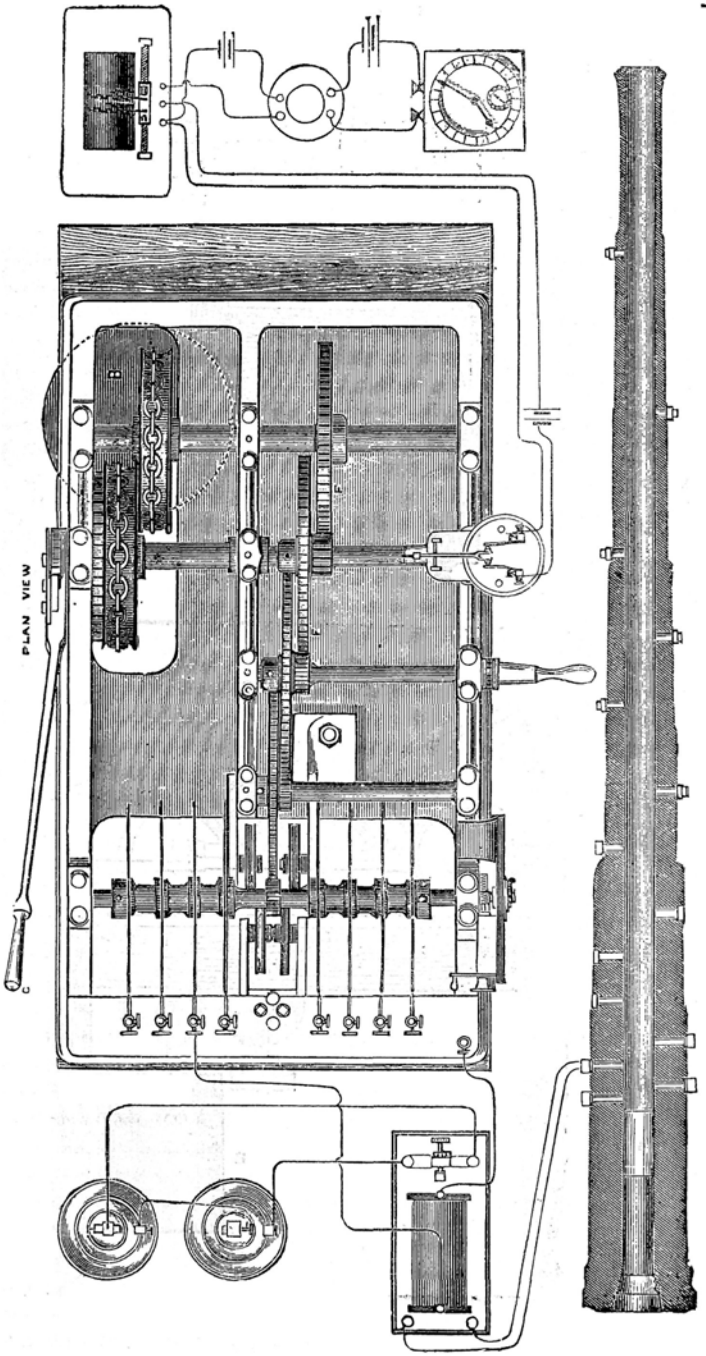
As a check upon the indications of the pressure gauges, Sir Andrew Noble inserts also a number of plugs, connected electrically with his chronoscope, and thereby determines experimentally the time occupied by the shot in its passage up the bore; thence the velocity at each point is inferred by calculation in exactly the same manner as the velocity outside from screen records, and the velocity curve can be drawn.

The energy curve is derived from this velocity curve, and thence the effective pressure accelerating the shot is determined; and these pressures are compared with the pressures recorded by the crusher gauges.

In this way Sir Andrew Noble finds that the crusher gauges record a higher pressure than the chronoscope records with modern explosives, such as cordite, but a lower pressure with the old-fashioned kinds of powder.

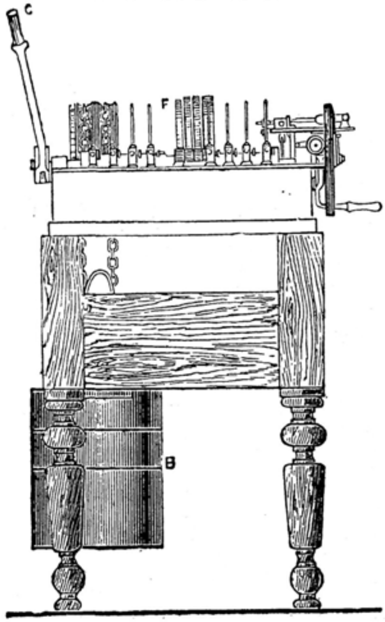
If the gun is free to recoil, there is a similar indicator diagram for the gun, representing the pressure or thrust on the base of the bore, or on the breech piece, at corresponding points of the length of the recoil.

The recoil can be measured at any instant by Colonel Sebert's velocimeter, consisting of a strip of smoked steel attached to the gun, on which, in recoiling, a wavy line is traced by a point on a fixed tuning fork, the period of which is known accurately, and this record is another independent check upon the previous methods.

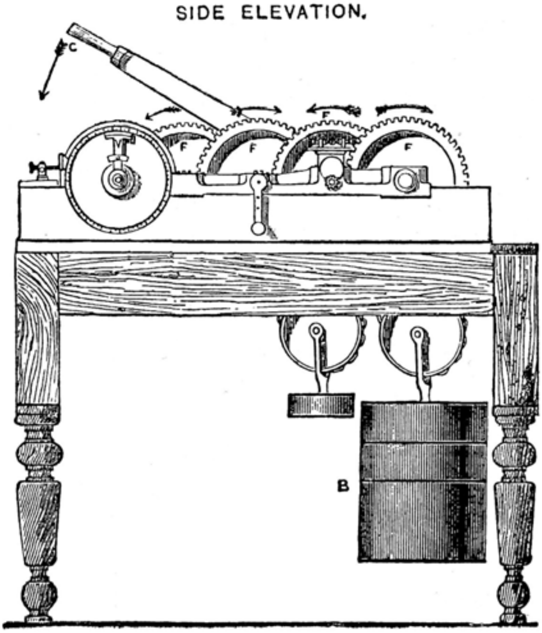


CHRONOSCOPE.

END ELEVATION.



SIDE ELEVATION.



The Crusher Gauge.

For guns devoted to experimental work or to proof of gunpowder, a number of holes are bored through the metal of the gun at definite intervals, commencing from the centre of the powder chamber up to within a few calibres of the muzzle, as in the figure on p. 87; into each of these is screwed a steel plug (see fig. 5, p. 90).

This shows only the end, for its length will depend on the thickness of metal; it is partially provided with a screw-thread of the same dimensions as a copper vent-bush, and the top has a square head for screwing it in and out of the gun.

The removable end, H, called the nozzle, is accurately bored through its centre with a hole one-sixth of a square inch in sectional area, and in this fits the piston, C.

By unscrewing the nozzle a chamber, B, is disclosed, into which the copper cylinder, A, is inserted, and there it is held tightly (but not prevented from expanding) by a small piece of watch-spring, F; this should keep the copper in a central position (see fig. 6) with one extremity in contact with the end of the chamber and the other with the head of the piston, C; a small gas-check, D, of copper is fitted in the nozzle after the plug has been got ready for use, so that its expansion prevents any penetration of gas into the chamber of the gauge.

On firing, the pressure of the gas acting upon the end of the piston compresses and shortens the copper cylinder. The crusher gauge is then taken out, the nozzle unscrewed, and the copper removed; its length is carefully measured to the thousandth of an inch by means of a micrometer.

The pressure in tons per square inch corresponding to any measured reduced length is then ascertained by reference to a table (p. 91) originally compiled from the compression of similar coppers in a statical pressing machine.

This table has been calculated to give the pressure in tons per square inch when a piston 0·461 inch in diameter (one-sixth of a square inch in sectional area) is used with a copper cylinder 0·5 inch long, and 0·326 inch in diameter (one-twelfth of a square inch sectional area).

Fig. 5.

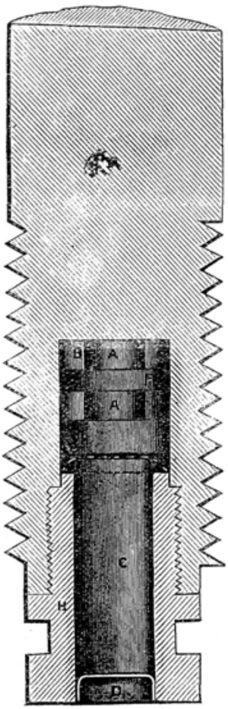
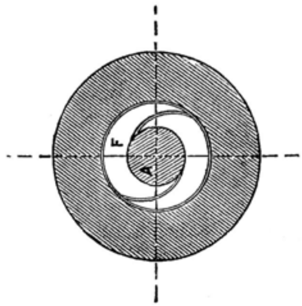


Fig. 6.



A. The copper.
B. The chamber.
C. The piston.
D. The gas-check.
F. The watch-spring.
H. The nozzle.

THE CRUSHER GAUGE.

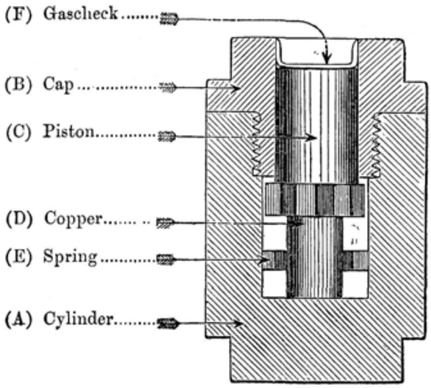
Table giving the lengths of copper cylinders, 0·326 inch in diameter ($\frac{1}{12}$ of a square inch sectional area), and corresponding pressure per square inch in a Crusher Gauge, the piston of which is 0·461 inch in diameter ($\frac{1}{6}$ of a square inch sectional area).

Length.	Pressure.	Length.	Pressure.	Length.	Pressure.	Length.	Pressure.
Inches.	Tons/in.^{2}	Inches.	Tons/in.^{2}	Inches.	Tons/in.^{2}	Inches.	Tons/in.^{2}
0·500	0·0	0·412	10·0	0·324	16·8	0·236	25·9
·498	1·6	·410	10·2	·322	17·0	·234	26·2
·496	2·2	·408	10·3	·320	17·1	·232	26·5
·494	2·7	·406	10·5	·318	17·3	·230	26·8
·492	3·0	·404	10·6	·316	17·5	·228	27·1
·490	3·2	·402	10·8	·314	17·6	·226	27·4
·488	3·5	·400	11·0	·312	17·8	·224	27·8
·486	3·7	·398	11·1	·310	18·0	·222	28·1
·484	4·0	·396	11·3	·308	18·2	·220	28·5
·482	4·2	·394	11·4	·306	18·3	·218	28·8
·480	4·4	·392	11·5	·304	18·5	·216	29·2
·478	4·6	·390	11·7	·302	18·7	·214	29·6
·476	4·8	·388	11·9	·300	18·8	·212	30·0
·474	5·0	·386	12·0	·298	19·0	·210	30·4
·472	5·1	·384	12·2	·296	19·2	·208	30·8
·470	5·2	·382	12·3	·294	19·3	·206	31·2
·468	5·5	·380	12·5	·292	19·5	·204	31·6
·466	5·6	·378	12·6	·290	19·7	·202	32·0
·464	5·8	·376	12·7	·288	19·8	·200	32·5
·462	5·9	·374	12·9	·286	20·0	·198	32·9
·460	6·1	·372	13·1	·284	20·2	·196	33·4
·458	6·3	·370	13·2	·282	20·4	·194	33·9
·456	6·4	·368	13·3	·280	20·6	·192	34·4
·454	6·6	·366	13·5	·278	20·8	·190	34·9
·452	6·8	·364	13·6	·276	20·9	·188	35·4
·450	6·9	·362	13·8	·274	21·1	·186	35·9
·448	7·1	·360	14·0	·272	21·3	·184	36·4
·446	7·3	·358	14·1	·270	21·5	·182	36·9
·444	7·5	·356	14·3	·268	21·7	·180	37·4
·442	7·6	·354	14·4	·266	21·9	·178	37·9
·440	7·8	·352	14·6	·264	22·2	·176	38·5
·438	8·0	·350	14·7	·262	22·4	·174	39·1
·436	8·1	·348	14·9	·360	22·6	·172	39·7
·434	8·3	·346	15·0	·258	22·9	·170	40·2
·432	8·4	·344	15·2	·256	23·2	·168	40·7
·430	8·6	·342	15·3	·254	23·4	·166	41·2
·428	8·7	·340	15·5	·252	23·6	·164	41·9
·426	8·9	·338	15·7	·250	23·9	·162	42·3
·424	9·1	·336	15·9	·248	24·2	·160	42·9
·422	9·2	·334	16·0	·246	24·5	·158	43·5
·420	9·4	·332	16·1	·244	24·7	·156	44·1
·418	9·6	·330	16·3	·242	25·0	·154	44·8
·416	9·7	·328	16·5	·240	25·3	·152	45·5
·414	9·9	·326	16·6	·238	25·6	·150	46·1

Table giving the Correcting Fraction for Hardness or Softness corresponding to certain lengths before firing, and to be applied to the pressure corresponding to the length after firing.

Nominal Pressure before Firing 15 tons/in.^{2}		Nominal Pressure before Firing 12 tons/in.^{2}		Nominal Pressure before Firing 9 tons/in.^{2}		Nominal Pressure before Firing 6 tons/in.^{2}	
Length before Firing.	Fraction Soft—S. Hard—H.	Length before Firing.	Fraction Soft—S. Hard—H.	Length before Firing.	Fraction Soft—S. Hard—H.	Length before Firing.	Fraction Soft—S. Hard—H.
Inches.		Inches.		Inches.		Inches.	
·338	$\frac{7}{150}$ S.	·378	$\frac{1}{20}$ S.	—	—	·458	$\frac{1}{20}$ S.
·339	$\frac{1}{25}$ S.	·379	$\frac{11}{240}$ S.	—	—	·459	$\frac{1}{30}$ S.
·340	$\frac{1}{30}$ S.	·380	$\frac{1}{24}$ S.	·419	$\frac{1}{18}$ S.	·460	$\frac{1}{60}$ S.
·341	$\frac{2}{75}$ S.	·381	$\frac{1}{30}$ S.	·420	$\frac{2}{45}$ S.	·461	Correct
·342	$\frac{1}{50}$ S.	·382	$\frac{1}{40}$ S.	·421	$\frac{1}{30}$ S.	·462	$\frac{1}{60}$ H
·343	$\frac{1}{60}$ S.	·383	$\frac{1}{48}$ S.	·422	$\frac{1}{45}$ S.	·463	$\frac{1}{40}$ H.
·344	$\frac{1}{75}$ S.	·384	$\frac{1}{60}$ S.	·423	$\frac{1}{60}$ S.	·464	$\frac{1}{30}$ H.
·345	$\frac{1}{150}$ S.	·385	$\frac{1}{120}$ S.	·424	$\frac{1}{90}$ S.	·465	$\frac{1}{20}$ H.
·346	Correct.	·386	Correct.	·425	Correct.		
·347	$\frac{1}{300}$ H.	·387	$\frac{1}{240}$ H.	·426	$\frac{1}{90}$ H.		
·348	$\frac{1}{150}$ H.	·388	$\frac{1}{120}$ H.	·427	$\frac{1}{45}$ H.		
·349	$\frac{1}{75}$ H.	·389	$\frac{1}{60}$ H.	·428	$\frac{1}{30}$ H.		
·350	$\frac{1}{50}$ H.	·390	$\frac{1}{40}$ H.	·429	$\frac{7}{180}$ H.		
·351	$\frac{7}{300}$ H.	·391	$\frac{1}{30}$ H.	·430	$\frac{7}{45}$ H.		
·352	$\frac{2}{75}$ H.	·392	$\frac{1}{24}$ H.	·431	$\frac{1}{18}$ H.		
·353	$\frac{1}{30}$ H.	·393	$\frac{11}{240}$ H.	—	—		
·354	$\frac{1}{25}$ H.	·394	$\frac{1}{20}$ H.	—	—		
·355	$\frac{13}{300}$ H.						
·356	$\frac{7}{150}$ H.						

Fig. 7.



With ordinary guns the chamber pressure only can be obtained, and the service pattern of gauge used for this is shown in fig. 7. It consists of a short steel cylinder, containing the same fittings as those already described. In guns loaded by hand, one or two of these gauges are placed at the rear end of the chamber, nozzle towards the muzzle, after loading B.L., and before loading M.L. guns; the object is to prevent their being blown out on firing.

They are generally found in the bore a little distance in front of the forward end of the chamber; but if, as may happen occasionally, they are blown out, they will be found a few yards from the muzzle of the gun.

Crusher gauges are not to be used when firing S.P. powder in 4·7 Q.F. guns, nor when firing E.X.E. powder in 6-inch Q.F. guns.

In guns loaded by hydraulic power, both M.L. and B.L., it is advisable to place the gauge inside the cartridge (taking care that it shall be at the extreme rear end of the chamber), removing a pebble or prism, if necessary, to enable this to be done.

In heavy Q.F. guns, when firing cordite, the lid of the cartridge case must be first carefully removed and the charge and wads taken out; the gauge should then be put in at the base of the case and the charge, rods and lid replaced. The cartridge should be carried base downwards and inserted carefully into the gun, to prevent the gauge shifting forward. A special tool is now supplied for securing the lid.

When taking pressures in 6-in. B.L. guns, and in 12·5-inch Mark II and 16-inch R.M.L. guns, two crusher-gauges are always used in each gun for each round, so that the results obtained may check each other.

The following points should be attended to when using crusher gauges:—

(*a*) That the copper is placed fair in the gauge, and is not tilted.
(*b*) That the piston is pressed down on the copper, and does not return again from the compression of the air in the gauge.
(*c*) That the piston is free to move—not tight. It should be capable of being moved to and fro by the finger; a little Russian tallow applied to the piston, and also to the gas-check, facilitates the action and helps to keep the gas out.

(*d*) The gauge should be examined from time to time, to see that the piston hole is not conical instead of cylindrical, as it is found that the pressure has a tendency to make the piston hole smaller at the outer end, and, when this happens, the hole should be brought to size by careful lapping with emery.

The micrometer used for measuring the coppers consists of a brass frame, in the upper part of which is a slide, moved laterally by means of a finely threaded micrometer screw; the copper to be measured is placed between two steel faces, one attached to the frame, and the other to the slide; the slide is fitted with a spring to prevent the instrument being strained by its being screwed down too hard.

The scale marked on the side of the frame is graduated in inches, subdivided into tenths as shown by the long lines; each tenth is further subdivided into five parts, the length of each small division being, therefore, one-fiftieth of an inch.

The vernier scale placed on the slide is divided into 20 equal parts, and, as its total length corresponds to 19 of the smaller divisions on the fixed scale, it enables twentieths of these to be read, or measurements of $\frac{1}{1000}$th of an inch to be made.

A small magnifying glass has to be used for reading the vernier.

Before measuring, care should be taken that the ends of the copper and faces of the instrument are wiped clean, and free from dust or grit, which might give an incorrect reading.

The length having been thus obtained, look out in the table under the heading "length," and the corresponding pressure will be found in the next column.

Copper cannot always be obtained in the market of the same uniform hardness as the sample for which the table for the crusher-gauge was drawn up. The half-inch cylinders are therefore subjected in a machine to pressures corresponding to 9, 12, and 15 tons per square inch, and those cylinders that after this treatment register a pressure differing from the tabular amount by more than 5 per cent. are rejected as unfit for use.

Thus no copper should be accepted that, having been pressed under 12 tons in the machine, recorded by the table either less than 11·4 tons or more than 12·6 tons.

Coppers should be selected that have been pressed to about a ton under the pressure expected in the gun.

For the coppers that fall between the limits of rejection a correction is made as shown in the two following examples:—

(1) A 12-ton copper before firing shows a length of 0·384 inch, and, after firing, a length of 0·348.

It will be seen from the table of corrective fractions that for a length of 0·384 the copper is $\frac{1}{60}$th soft, and a reduction of $\frac{1}{60}$th per ton must therefore be made from the pressure recorded after firing. From the length pressure table we see that a length of 0·348 corresponds to a pressure of 14·9 tons in.^{2}, then 14·9 tons less $\frac{1}{60}$th of 14·9 gives the true pressure, viz., 14·7 tons in.^{2} (neglecting hundredths).

(2) A 12-ton copper before firing shows a length of 0·390 inch, and after firing 0·348 inch. In this case the copper is $\frac{1}{40}$ hard, and the true pressure becomes 15·2 tons in.^{2}.

As a preliminary step to the determination of the pressure of fired gunpowder in the bore of a gun, it is desirable to record the pressure obtained by exploding charges of powder in a closed explosion vessel, varying the gravimetric density (p. 97).

The earliest experiments of this nature on the pressure of fired gunpowder are due to Benjamin Robins in 1743, and similar investigations were carried out subsequently by the Chevalier D'Arcy, 1760, and by Count Rumford in 1792. Recently the methods of Robins and Rumford have been revised by Dr. Kellner, War Department Chemist, who employed the steel spheres of bicycle ball bearings as safety valves, loaded to register the pressure at which the powder gases will blow off, and thereby check the indications of the crusher gauge (Proc. R. S., March, 1895).

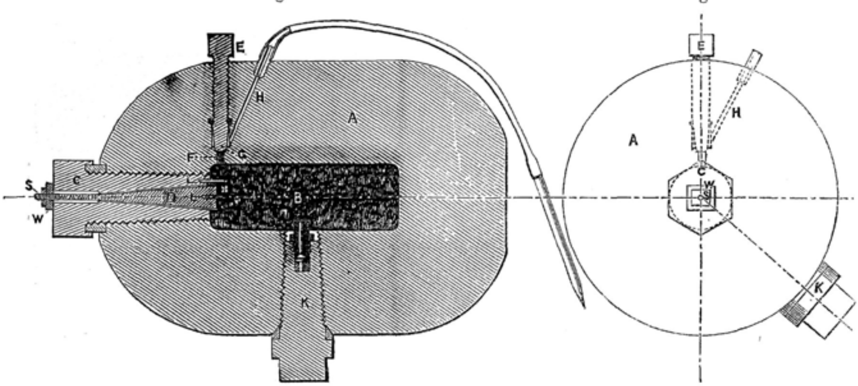
But the most modern results employed with gunpowder are based on the experiments of Sir Andrew Noble and Sir Frederick Abel (Phil. Trans., 1875, 1880, 1892, 1894). They proceeded as follows:—Charges of powder, whose different gravimetric densities were known, were exploded in a very strong chamber of mild steel, and the pressure each time was noted by means of an enclosed crusher gauge, and recorded, and the permanent gases were afterwards drawn off and examined.

The principal apparatus used by Captain Noble and Sir F. Abel for their experiments on fired gunpowder held some $2\frac{1}{4}$ lbs. of gunpowder, and is best described in their own words as follows:—

In figs. 8 and 9 (A) is a mild steel vessel of great strength, carefully tempered in oil, in the chamber of which (B) the charge to be exploded is placed.

The main orifice of the chamber is closed by a screwed plug (C), called the firing plug, which is fitted and ground into its place with great exactness.

Fig. 8. Fig. 9.



In the firing plug itself is a conical hole, which is stopped by the plug D, also ground into its place with great accuracy. As the firing plug is generally placed on the top of the cylinder, and as, before firing, the conical plug would drop into the chamber if not held, it is retained in position by means of the set-screw S, between which and the cylinder a small washer (W) of ebonite is placed. After firing the cone is, of course, firmly held, and the only effect of internal pressure is more completely to seal the aperture. At E is the arrangement for letting the gases escape; the small hole (F) communicates with the chamber where the powder is fired, and perfect tightness is secured by means of the mitred surface (G).

When it is wished to let the gases escape, the screw (E) is slightly withdrawn, and the gas passes into the passage H.

At K is placed the crusher apparatus for determining the pressure at the moment of explosion.

When it is desired to explode a charge, the crusher apparatus, after due preparation, is first carefully screwed into its place, and the hole (F) closed. The cone in the firing plug is covered with the finest tissue paper, to act as an insulator.

The two wires (L, L), one in the insulated cone, the other in the cylinder, are connected by a very fine platinum wire passing through a small glass tube filled with mealed powder. Upon completing connection with a Daniell's battery the charge is fired.

The only audible indication of the explosion is a slight click; but frequently, upon approaching the nose to the apparatus, a faint smell of sulphuretted hydrogen is perceptible.

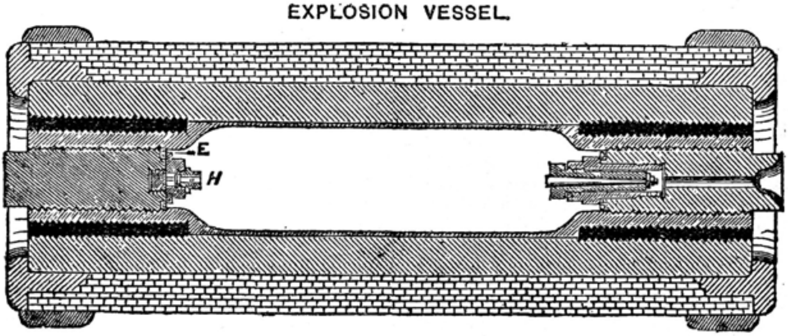
Great care was necessary in exploding the powder in this chamber, and any looseness of screws at once gave an exit to the gas, which washed away the metal of the threads in its rapid rush. When such a state of things occurred, the metal had apparently been fused.

The use of improved carefully tempered mild steel gave these experimenters an advantage over their predecessors, as it enabled them to explode larger charges and obtain higher pressures without risk of breaking the apparatus.

Fig. 10 shows an explosion vessel, which is even stronger made, being wound with steel wire.

Fig. 10.

EXPLOSION VESSEL.



The method of deducing the temperature of explosion from the data obtained by experiment is explained in the authors' paper; the calculations were roughly verified by the following observed facts:—

(1.) The explosion chamber was put into a water calorimeter, and the quantity of heat developed on firing was determined in the usual manner. The composition of the gases and residue being found from analysis, and the specific heats of all the constituents being known, a calculation of the temperature of explosion was made, which, however, gave a much higher result than that previously obtained. But the experimenters explain that (judging from analogy) the specific heat of the solid residue, which they examined when cold, would probably be greatly increased when it assumed the liquid form under the heat of explosion; they had no means of determining this point with certainty. Taking this into consideration, the agreement seemed good.

(2.) Thin platinum wire and foil were put into the chamber, and after explosion small parts showed signs of the beginning of fusion; but there was no appearance of volatilisation, which can be effected by the blowpipe at about 3700° C. (6692° F.). Platinum melts at about 2000°C. (3632° F.).

Gravimetric Density.

The **gravimetric density** of a charge of powder in the chamber of a gun is the ratio of its weight to the weight of that volume of water which would fill the space behind the projectile in the gun. It is the mean specific gravity of the grains of powder and of all the interstitial and other spaces; or it is the specific gravity of the gaseous products which fill the chamber when the gunpowder is fired.

When a charge of P lb. is placed in the chamber of volume C cubic inches, the density of the loading is $P \div C$, in pounds per cubic inch ($lb./in.^3$); and to convert this density into specific gravity, we must multiply by 27·73, since the gallon, of 10 lb. of water at 62° F., is 277·3 cubic inches; thus the gravimetric density, or

$$G.D. = 27{\cdot}73\,\frac{P}{C}.$$

The reciprocal of the G.D. is employed in Table XIV, where it was called the number of volumes, or the volumes of expansions; this may be called the *gravimetric* volume (G.V.); thus

$$G.V. = \frac{C}{27{\cdot}73\,P}.$$

With metric units, if P is in grammes and C is cubic centimetres,

$$G.D. = \frac{P}{C},\ G.V = \frac{P}{C},$$

and no factor, like 27·73, is required.

A gun charge is expressed thus at the head of a Range Table:

$$75\ P^2 \frac{33}{0{\cdot}840},$$

which means a charge of 75 pounds of P^2 powder, with 33 cubic inches allotted to each pound of gunpowder when in the chamber, and a consequent gravimetric density of 0·840; and

$$C = 75 \times 33 = 2475\ in.^3,$$

while the $$G.V = 33 \div 27{\cdot}73 = 1{\cdot}19,$$

the reciprocal of the G.D. 0·84.

From the observed pressure in the explosion chamber corresponding to given air-spacing or to a given G.D. (gravimetric density) of the powder, Captain Noble has plotted a curve of pressure (figs. 15, 16, p. 108), and thence deduced the amount of work in foot-tons capable of being done by one pound of powder, as the G.D. changes from unity to the G.D. of the products of combustion which fill the bore or any fraction of the bore (Table XIV), or as the volume changes from unity to the reciprocal of the corresponding G.D, which is called the *gravimetric volume* (G.V.).

The G.D. of the products of combustion which fill the bore is

$$\frac{\text{number of pounds of powder in the charge} \times 27{\cdot}73}{\text{volume of the whole of the bore in cubic inches.}}$$

Suppose, for instance, that the cross section of the bore of a gun is 27·73 square inches, corresponding to a calibre of nearly 6 inches.

Then 1 lb. of powder of unit G.D. would occupy 1 inch length of the bore; and in expanding to 15 times its volume, it would drive the projectile 14 inches, and the G.D. of the products of combustion would fall from 1 to $\frac{1}{15} = 0·067$; and, according to Table XIV, the work done by the expansion of the powder would be 131·97 foot-tons.

In expanding through five times its original volume, from 4·9 to 5·1 volumes, the projectile advances 0·2 inch, or $\frac{1}{60}$ of a foot; and the work done is, according to the Table XIV,

$$92·186 - 90·565 = 1·621 \text{ foot-tons.}$$

If P denotes the average thrust of the powder in tons, then

$$Ps = \frac{P}{60} = 1·621,$$

so that $$P = 97·26 \text{ tons};$$

and if this thrust is due to a pressure of p tons/in.², exerted over an area $A = 27·73$ in.²,

$$p = P/A = 3·5, \text{ tons/in.}^2;$$

and this is the pressure recorded in the experiments of Noble and Abel, when gunpowder is exploded in a closed vessel, at G.D. 0·2.

Conversely, from the experimental values of p, the value of the work done by the expansion of powder was calculated and tabulated in Table XIV.

Pressures in Closed Vessels Observed and Calculated (figs. 14, 15, *p.* 108).

Density of products of combustion.	Volume.	Pressure observed in explosion vessels.	Pressure calculated.
		Tons per square inch.	
·90	1·11	32·46	32·460
·80	1·25	25·03	25·525
·70	1·43	19·09	20·024
·60	1·66	14·39	15·554
·50	2·00	10·69	11·851
·40	2·50	7·75	8·732
·30	3·33	5·33	6·071
·20	5·00	3·26	3·771
·10	10·00	1·47	1·765
·05	20·00	0·70	·855

If the powder charge has a G.D. less than unity, the corresponding number of foot-tons must be deducted from the number of foot-tons against the G.D. of the products of combustion which fill the bore, to obtain the work done per pound of the powder-charge in the bore.

The methods of using Table XIV are best illustrated by examples:—

In the following examples let

C denote the volume of chamber in cubic inches.

B " " bore " "

G.D. denote the gravimetric density, and G.V. the gravimetric volume, the reciprocal of the G.D.

P the weight of the charge in lbs.

Then G.D. of powder charge $= 27{\cdot}73\,\frac{P}{C}$.

G.D. of products of combustion $= 27{\cdot}73\,\frac{P}{B}$.

Let w denote the weight of the projectile in lbs., and V the muzzle velocity in f/s.

These examples are worked out for gunpowder, now obsolete, but the method will be the same with tables for modern explosives, such as cordite, when these tables are published.

But it is found that the results can be applied to cordite, by using a factor of effect of 3 to 4; so that a charge of gunpowder can be reduced to one-third to one-quarter of its weight of cordite, to produce the same effect.

EXAMPLE 1.

Suppose the volume of the bore of a gun to be 1386·5 cubic inches, and the charge 10 lbs. of powder with G.D. 0·8. Find the total theoretical work which can be put into the projectile.

Here the G.D. of the products of combustion

$$= \frac{27{\cdot}73 \times 10}{1386{\cdot}5} = \frac{1}{5},$$

so that the G.V. of the charge increases from $1 \div 0{\cdot}8 = 1{\cdot}25$ to 5; and the work done per lb. of powder is, from Table XIV,

$$91{\cdot}385 - 19{\cdot}226 = 72{\cdot}16 \text{ foot-tons.}$$

This must be multiplied by the number of pounds in the charge to obtain the total theoretical work which can be put into the projectile; in this case it is

$$72{\cdot}16 \times 10 = 721{\cdot}6 \text{ foot-tons.}$$

Only a fraction of this, called the **factor of effect**, is, however, really obtained. According to Mr. Longridge, the factor of effect is due principally to the time required for the pressure of the gas to reach a maximum, and for complete combustion of the charge, so that the actual pressure curve is like CPD in fig. 4, instead of like the upper dotted curve implied in the direct employment of Table XIV, which assumes that the charge was completely consumed before any appreciable movement of the shot.

Thus the area between these two pressure curves must be deducted to obtain the work realised by the expansion of the powder; and this work deducted by cutting off the tip of the theoretical curve, may amount to 20 or 30 °/ₒ of the theoretical work given by Table XIV.

Suppose in the case we are considering the factor of effect is 0·7, the total work realised is

$$0{\cdot}7 \text{ of } 721{\cdot}6 = 505{\cdot}12 \text{ foot-tons.}$$

EXAMPLE 2.

A 10-inch B.L. gun was fired with a full charge of 252 lbs. and then with a three-quarter charge of 189 lbs. of powder; gravimetric densities 0·835 and 0·626 respectively. The gravimetric volumes of the powder charges were

$$\frac{1}{0\cdot835} \text{ and } \frac{1}{0\cdot626}, \text{ or } 1\cdot2 \text{ and } 1\cdot6,$$

The capac. ty of bore being 29,300 cubic inches, B = 29,300, the G.V. of the products of combustion is

$$\frac{29300}{252 \times 27\cdot73} \text{ and } \frac{29300}{189 \times 27\cdot73}, \text{ or } 4\cdot2 \text{ and } 5\cdot6.$$

The work done per lb. of powder charge was thus, in the two cases—

$$84\cdot176 - 16\cdot063 = 68\cdot113 \text{ foot-tons,}$$
$$95\cdot925 - 36\cdot086 = 59\cdot839 \quad \text{,,}$$

The total theoretical amount of work would therefore be in each case—

$$\left.\begin{array}{l} \text{for the 252-lb. charge, } 68\cdot113 \times 252 = 17160 \text{ tons} \\ \text{,,} \quad 189 \quad \text{,,} \quad 59\cdot839 \times 189 = 11310 \quad \text{,,} \end{array}\right\} \text{(i).}$$

The velocities with each charge, viz., 2040 and 1735 f/s, were found from experiment, and the weight of the projectiles being 500 lbs., the energy actually developed in each was found from the expression

$$\frac{wV^2}{2g \times 2240}$$

to be, taking $g = 32$—

$$\left.\begin{array}{l} \text{for the 252 lbs. charge} = 14520 \text{ foot-tons;} \\ \text{and for the 189 lbs. charge} = 10500 \quad \text{,,} \end{array}\right\} \dots \text{(ii)}$$

dividing the realized work in (ii) by the theoretical work in (i) in each case, factors of effect are obtained of

0·85 and 0·93.

Knowing the work put into the projectile, we can calculate its velocity from the relation—

$$\frac{wV^2}{2g \times 2240} = \text{energy in foot-tons,}$$

and thus the probable velocity in experimental guns has been estimated beforehand with fair accuracy.

EXAMPLE 3.

At what velocity must a projectile move to have half the energy which it had when travelling at 1000 f/s?

If v be the required velocity, then

$$\frac{w\,(1000)^2}{2g} = \frac{2wv^2}{2g},$$

whence $$v^2 = \frac{1000^2}{2} = 50{,}000;$$

or $$v = 707{\cdot}1 \text{ f/s}.$$

EXAMPLE 4.

Suppose a 12-pr. projectile has M.V. of 1710 f/s,
and a 15-pr " " 1569 f/s;

compare their energies.

For 12-pr. For 15-pr.

$$\frac{12{\cdot}5(1710)^2}{2g \times 2240} : \frac{14{\cdot}06(1569)^2}{2g \times 2240},$$

or $$253{\cdot}5 \quad : \quad 240 \text{ foot-tons.}$$

$\therefore$ Hence the 12-pr. has rather higher muzzle energy than the 15-pr.

EXAMPLE 5.

If the M.V. of a filled common shell of 25 lbs. weight is 1900 f/s, what will be the M.V. if the shell is fired empty? Weight of bursting charge 2·875 lbs.

The weight of empty shell $= 25 - 2{\cdot}875$ lbs.
$= 22{\cdot}125$ lbs.

Assume the amount of work given to the projectile to be the same in each case as the expansions are the same.

$$\therefore \frac{25(1900)^2}{2g \times 2240} = \frac{22{\cdot}125\, V^2}{2g \times 2240},$$

whence $$V = 2019 \text{ f/s},$$

an increase of 119 f/s.

EXAMPLE 6.

A certain charge with an experimental field gun gave a muzzle velocity of 1670 f/s to its 12-lb. projectile, when the calibre was 3 inches; but when the calibre was increased to 3·2 inches (with the same weight of projectile and charge) the M.V. was 1700 f/s. Why was this increase?

The capacity of the bore was enlarged, and the number of expansions of the powder charge increased: hence more work, involving a greater velocity, was given to the projectile.

EXAMPLE 7.

If a charge of 48 lbs. of gravimetric density 0·976 is allowed 3·82 expansions in the bore of a gun, what wil be the M.V. of the projectile if its weight is 100 lbs. and the factor of effect of the gun is 0·715?

The G.D. of the charge diminishes from

$$0{\cdot}976 \text{ to } 1 \div 3{\cdot}82,$$

or the volume increases from

$$1 \div 0{\cdot}976 = 1{\cdot}025 \text{ to } 3{\cdot}82;$$

so that the work done per lb. of powder is

$$80{\cdot}110 - 2{\cdot}403 = 77{\cdot}707 \text{ foot-tons.}$$

∴ for 48 lbs. the total theoretical work is 3730 foot-tons.

The factor of effect being taken as 0·715, the work actually realised is

$$3729 \times 0{\cdot}715 = 2667 \text{ foot-tons.}$$

$$\therefore 2667 = \frac{wV^2}{2g \times 2440}.$$

Take $g = 32{\cdot}19, \log g = 1{\cdot}5077,$

then $V = 1960$ f/s,

the muzzle velocity of a 6-inch B.L. gun.

EXAMPLE 8.

Suppose, in the last example, that the projectile was not rammed home, and that consequently the space for the cartridge was doubled: find the M.V. to be expected.

The volume now increases from 2·05 to 3·8, so that the work done per lb. of powder is

$$80{\cdot}110 - 50{\cdot}383 = 29{\cdot}727 \text{ foot-tons,}$$

and by 48 lb. of powder is 1427 foot-tons.

Using the same factor of effect we get

$$1427 \times 0{\cdot}715 \text{ or } 1021 \text{ foot-tons of work realised.}$$

$$\therefore 1021 = \frac{100\ V^2}{2 \times 32{\cdot}19 \times 2240}$$

$$\therefore V = 1213 \text{ f/s.}$$

—a considerable decrease of velocity to that attained in Example 1 with the same weight of projectile and charge.

EXAMPLE 9.

An experimental gun of 9·2-inch calibre is to be designed to fire a projectile of 380 lbs. with M.V. of 2000 f/s.

How can the charge and length of bore be determined?

Find $\frac{wV^2}{2g \times 2240}$; it is 10,600 ft.-tons, taking $g = 32$.

Assume a factor of effect from previous experience with other guns of about the same calibre with the same powder, suppose it is 0·8.

Then the theoretical amount of work furnished by the charge is $10600 \div 0·8 = 13250$ foot-tons.

Now, suppose it is assumed that five expansions shall be given to the charge (consult Table XIV), we find that, if the gravimetric density of loading is unity, each lb. of powder then gives 91·385 foot-tons of work.

$$\therefore \frac{13250}{91·385} = 145 \text{ lbs. will be required.}$$

The length of bore of course follows: and if this is found to be inconvenient a different number of expansions must be assumed and fresh calculations made until the necessary conditions are fulfilled.

Further investigations are now in progress, carried out by Sir Andrew Noble, Sir Frederick Abel, and Professor Dewar, with the object of determining a corresponding Table of Work for different expansions, when cordite and other modern explosives are employed.

The results obtained by Sir Andrew Noble are shown diagrammatically in figs. 11, 12, 13, pp. 105–107.

In fig. 13 the effect of fouling in increasing the friction is very clearly shown. Round I was fired in a clean bore with a charge of R.L.G. powder, and the diminution of velocity in Rounds II and III is very manifest, but only when the length of bore exceeds 40 calibres.

The annexed Tables, extracted from Sir Andrew Noble's paper (Proc. R.S.,' June, 1894), show our latest knowledge of the energy and velocity realised in the experimental 6-inch gun, which could be lengthened as required from 40 up to 100 calibres, a length of 50 feet. also the pressures observed in the explosion chamber shown in figs. 14, 15, p. 108, according to the latest experiments.

It is found that the temperature of explosion is now much higher, but that this temperature is rapidly diminished by the communication of heat to the surrounding walls.

Thus Sir Andrew Noble finds that a charge of $1\frac{3}{4}$ lbs. of cordite, exploded in a closed vessel to a pressure of 6 tons/in.^{2}, or say 1000 atmospheres, reaches this pressure in about 0·07 second after explosion, but falls to 5 tons/in.^{2} in 0·171 second, to 4 in 0·731 second, to 3 in 1·764 seconds, to 2 in 3·323 seconds, and to 1 ton/in.^{2} in 7·08 seconds.

The high temperature of cordite has unfortunately a very powerful effect in the erosion of the gun; the metal of the surface of the bore appears to be washed away, as if melted by the high temperature: and the means to obviate this erosion are engaging at present the serious attention of artillerists.

TABLE showing the Velocity and Energy realised in a 6-in. Gun with the undermentioned Explosives.

Nature of explosive and weight of charge.	Length of bore, 40 calibres.		Length of bore, 50 calibres.		Length of bore, 75 calibres.		Length of bore, 100 calibres.	
	Velocity in f/s.	Energy in ft.-tons.	Velocity in f/s.	Energy in ft.-tons.	Velocity in f/s.	Energy in ft.-tons.	Velocity in f/s.	Energy in ft.-tons.
Cordite, 0·4-in. dia., 27·5 lbs.	2794	5413	2940	5994	3166	6950	3284	7478
Cordite, 0·35-in. dia., 22 lbs.	2444	4142	2583	4626	2798	5429	2915	5892
Cordite, 0·3-in. dia., 20 lbs.	2495	4316	2632	4804	2821	5518	2914	5888
Ballistite, 0·3-in. cubes, 20 lbs.	2416	4047	2537	4463	2713	5104	2806	5460
French B.N., 25 lbs.	2422	4068	2530	4438	2700	5055	2786	5382
Amide prismatic, 32 lbs.	2225	3433	2331	3768	2486	4285	2566	4566
R.L.G.$_2$, 23 lbs.	1533	1630	1592	1757	1668	1929	1705	2016

TABLE OF PRESSURE in Explosion Vessel.

GD = gravimetric density of products of combustion.	Volume.	Pressure in tons per square inch.	
		Pebble powder.	Cordite.
0·05	20·00	0·855	3·00
0·06	16·66	1·00	3·80
0·08	12·50	1·36	5·40
0·10	10·00	1·76	7·10
0·12	8·33	2·06	8·70
0·14	7·14	2·53	10·50
0·15	6·66	2·73	11·36
0·16	6·25	2·96	12·30
0·18	5·55	3·33	14·20
0·20	5·00	3·77	16·00
0·22	4·54	4·26	17·90
0·24	4·17	4·66	19·80
0·25	4·00	4·88	20·63
0·26	3·84	5·10	21·75
0·30	3·33	6·07	26·00
0·35	2·85	7·35	31·00
0·40	2·50	8·73	36·53
0·45	2·22	10·23	42·20
0·50	2·00	11·25	48·66
0·55	1·81	13·62	55·86
0·60	1·66	15·55	63·33
0·65	1·54	17·68	—
0·70	1·43	20·02	—
0·80	1·25	25·52	—
0·90	1·11	32·46	—
1·00	1·00	41·48	—

Fig. 11.

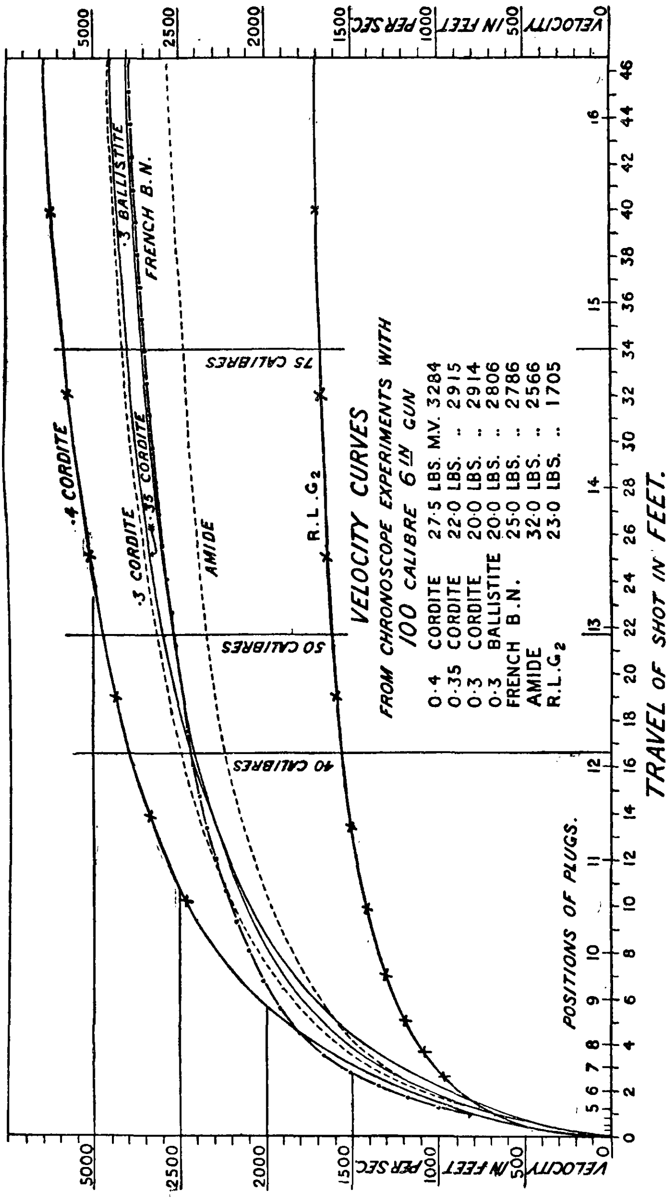

VELOCITY CURVES
FROM CHRONOSCOPE EXPERIMENTS WITH
100 CALIBRE 6 IN GUN
0·4 CORDITE 27·5 LBS. M.V. 3284
0·35 CORDITE 22·0 LBS. " 2915
0·3 CORDITE 20·0 LBS. " 2914
0·3 BALLISTITE 20·0 LBS. " 2806
FRENCH B.N. 25·0 LBS. " 2786
AMIDE 32·0 LBS. " 2566
R.L.G2 23·0 LBS. " 1705
·4 CORDITE
·35 CORDITE
·3 CORDITE
·3 BALLISTITE
FRENCH B.N.
AMIDE
R.L.G2
40 CALIBRES
50 CALIBRES
75 CALIBRES
VELOCITY IN FEET PER SEC
POSITIONS OF PLUGS.
TRAVEL OF SHOT IN FEET.

Fig. 12.

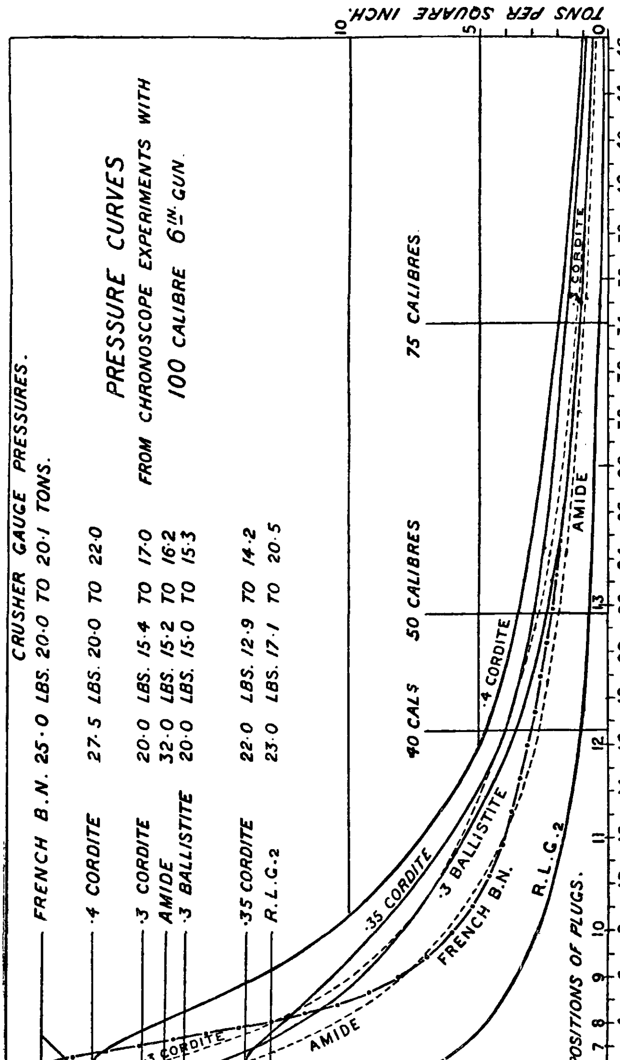

CRUSHER GAUGE PRESSURES.
FRENCH B.N. 25·0 LBS. 20·0 TO 20·1 TONS.
·4 CORDITE 27·5 LBS. 20·0 TO 22·0
·3 CORDITE 20·0 LBS. 15·4 TO 17·0
AMIDE 32·0 LBS. 15·2 TO 16·2
·3 BALLISTITE 20·0 LBS. 15·0 TO 15·3
·35 CORDITE 22·0 LBS. 12·9 TO 14·2
R.L.G.2 23·0 LBS. 17·1 TO 20·5
PRESSURE CURVES
FROM CHRONOSCOPE EXPERIMENTS WITH
100 CALIBRE 6IN. GUN.
40 CALS
50 CALIBRES
75 CALIBRES
·3 CORDITE
AMIDE
·35 CORDITE
·3 BALLISTITE
FRENCH B.N.
R.L.G.2
·4 CORDITE
POSITIONS OF PLUGS.
TRAVEL OF SHOT IN FEET.
PRESSURE IN TONS PER SQUARE INCH
TONS PER SQUARE INCH.

Fig. 13.

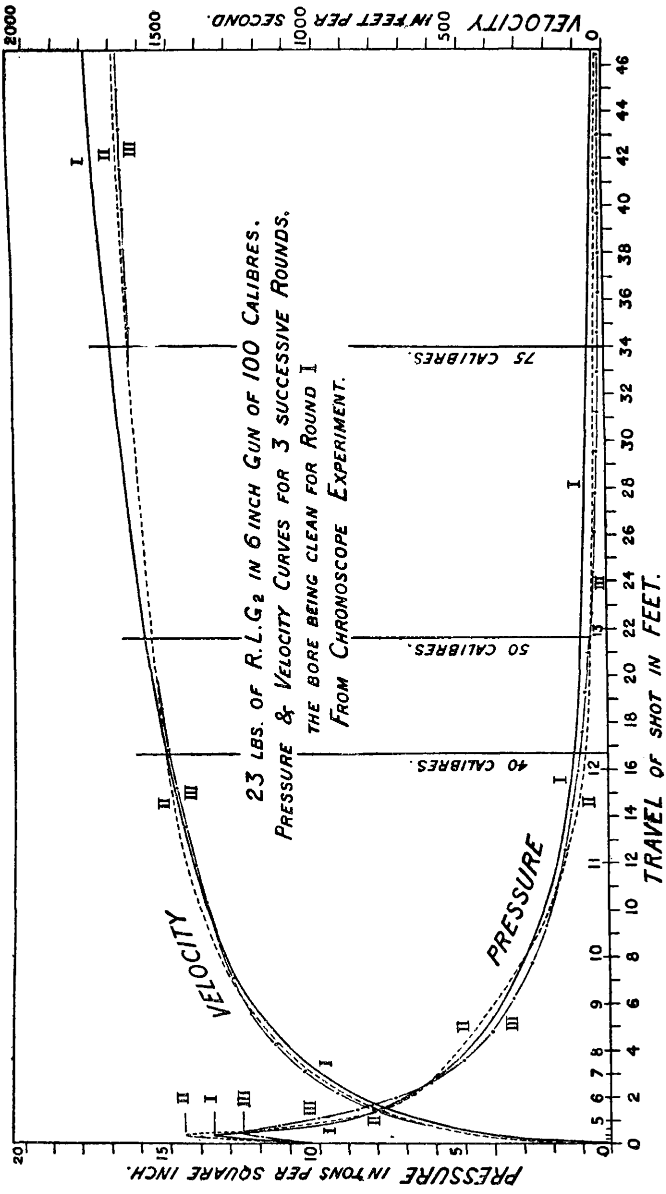

23 LBS. OF R.L.G2 IN 6 INCH GUN OF 100 CALIBRES.
PRESSURE & VELOCITY CURVES FOR 3 SUCCESSIVE ROUNDS.
THE BORE BEING CLEAN FOR ROUND I
FROM CHRONOSCOPE EXPERIMENT.
VELOCITY IN FEET PER SECOND.
PRESSURE IN TONS PER SQUARE INCH.
TRAVEL OF SHOT IN FEET.
VELOCITY
PRESSURE
40 CALIBRES.
50 CALIBRES.
75 CALIBRES.

Fig. 14.

PRESSURES IN CLOSED VESSELS OBSERVED AND CALCULATED.

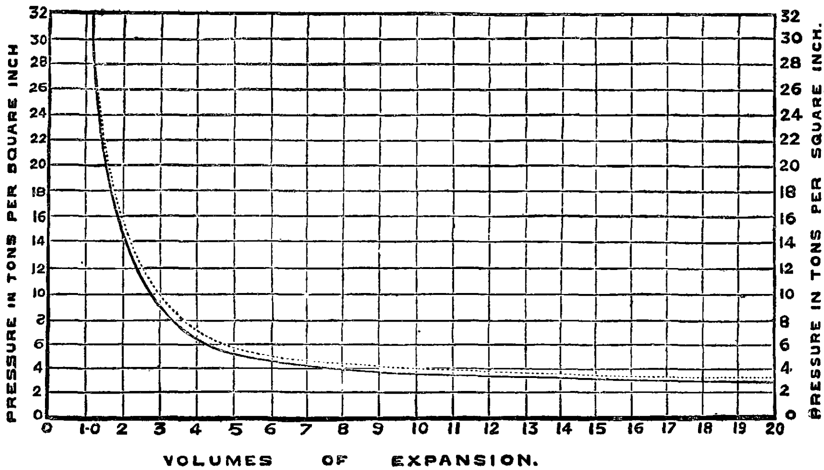
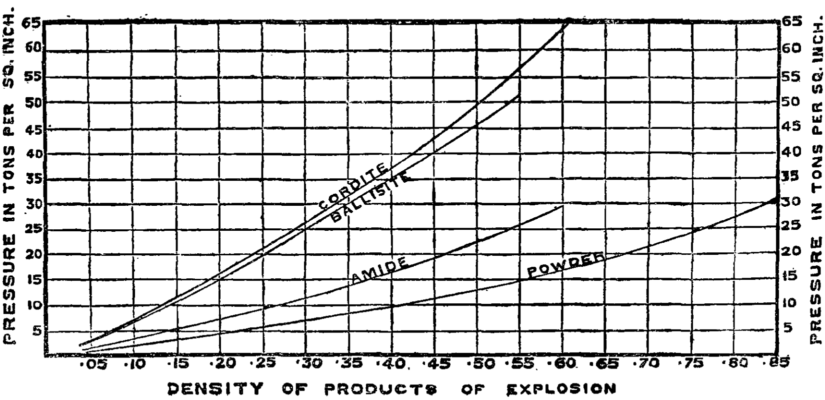


Fig. 15.

PRESSURES OBSERVED IN CLOSED VESSELS WITH VARIOUS EXPLOSIVES.



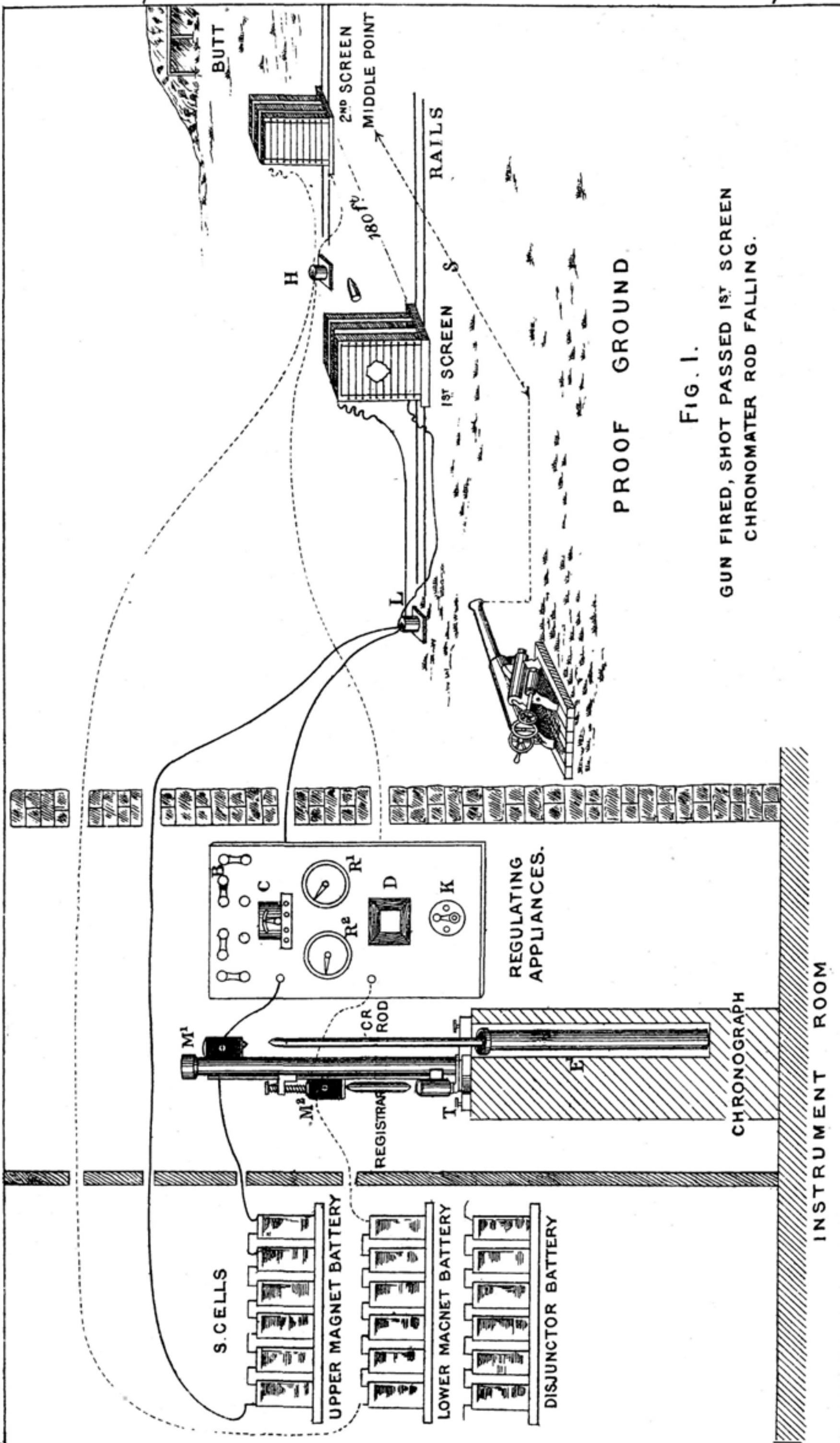


FIG. I.
GUN FIRED, SHOT PASSED 1ST SCREEN
CHRONOMATER ROD FALLING.

SECTION II.—MEASUREMENT OF VELOCITY.

For ordinary purposes, such as proving gunpowder, by finding what velocity a given charge will impart to a given projectile in gun, the **Boulengé chronograph** is employed.

In order to see to what accuracy time has to be measured in order to obtain the velocity of a projectile to a foot per second, take, for example, a shot whose mean velocity between two screens placed 180 feet apart is 1,800 feet per second; a variation of one above or below 1,800 feet per second is represented by a decrease or increase in time of only 0·00005 (five hundred thousandths or fifty millionths) of a second.

Such accuracy can only be obtained by a careful elimination of the sources of error in the instrument.

The original pattern of Le Boulengé was subsequently improved upon by Captain Bréger, of the French marine artillery, with a view to reduce the mechanical and electrical sources of error.

The general principle of its action remaining as before, the Boulengé-Bréger instrument has been improved recently by Major H. C. L. Holden, R.A.; besides alterations in the instrument itself, all the regulating appliances are now grouped together on a board; better forms of rheostats, and a much more accurate disjunctor have been fitted, and a commutator has been introduced, the use of which will be explained later on.

Fig. 2 shows the instrument as improved by Major Holden; fig. 3, is the new disjunctor; and fig. 4 is a diagram of the connections on the switch or instrument board.

The whole arrangement consists of two separate parts:—

(1.) That on the proof ground consisting of gun position, butt, and the mechanical arrangement whereby the electrical circuits are broken by the projectile, viz., screens and the system of electrical circuits (*see* the right hand side of fig. 1).

(2.) The instrument room containing the chronograph, batteries, regulating and testing appliances, also electric circuits in connection with those on the proof ground (as shown on the left).

The proof ground arrangements will now be described:—

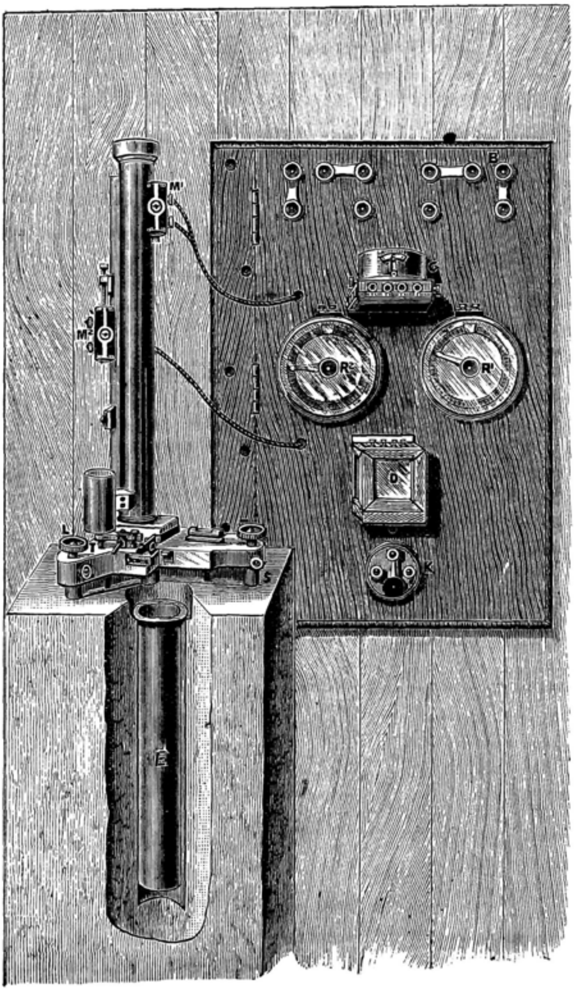
From the instrument room, about 300 yards distant, there are laid in iron pipes underground, a number of copper wires insulated with gutta-percha; these wires are severally connected to the instruments at one end, and on the proof ground are led up into strong cast-iron boxes H and L (fig. 1) above the surface of the ground and fixed to terminals on ebonite bars in the boxes; the terminals of the outgoing wires being fixed to one bar and those of the return wires to another, so as to secure the maximum insulation possible. From these terminals are taken short pieces of wire to the screens where the circuit is broken by the projectile. Thus for the two screens connected to each chronograph instrument there are necessarily four wires in all. The screens themselves consist of upright oblong wooden frames, about 8 feet high by 4 feet wide between the sides, and they are combined in sets of two or three on one stand, so that the records for two or three instruments may be obtained simultaneously and independently of each other.

The frames are provided on their two vertical faces with a series of insulated pegs, so that a wire can be stretched continuously backwards and forwards across the frame from top to bottom of it, for the shot to cut, the distance between each return of wire being always less than the diameter of the shot fired.

One end of the wire wound on the screen is attached to the outgoing wire from the terminal box, and the other end to the return wire. The general arrangement will be clearly seen by fig. 1 ; only, for the sake of clearness, the wires from the instruments to the terminal boxes are shown in the air instead of underground.

The screens slide sideways on rails fixed accurately 180 feet apart. The distance of the first screen from the muzzle of the gun being from 190 to 150 feet with guns of large calibre, though a much smaller distance is ample for light guns, on account of the disturbance due to their muzzle blast being less.

Fig. 2.



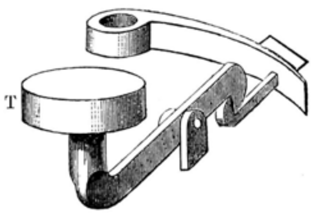
Passing now to the instrument room, the chronograph (fig. 2) consists of a substantial vertical brass pillar which supports two electro-magnets, M_1 on its right-hand side, and M_2 on its left. Of these two magnets, M_1 is fixed permanently, whilst M_2 can be moved up and down the pillar by means of quick and slow motions, and can be clamped temporarily in any position. The pillar stands on a triangular base which can be levelled by means of the levelling screws and two spirit levels at right angles to each other.

In the plate, the concrete pillar on which the instrument stands is cut away to show the full tube or receptacle, E, into which the rod falls on which the record is made, the "chronometer rod," as it is called.

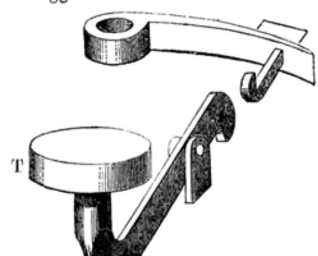
The concrete pillar should not touch the floor or walls of the building in which it is built, as if it did so, it would be liable to participate in the vibration.

The electro-magnets, M_1 and M_2, are exactly similar, mechanically, electrically, and magnetically, and they are capable of supporting from their conical-ended projecting iron cores two rods weighing 14 ounces each. These rods, though having exactly similar conical iron tips, or armatures, by which they are supported from the magnets, are, however, very different in appearance. The one that is supported by the magnet M_1 being some 22 inches long, and the other 5 inches long. They are both provided with bobs at their lower end to keep the centre of gravity as low as possible.

Rough sketch of Trigger.



(*a*) Cocked.



(*b*) Spring released so that the knife on it may mark the chronometer rod.

The longer, or "chronometer rod," when released from the magnet M_1 on the rupture of the circuit, falls vertically downward until arrested by the bottom of the fall tube; but the shorter rod or "registrar," supported by the magnet M_2 falls through a guard tube on to a trigger table, T, which is thereupon pressed down, releasing a spring which has affixed to it a cutting edge, and is situate on the right-hand side of the base of the pillar. This spring, carrying the cutting edge, moves to the right in a horizontal plane, and when it comes in contact with the falling chronometer rod makes a mark thereon which constitutes the record. On a board shown to the right of the instrument, in figs. 2, 4, the necessary instruments are fixed for adjusting it and checking its accuracy.

These consist of the commutator C, the two adjustable resistances R_1, R_2, the disjunctor D, and the disjunctor key K. The connections from the board to the instrument are made by twin flexible wires, such as used for incandescent electric lighting work.

Besides these, there are, of course, the batteries supplying the electric current, which are all installed separately in glass cupboards, and connected up to the back of the board, the terminals on which, seen at the top of the illustration, are used merely for testing purposes.

Each battery is composed of six cells of a special secondary element. As shown in fig. 1, separate batteries are employed for each magnet and also for each disjunctor.

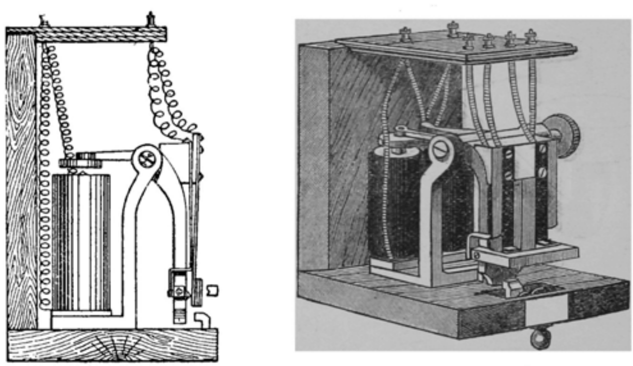
The accessory instruments for regulating and adjusting the chronograph are the next point. These have been enumerated, but we will now describe them and their functions in detail.

1. *The Rheostats R_1, R_2, or Adjustable Resistances.*—These are of circular form, and so arranged that, by moving the radial arm, resistance can be interposed or taken out of the circuit without breaking it. One rheostat is included in the circuit of each electro-magnet, and serves to regulate the current through the coil to a nicety. The maximum resistance that can thus be included in either circuit is 20 ohms, which gives an ample margin for practical requirements.

The use of the rheostats is described more fully under the head of "Adjusting the Instrument."

2. *The Disjunctor D.*—This is shown, with the cover removed, in Fig. 3. It consists of an electro-magnet, the armature of which forms one end of a swinging ˥-shaped frame, and is situate horizontally

Fig. 3.



above the poles of the magnet. The vertical, or other leg of the swinging frame, carries two flat steel springs of equal strength, separately insulated, and loaded at their lower extremities by brass weights, which in the position of rest make contact through iridium contact points with two fixed contacts also attached to the frame. A guardpiece extends from one side of the frame to the other, and prevents the springs moving more than a certain distance away from the contacts.

Thus we have two circuits—one through the one spring to the fixed contact, and the other through the second spring to the second fixed contact. These two circuits are terminated at the top of the wooden case by four binding screws, the connections from the swinging frame being made by four phosphor bronze spiral springs. The movement of the swinging frame is regulated by two stops, one of which is fixed and the other adjustable, so as to limit the arc through which it moves. By means of the key K, the current from a battery of six secondary cells can be sent through the electro-magnet's coils, whereupon the armature is attracted and the swinging frame moves suddenly until brought up by the fixed stop, when it is thus suddenly arrested. The springs, acting under the momentum acquired by their weights, continue to move on until arrested by the guard-piece, having at their first movement broken their respective circuits by leaving the fixed contacts.

Now, one of these circuits forms part of one of the chronograph magnet circuits, and the other one in the same manner forms part of the circuit of the other magnet.

We thus have the power in this piece of apparatus of breaking the two circuits suddenly and simultaneously. The object of this will be seen later on.

3. *The Commutator C.*—The commutator comes next under our consideration. This is a most important accessory, and fulfils two functions.

(1.) The cutting off of the current from the instrument when not actually in use.

(2.) The changing of the two magnet circuits through the disjunctor above described, so that either spring or contact may be in the circuit of either magnet, or *vice versâ*, the object of this being to check and be able to correct for any small error in the working of the disjunctor, such as dust in the contacts, unevenness in the power of the springs, &c., &c.

It consists of a horizontal board, with 12 holes in 3 rows of 4 each. Each of these holes contains mercury, and is in connection with a binding screw on the outside of the case. The four front screws are in connection with the central row of holes, and these are thus in direct connection with the two magnet circuits. The other connections will be clear from a consideration of the diagram of connections (fig. 4).

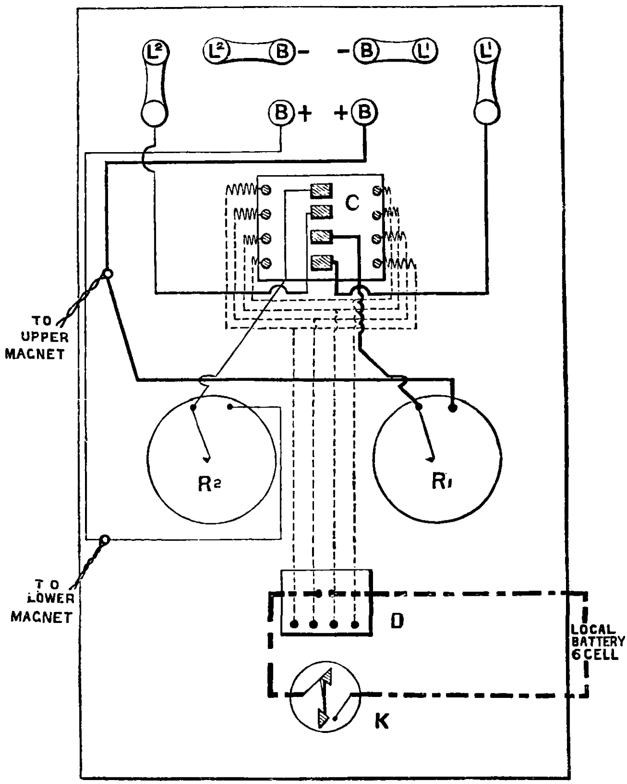
An insulated rocking arm, carrying four suitably-shaped pieces of copper, also insulated from each other, is placed immediately above the central line of mercury contacts; and by moving this to the right or left by means of the lever arm and indicator outside the case, the central holes can be connected to those on either side, or by placing the lever in the centre disconnected from either.

The wires leading from this piece of apparatus to the disjunctor are carefully arranged so that they are of the same length and resistance, with a view to the conditions of the circuit resistance remaining the same, which ever side the lever may be down.

The only other things on the board are the terminals; these are, as has been mentioned before, chiefly for testing purposes, so that the lines, batteries, or instrument magnets may be tested without disturbing the connections, which are securely made behind the board.

To the terminals, L_1 L_1, are connected the line wires to the screen nearest the gun—those of the farther screen being connected to the terminals L_2 L_2. By removing the connecting straps, which are seen between L_1 and $-B$ (the negative pole of the battery), and also between L_1 and the other terminal, the line wires are quite disconnected from the rest of the instrument, and can be tested. The condition of the battery can be tested by the application of a voltmeter to the terminals $-B$ and $+B$. The diagram of connections (fig. 4) almost explains itself; but, to make it even clearer, we will trace one circuit, say that through the first screen and instrument.

Fig. 4.—Diagram of connections.



In fig. 4 the circuit of the chronometer (upper magnet) is shown by a thick line, and that of the registrar (lower magnet) by a thin line.

The current from the battery enters at +B, and, after passing through the coil of the upper magnet, goes to the right-hand rheostat; from this it goes to the commutator, and afterwards, according to the side on which the lever of the commutator is down, to either the right or left-hand spring of the disjunctor. Leaving the disjunctor, and going through the commutator again, it leaves the instrument house by the strap connected to the right-hand L, which, as we have already said, is connected to the wire to the first screen, from which it returns by the other wire to the terminal, the left L_1, and thence by the strap connecting L_1 to −B, to the negative pole of the battery—its destination.

This, with the exception of some small working details, which are beyond our province, concludes the description of the chronograph instrument, and we will next treat of the *adjustment* before working, as well as the manner of using it.

Having in the first instance moved the lever arm of the commutator to the right or left, as the case may be (the screens are supposed to be prepared correctly and all other preparations made), the fact of the current passing or not, will be known by the small visual indicators, in the centre of each magnet, assuming a vertical position instead of a horizontal one as before. If either of them remains horizontal, it shows that there is a break in the circuit somewhere. It windy weather it not infrequently happens that there an intermittent break in the screen wire; this is immediately detected by the indicator, but it would be troublesome to discover it were the indicator not there. Assuming that so far everything is correct, the next thing to be done is to adjust the magnets, so that their strength is precisely the same, in the following manner:—The rheostats having both been set to zero, or their position of lowest resistance, the chronometer or registrar rods are hung up to their respective magnets, each having had in the first instance a small tubular weight slipped over them. The resistance of each circuit is then gradually increased, the chronometer circuit first, till the rods fall. The tubular weights having then been taken off, it will be found that the magnets are just capable of sustaining their respective rods, and that they are necessarily, from the mode of adjustment adopted, of exactly the same power.

Having suspended both the rods again, the disjunctor now comes into play; and here we must make a slight digression to explain one or two points that have hitherto not been touched upon. It has been already explained that when the first screen is broken the chronometer rod commences to fall; but this action is not instantaneous, as on the cessation of the electric current in the coils, supposing the latter were instantaneous (which it cannot be), the magnetism of the core has still to fall below a certain strength before the chronometer rod is released, and, therefore, there is an evident delay. A similar delay occurs in the release of the registrar rod; but in this case there are yet other effects to be taken into account. In the first place, the registrar rod falls some distance before it strikes the trigger table; then the trigger table has to release the spring and knife, and the latter has to move a certain distance before coming into contact with the chronometer rod.

To remove all these time errors mechanically is the function of the disjunctor. As we have seen, by pressing the disjunctor key we can break both the circuits instantaneously, and, moreover, simultaneously; this causes both the rods to fall, and the height up the chronometer rod where the mark is made when this action is performed, forms the means of eliminating the time errors, which would otherwise be caused.

For the sake of convenience, as will be seen hereafter, when we come to the question of the scale for reading velocities, the position of the disjunction line on the chronometer rod is made a fixed one, viz., 4·345 inches above the zero, equivalent to a time of fall of 0·15 of a second, from

$$h = 6g\,(0{\cdot}15)^2 \text{ in inches};$$

and the disjunction mark made by the instrument is adjusted by raising or lowering the registrar magnet M_2 on the pillar, so as to coincide exactly with the mark previously made on the rod.

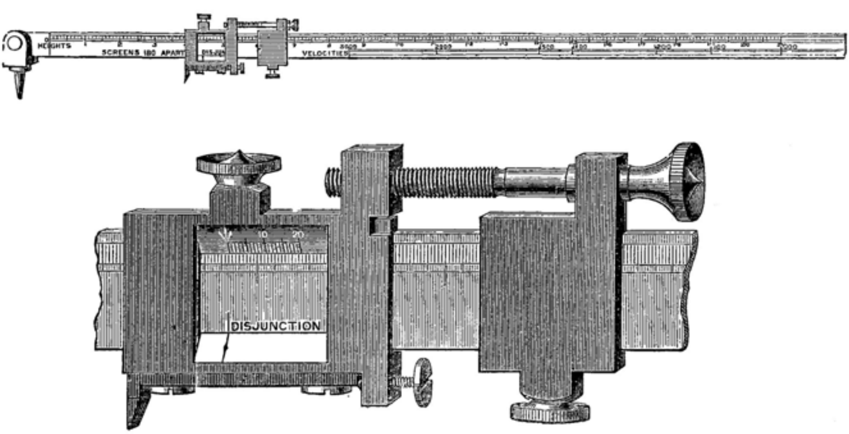
To check the disjunctor, all that is necessary is to repeat the above operation with the lever of the commutator on the opposite side. If the disjunctor is in correct adjustment, the two marks will be at the same height on the chronometer rod; if they are not, the disjunctor requires adjusting. The correct position, however, will be midway between the two marks.

We may here remark that the records are taken, not actually on the chronometer rod itself, but on a silvered copper tube, which fits it sufficiently tight so as not to slip about on it, and so change its position. Around this copper tube, which lasts some 50 rounds or more, and is then replaced by another, a fine line is made at a height of 4·345 inches above the zero, by means of a special instrument for the purpose.

The instrument having now been adjusted, and the disjunctor line, as it is called, established correctly, the gun is fired, the projectile passes through the screens, and the chronometer and registrar rods fall one after the other. The chronometer rod having been taken out of the fall tube and examined, it will be found that the mark on the rod is made some distance above the disjunctor line, and the distance of this mark above zero is a measure of the time the projectile has taken to pass between the screens plus the time the registrar takes to fall and the knife takes to act.

The distance is measured accurately by means of a scale (see fig. 5) which can be applied to the rod. It consists of a metal bar with a hinged end carrying a conical plug which fits into a socket prepared for it at the lower end of the chronometer rod, and having a zero point coinciding exactly with that of the instrument with which it is used; *i.e.*, with the point on the chronometer rod opposite to the knife when the rod is hung up. On the scale are two slides, either of which can be clamped with it. One is fitted with a micrometer screw, by which the other can be moved short distances for final adjustment of reading.

Fig. 5.



The scale is graduated on its upper edge in inches, and can be read to thousandths by means of a vernier; then, by measuring the distance of the mark up the rod from the zero of the scale, the time taken by the shot in traversing the distance between the screens, can be calculated as follows:—

Let

T = time in seconds between the commencement of fall of the chronometer rod and the mark made on the rod.

t = actual time in seconds of the shot between screens, so that $T-t$ is the disjunctor time;

h = distance in inches from zero to mark on chronometer rod;

g = acceleration of gravity;

then, since h is measured in inches,

$$h = 6gT^2, \quad T = \sqrt{\frac{h}{6g}},$$

and $\frac{l}{t}$, is the average velocity between the screens, l feet apart.

As an example, suppose the mark made on the chronometer rod was at 10·644 inches above the zero; putting $h = 10·644$ in the above, gives $T = 0·235$ of a second.

Now from this we must deduct 0·15 of a second, the time taken for the registrar rod, trigger, and knife to act, giving $t = 0·085$ of a second as the time taken to pass from one screen to the other.

Since the distance between the screens is 180 feet, the mean velocity must be

$$\frac{180}{0·085} = 2118 \text{ f/s.}$$

This is assumed to be the *actual* velocity at the middle point, and when the resistance varies with the cube of the velocity it is absolutely true; even when the velocity is such that the resistance varies with some other power, the difference would not be practically appreciable when the distance considered is so short as 180 feet.

The lower edge of the scale being graduated in velocities, these can be read off directly, thus saving calculation.

With screens placed 180 feet apart, and time to the disjunctor mark fixed at 0·15 of a second, the formula required for graduating the velocity scale is

$$h = 6g\left(\frac{180}{v} + 0·15\right)^2.$$

Putting $v = 1000$ f/s gives $h = 21·033$ inches,
and $v = 2000$,, ,, $h = 11·125$,,

In order to obtain the muzzle velocity V we must make use of the formula given on p. 24.

$$S_V = S_v + \frac{s}{C}$$

EXAMPLE 1—SERVICE PROJECTILE.

Suppose a 6-inch Q.F. gun to have been fired with a 100-lb. projectile of service pattern at 60 yards from the first screen, then the distance s from the muzzle to the middle point between the screens where the velocity $v = 2118$ f/s. has been observed by means of the chronograph, is $180 + 90 = 270$ feet, and $C = \frac{w}{d^2} = 2·778$.

Then

$$\begin{aligned} S_V &= S_{2118} + \frac{270}{2·778} \\ &= 45616·2 + 97·2 \\ &= 45713·4 \\ V &= 2148 \text{ f/s.} \end{aligned}$$

EXAMPLE 2 PROOF PROJECTILE.

At the Proof Butts projectiles of the same weight as the service projectile, but with flat heads, are used, and supposing the same middle point velocity v as in (1) to have been observed, but a proof shot to have been fired.

On account of the flat head, we must introduce the factor $\kappa = 2$, then

$$C = \frac{w}{\kappa d^2} = \frac{100}{2 \times 36} = 1{\cdot}389$$

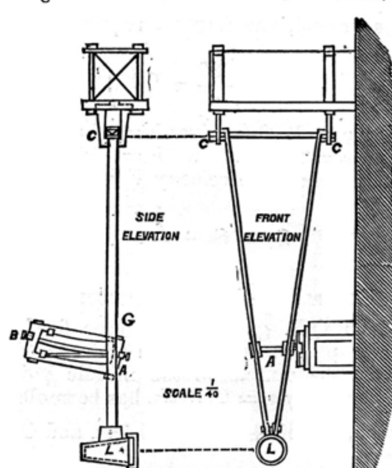
The coefficience of shape is commonly taken as $\kappa = 2$, but 1·817 is better for flat-headed projectiles (p. 25).

Now
$$\frac{S}{C} = 194{\cdot}4,$$
$$S_V = 45810{\cdot}6,$$
$$V = 2178 \text{ f/s}.$$

Before the adoption of electricity, the instrument employed for determining the velocity of shot was the ballistic pendulum, invented by Benjamin Robins in 1740.

This consists essentially of a large pendulum provided with an iron plate or a box filled with sand; the bullet strikes the plate and is shattered, or else the cannon ball is imbedded in the box of sand and now the *diluted* velocity of the ball and pendulum causes the pendulum to swing back through a certain angle from which the striking velocity of the ball is inferred.

Fig. 6.—Musket Pendulum for Small Arms.



The musket pendulum is shown in fig. 6; and when, as here, the pendulum consists of a rigid framework, swinging bodily about the axis of suspension, it is important that the bullet should strike at or near a certain point called the centre of percussion, to minimise the shock on the axis of suspension.

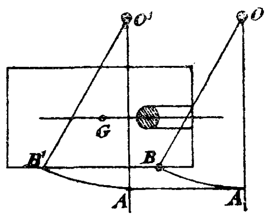
The theoretical determination of this centre of percussion depends upon advanced dynamical considerations, although the position can be determined experimentally with great ease by noting the point where a plummet, suspended from the same axis, swings in the same period, and has no tendency to separate from the pendulum.

To avoid, however, all considerations of this centre of percussion, the rigid framework of the pendulum may be supposed replaced by four equal chains or cords, hanging vertically at rest, fig. 7; and now if the box of sand or the block is struck by the ball in a line passing horizontally through the centre of gravity, the box will recoil, without rotation, through a certain height h feet, such that

$$h = \frac{U^2}{2g},$$

if the box and imbedded ball acquire a common velocity U f/s during the penetration.

FIG. 7.



If the ball, weighing w lbs., strikes with velocity V f/s, and is imbedded in the box weighing W lbs., then the velocity of the ball is *diluted* from V to U, such that

$$wV = (W + w)U,$$

in accordance with the principle of momentum; and thence

$$V = \frac{W + w}{w} \sqrt{(2gh)}.$$

If a point A on the box in recoiling draws out a tape to a length c, the cord of the circular arc AB described by the point A, and if the chains or cords are l feet long, so that the radius of the circular arc is l, then

$$c^2 = 2hl,$$

so that

$$V = \frac{W + w}{w} \sqrt{\frac{g}{l}} \; c,$$

or V is proportional to c, so that this tape can be graduated uniformly for equal increments of velocity.

The striking energy of the ball, $\frac{wV^2}{2g}$ ft.-lbs., is reduced by the impact to

$$(W + w)\frac{U^2}{2g} = \frac{w^2}{W + w}\frac{V^2}{2g} \text{ ft.-lb.},$$

shared between the block and the ball; so that

$$\frac{Ww}{W+w}\,\frac{V^2}{2g}\text{ ft.-lb.}$$

of energy is dissipated or liberated by the blow.

If we suppose this energy is used up in penetrating the sand of the box to a distance a feet against an average resistance of R pounds, then

$$Ra = \frac{Ww}{W+w}\,\frac{V^2}{2g} = \tfrac{1}{2}(W+w)\,\frac{W}{w}\,\frac{c^2}{l}\text{ ft.-lb.},$$

whence R can be determined from an observation of a.

If the shot occupies t seconds in penetrating the sand, then Rt = momentum in second-pounds lost by shot or gained by box,

$$Rt = \frac{w}{g}(V-U) = \frac{W}{g}U;$$

so that

$$t = \frac{2w}{W+w}\,\frac{a}{c}\sqrt{\frac{l}{g}},$$

seconds, during which the box will have moved with average velocity $\frac{1}{2}U$, and therefore through a distance

$$\frac{aW}{W+w}\text{ feet}$$

this distance is so small in practice that we are justified in ignoring the curvature in the motion as assumed above.

Thus, for example, if $W = 2{,}000$ lbs., $L = 8$ feet, $c = 6$ feet, and $w = 20$ lbs., then we find $v = 1{,}212$ f/s; and if the penetration of the shot into the sand is 2 feet, $a = 2$; and then the mean resistance $R = 227{,}250$ lbs., and the time of penetration about 0·0033 or $1 \div 300$ of a second, during which the box will have moved about 0·2376 inch, say, one-quarter of an inch.

Sometimes the gun or rifle is mounted on a pendulum, thence called the gun pendulum; in this case the pendulum measures the recoil, as felt on the shoulder when the rifle is fired; for this purpose the rifle can be suspended by two cords about 3 or 4 feet long; but the additional recoil, due to the blast of powder, prevents the gun pendulum from giving accurate records of the muzzle velocity of the shot.

A bullet-proof steel cuirass illustrates in a popular manner the principles of the ballistic pendulum.

If the cuirass weighed, for instance, 12 lbs., and was struck by a $\frac{1}{2}$-oz. bullet with velocity 2,000 f/s, it would deliver a blow to the body as if let fall about 5 inches; this is about the same blow as is felt on the shoulder when firing the rifle, supposed of the same weight 12 lbs.

SECTION III.—RECOIL.

Consider a gun and carriage (either field, siege, or garrison) as one system. On firing, a resultant force acts along the axis of the piece and tends to produce two kinds of motion.

(1.) Motion of translation of the centre of gravity of the system.

(2.) Rotation round the centre of gravity of the system, or some fixed point, the spade for instance in a field gun.

We can consider these tendencies to motion separately.

1.) The tendency to motion of the centre of gravity of the system produced by the act of firing can be resolved—

(*a*) Vertically, and
(*b*) parallel to the ground or platform.

The first of these tendencies causes a stress in the axletree, and eventually a downward thrust on the wheels of a field carriage.

A downward blow is also produced on the sides of a garrison carriage and on the slide. This downward blow is very destructive at high angles of elevation.

The component parallel to the ground is the more considerable and causes the motion of recoil, which is checked by various means.

This force produces a horizontal stress on the axletree of a field carriage. Tensile stays transfer the pull from near the middle to parts nearer to its points of support.

The form of axletree which appears to be best fitted to resist these various stresses is one which is circular in section, lightness being obtained by the uses of tubular steel.

In modern gun carriages the gun is allowed to recoil some distance in a cradle, the movement being controlled by a hydraulic buffer. This movement of the gun reduces very much the stress on the carriage, so that in a field gun the wheels need not move on the ground.

If we consider the gun and carriage as forming one rigid body, and the gun be supposed to be fired with no elevation from a smooth horizontal platform, and, further, suppose that the density of the powder gas is uniform, then, while the shot is in the bore, the velocity of recoil of the gun and carriage, and the velocity of the shot are connected at any point by the equation

$$(1) \qquad (W + \tfrac{1}{2}w_1)U = (w + \tfrac{1}{2}w_1)V,$$

where

W = weight of gun and carriage in pounds.
w = weight of shot in pounds.
w_1 = weight of powder charge in pounds.
U = velocity of gun and carriage in f/s.
V = velocity of projectile in f/s.

For the forward velocity of the C.G. of the powder gases, taking their density as uniform, is $\frac{1}{2}(V-U)$; so that the forward momentum of the shot and of the powder is

$$\frac{wV}{g} + \frac{w_1}{g}\,\frac{V-U}{2}$$

and this, in accordance with the Third Law of Motion, " Action and Reaction are equal and opposite," must be equated to the backward momentum $\frac{WU}{g}$ of the recoiling gun and carriage; so that

$$\frac{WU}{g} = \frac{wV}{g} + \frac{w_1}{g}\frac{V - U}{2},$$

or

$$(W + \tfrac{1}{2}w_1)U = (w + \tfrac{1}{2}w_1)V.$$

If x denotes the recoil of the gun, and y the advance of the shot, while the shot has passed up the length l of the rifled portion of the bore,

$$x + y = l,$$

and

$$\frac{x}{U} = \frac{y}{V} = \frac{l}{U + V},$$

so that

(2) $$x = \frac{Ul}{U + V} = \frac{w + \frac{1}{2}w_1}{W + w + w_1} l$$

(3) $$y = \frac{Vl}{U + V} = \frac{W + \frac{1}{2}w_1}{W + w + w_1} l.$$

We may take the average velocity of the shot through the bore $\frac{1}{2}(U + V)$; so that the shot takes

(4) $$\frac{l}{\frac{1}{2}(U + V)} = \frac{2W + w}{W + w + w}\frac{V}{l}$$

seconds to pass up the bore.

If P pounds denotes the average thrust of the powder on the base of the shot, and Q pounds on the base of the bore,

$$Py = \frac{wV^2}{2g}, \text{ the energy of the shot in ft.-lb.}$$

$$Qy = \frac{WU^2}{2g}, \text{ the energy of the gun in ft.-lb.},$$

so that

(5) $$P = \frac{W + w + w}{W + \frac{1}{2}w}\frac{wV}{2gl},$$

(6) $$Q = \frac{W + w + w}{w + \frac{1}{2}w}\frac{WU^2}{2gl}$$

It is found practically that the gun and carriage have moved only a very short distance when the projectile has just left the muzzle; and that the maximum velocity of recoil is not attained till a short time afterwards.

For after the shot has left the muzzle the powder gases escape with some unknown high velocity and mingle with the surrounding air, imparting to it a considerable momentum; this is well exhibited in some photographs of Krupp guns.

Meanwhile the pressure on the base of the bore must last for an appreciable time longer, so that the gun receives an additional recoil after the shot has left the muzzle; and this recoil is greater as the weight of the charge and the powder pressure up to the muzzle is greater.

This extra recoil is very considerable with slow burning powder, and may amount to about 30 % increase.

To allow for this in practice the empirical formula, which is now used for calculating the velocity of recoil is—

$$(7) \qquad \mathrm{WU} = (w + \mathrm{C}w)\mathrm{V},$$

where C is a constant determined by experiment.

The value of C is usually taken at from 1·5 to 2, according to the nature of the powder.

From formula (7),

$$\frac{\text{The velocity of the shot}}{\text{,, \quad ,, \quad gun}} = \frac{\mathrm{V}}{\mathrm{U}} = \frac{\mathrm{W}}{w + \mathrm{C}w} = \frac{\mathrm{W}}{w} \text{ approximately};$$

$$\frac{\text{the momentum of the shot}}{\text{,, \quad ,, \quad gun}} = \frac{w\mathrm{V}}{\mathrm{WU}} = \frac{w\mathrm{W}}{w\mathrm{W} + \mathrm{CW}w_1} = 1 \text{ approximately};$$

$$\frac{\text{the energy of the shot}}{\text{,, \quad ,, \quad gun}} = \frac{w\mathrm{V}^2}{\mathrm{WU}^2} = \frac{w}{\mathrm{W}}\left(\frac{\mathrm{W}}{w + \mathrm{C}w}\right)^2 = \frac{\mathrm{W}}{w} \text{ approximately},$$

so that the energy of recoil diminishes as the weight of the gun and carriage is increased.

In the most modern systems of field artillery of Schneider-Canet and Ehrhardt, the gun has a long recoil in its cradle, and the wheels and trail remain stationary on the ground. The gun is brought up in its recoil on the cradle by a hydraulic buffer, and is run out again immediately and automatically either by the action of compressed air as in the Schneider-Canet method, or by the resilience of a number of spiral springs in the Ehrhardt system.

In this way the carriage does not jump about on the ground, and the pointer has plenty of time between the shots, with the fine adjustments at his command, to keep the gun laid accurately on its object.

The following numbers are taken from an article in the "Revue d'Artillerie," February, 1901:—

The average pressure in the bore of the powder gas being taken as 2,000 atmospheres, the total thrust on the base of the bore, 7·5 cm. (2·94 inches) in diameter, will be about 100,000 kg. (say 100 tons).

The weight of the shot being 6·5 kg. (14·25 lbs.), and the weight of the charge 0·5 kg. (1·1 lb.), doubling the weight of the charge to allow for the blast of powder from the muzzle, and adding this to the weight of the shot, moving with the initial velocity 500 m/s. (1640 f/s) gives a forward momentum of

$$\frac{7{\cdot}5 \times 500}{g} = \frac{3750}{g} \text{ (second-kg.)}.$$

Taking the weight of the recoiling gun as 400 kg. (882 lbs.), the velocity u of the recoil will be given by

$$\frac{400u}{g} = \frac{3750}{g},$$

so that $$u = 9{\cdot}375 \text{ m/s } (30{\cdot}75 \text{ f/s}).$$

The kinetic energy of the gun will then be

$$\frac{400u^2}{2g} = 1790 \text{ kg.-m. } (5{\cdot}78 \text{ ft.-tons}).$$

If the length of recoil of the buffer is 1 mètre (3·28 feet, or 39·37 inches), the average force of resistance of the buffer must be 1790 kg. (say 1·79 tons). If, then, the line of action of the buffer is at a height of 1 mètre from the ground, and if the cèntre of gravity of the gun and carriage is 2·5 mètres in front of the spade, the wheels will not be lifted off the ground if the weight of the gun and carriage exceeds W kg., where

$$w \times 2{\cdot}5 = 1790,$$

or $$W = 716 \text{ kg. (say } 0{\cdot}7 \text{ ton, or } 14 \text{ cwt.)}.$$

This value of W can be still further diminished by a suitable arrangement of the variation of resistance in the hydraulic buffer.

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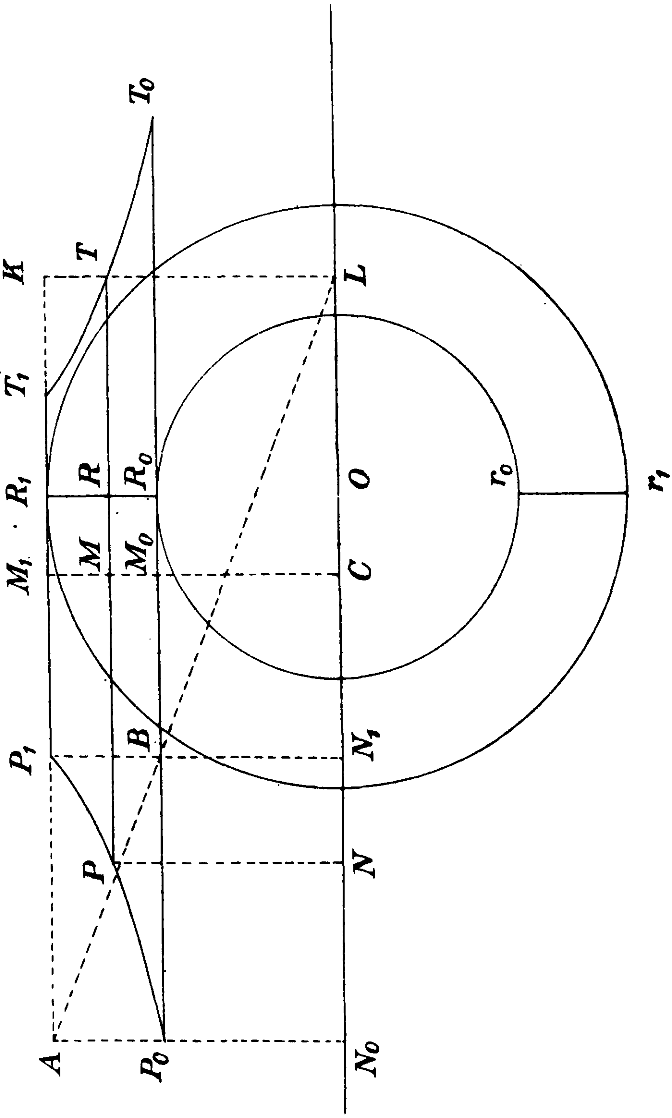

Fig. I.

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CHAPTER V
PRINCIPLES OF GUN CONSTRUCTION.

SECTION I.—STRESSES IN THE MATERIAL OF A GUN.

A GUN is essentially a thick tube, reinforced in those parts where the internal pressure is likely to be greatest.

After the determination of the maximum pressure to be expected in the chamber of a gun and at various distances up the bore, the gunmaker is required to proportion the thickness of the metal of the gun so as to withstand the pressure with safety.

The theoretical problem is then the determination of the state of stress set up in the metal of a thick cylindrical tube, due to arbitrary internal and external pressure; and to determine this the tube is supposed cut in half by a diametral plane R_1Or_1 (fig. 1), and the equilibrium of either half is considered.

The inch and ton are the units employed in practice, or else the centimetre and kilogramme in metric units.

Suppose then that the internal and external radii of the tube are r_0 and r_1 inches (cm.), and that the tube is subject to steadily applied internal and external pressures of p_0 and p_1 tons/in.^{2} (kg./cm.^{2} or atmospheres).

Considering an inch length of the tube, the hydrostatic thrust of the interior pressure p_0 over the interior semicircular surface is the same as the thrust across the diametral plane R_0r_0, and is therefore $2r_0p_0$ tons, so also the thrust of the pressure p_1 over the exterior semicircle is $2r_1p_1$ tons, in the opposite direction.

The resultant thrust on the half tube is

(1) $$2r_0p_0 - 2r_1p_1 \text{ (tons)},$$

and this is balanced by the tension set up in the circumferential fibres of the metal; so that if X denotes the pull in tons across each section of the tube,

(2) $$2X = 2r_0p_0 - 2rp_1,$$

or, considering only one section, R_0R_1, of the tube,

(3) $$X = r_0p_0 - r_1p_1.$$

Ordinates such as RT are drawn to represent to scale the tension in tons/in.^{2} at the radius OR of the circumferential fibre of radius r, and the tops of these ordinates are joined by the curve T_0 T T_1', called the *curve of circumferential or hoop tension* (fig. 1), and then X is represented to scale by the area R_0 T_0 T_1' R_1.

So also the ordinates R_0P_0 and R_1P_1 are drawn to represent to the same scale the radial pressure p_0 and p_1 at the radius r_0 and r_1; and the curve P_0PP_1, called the *curve of radial pressure*, is drawn to represent graphically the radial pressure RP or p across any concentric cylinder of radius OR or r.

Interpreted geometrically on fig. 1, equation (3) may be written

(4) $$\text{Area } R_0T_0T_1'R_1 = \text{rectangle } OP_0 - \text{rectangle } OP_1 = \text{rectangle } N_1P_0 - \text{rectangle } R_0P_1$$

(taking away the common part OB); or

(5) $$\text{Area } BT_0T_1'P_1 = \text{rectangle } N_1P_0;$$

so that, drawing the diagonal ABL of the rectangle AN_0LK_1

(6) $$\text{Area } BT_0T_1'P_1 = \text{rectangle BK}.$$

The *average* hoop tension, $\frac{X}{R_0R_1}$, across the section R_0R_1 is thus represented graphically by RT, obtained geometrically by producing the diagonal AB to L on ON, and drawing LTK parallel to OR.

When the thickness of the tube is small compared with the diameter, the maximum and minimum hoop tension differ only slightly from the average; so that this average need only be considered, as in the case of the cylindrical shell of a steam boiler.

But in gun construction the *maximum* hoop tension must be carefully considered, so as to keep it well below the *elastic limit* (p. 6); the shape of the curve of hoop tension T_0TT_1' must therefore be determined, and at the same time the curve of radial pressure P_0PP_1.

In Part II it will be shown that in a homogeneous tube the curves T_0TT_1' and P_0PP_1 (called *Barlow curves*, from Peter Barlow, of the Royal Military Academy, who first investigated this problem) are symmetrical with regard to an axis CM, so that MT = MP, and also that each varies inversely as the square of OR, so that we may put

(7) $$MT = MP = \frac{a}{r^2},$$

where a denotes some constant; and then, if OC is denoted by b,

(8) $$t = RT = \frac{a}{r^2} - b, \qquad p = RP = \frac{a}{r^2} + b;$$

so that

(9) $$p + t = \frac{2a}{r^2}, \qquad p - t = 2b,$$

where a and b are two disposable constants, positive or negative, which can be determined from any two arbitrary imposed conditions.

For instance, the pressures p_0 and p_1 may be assigned; then

(10) $$p_0 = ar_0^{-2} + b, \qquad p_1 = ar_1^{-2} + b;$$

so that

(11) $$a = \frac{p_0 - p_1}{r_0^{-2} - r_1^{-2}}, \qquad b = \frac{p_1r_0^{-2} - p_0r_1^{-2}}{r_0^{-2} - r_1^{-2}};$$

and then

(12) $$t = \frac{p_0(r^{-2} + r_1^{-2}) - p_1(r_0^{-2} + r^{-2})}{r_0^{-2} - r_1^{-2}},$$

(13) $$p = \frac{p_0(r^{-2} - r_1^{-2}) + p_1(r_0^{-2} - r^{-2})}{r_0^{-2} - r_1^{-2}}$$

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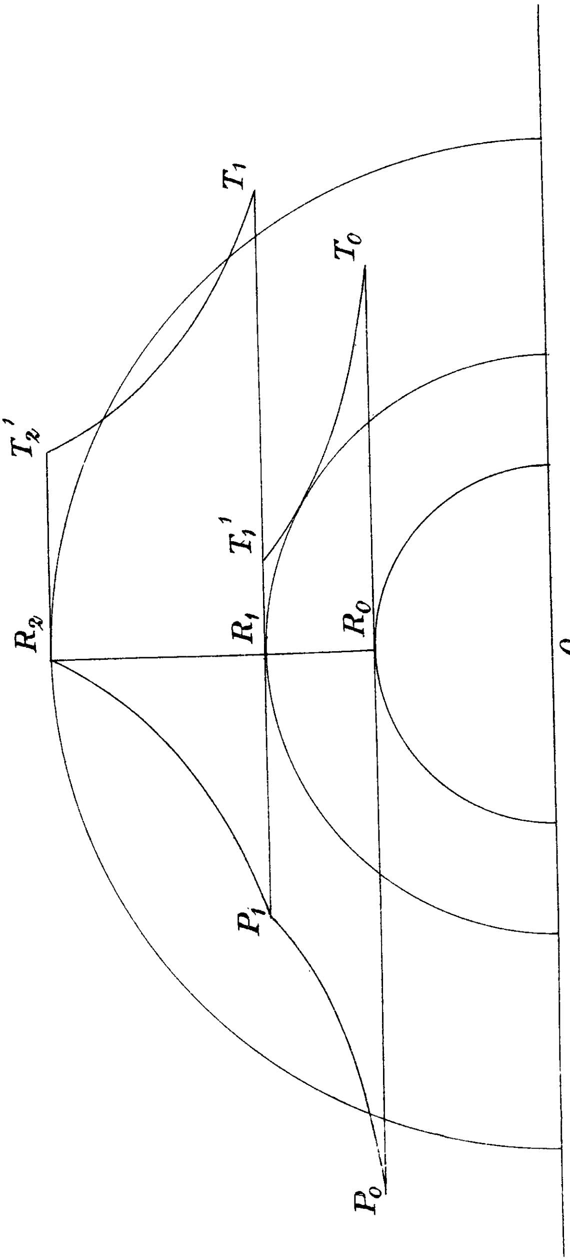

FIG 2 A.

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this is required in the sequel for the calculation of the powder stresses, with zero exterior pressure, as shown in figs. 2B and 3B.

But the usual problem which first presents itself in gun construction is the determination of p_0, when p_1, the exterior applied pressure, is given (due to the pressure of the shrinkage of an exterior hoop or jacket), and t_0, the maximum allowable hoop tension, is assigned.

According to rules laid down by the Ordnance Committee, the maximum allowable hoop tension is 18 tons/in² in a hoop or jacket, reduced to 15 tons/in² for an internal A tube, to allow for the diminution in strength due to rifling and erosion.

Now if p_0 and t_1 are the data in the above equations (8) and (9),

$$(14) \qquad p_0 - p_1 = a(r_0^{-2} - r_1^{-2}),$$

$$(15) \qquad t_0 + p_1 = a(r_0^{-2} + r_1^{-2}),$$

so that

$$(16) \qquad \frac{p_0 - p_1}{t_0 + p_1} = \frac{r_0^{-2} - r_1^{-2}}{r_0^{-2} + r_1^{-2}} = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2},$$

the formula employed by the gunmaker to calculate p_0, in the form

$$(17) \qquad p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1.$$

Fig. 1 shows the cross-section of the A tube of the 4·7-inch gun at the cartridge chamber, where the bore is 5 inches in diameter, and the thickness of the metal is 1·6 inches, so that $r_0 = 2{\cdot}5$, $r_1 = 4{\cdot}1$ ins.

Suppose we are given that $p_1 = 9{\cdot}72$ tons / in.²; then, with $t_0 = 15$ tons/ins.,

$$(18) \qquad p_0 = \frac{(4{\cdot}1)^2 - (2{\cdot}5)^2}{(4{\cdot}1)^2 + (2{\cdot}5)^2}(15 + 9{\cdot}72) + 9{\cdot}72 = 21{\cdot}04.$$

But with $p_1 = 0$, we find $p_0 = 6{\cdot}87$, the maximum pressure the A tube can withstand if unsupported.

Now suppose the exterior pressure p_1 on the tube has been applied by shrinking on a single jacket or hoop, of external radius r_2, and therefore of thickness $r_2 - r_1$ (fig. 2A).

The gun designer has to calculate p_1 from the conditions that $p = p_2 = 0$ at the exterior where $r = r_2$, and that $t = t_1 = 18$ where $r = r_1$, in accordance with the rules of the Ordnance Committee.

Changing the suffixes, the gunmaker's formula (17) becomes

$$(19) \qquad p_1 = \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}(t_1 + p_2) + p_2,$$

with $\qquad p_2 = 0.$

Thus in fig. 2A, representing the cross-section of the 4·7-inch gun, in which the A tube is reinforced by a jacket 3·4 inches thick, then

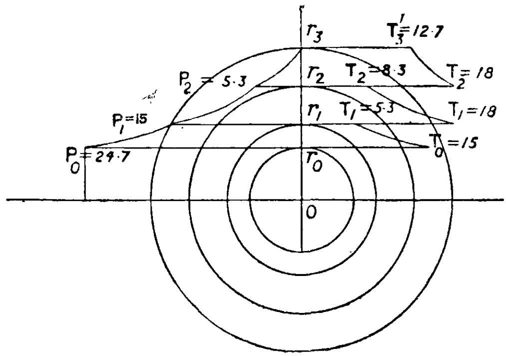
$$r_2 = 7{\cdot}5 \text{ inches}, p_2 = 0,$$

and $\qquad r_1 = 4{\cdot}1$ inches as before, but $t_1 = 18$,

$$(20) \qquad p_1 = \frac{(7{\cdot}5)^2 - (4{\cdot}1)^2}{(7{\cdot}5)^2 + (4{\cdot}1)^2} \times 18 = 9{\cdot}72,$$

the value of p_1 adopted in the calculation of fig. 1, so that proceeding with the calculation, $p_0 = 21{\cdot}04$, the maximum allowable pressure inside this gun; and two-thirds of this, or 14 tons/in.^{2}, is sometimes called the *normal pressure.*

Fig. 3A.



With a gun built up of three parts, the tube A, the breech-piece B, and the jacket C, as shown in fig. 3A, the gunmaker's formula to employ in successive order, beginning from the outside, gives

$$(21) \qquad p_2 = \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2}(t_2 + p_3) + p_3,$$

with $\qquad p_3 = 0, \ t_2 = 18;$

$$(22) \qquad p_1 = \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}(t_1 + p_2) + p_2,$$

with p_2 from (20), and $t_1 = 18$;

$$(23) \qquad p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1,$$

with p_1 from (21) and $t_0 = 15$; and $\frac{2}{3}p_0$ is called the *normal* chamber pressure.

A similar procedure will apply for a gun built up of four or more parts.

It will be noticed that in crossing a surface of separation, for instance between the A tube and breech-piece, or the breech-piece and jacket, there can be no sudden change in radial pressure, but that the hoop-tension can change suddenly; and to distinguish the two values of t at a surface of separation, an accent will be employed with the t which refers to the inner substance; the value of t' is easily calculated from the formulas

$$(24) \quad p_1 - t_1', = p_0 - t_0, \quad p_2 - t_2' = p_1 - t_1, \quad p_3 - t_3' = p_2 - t_2, \ \&\text{c.},$$

derived from equation (9).

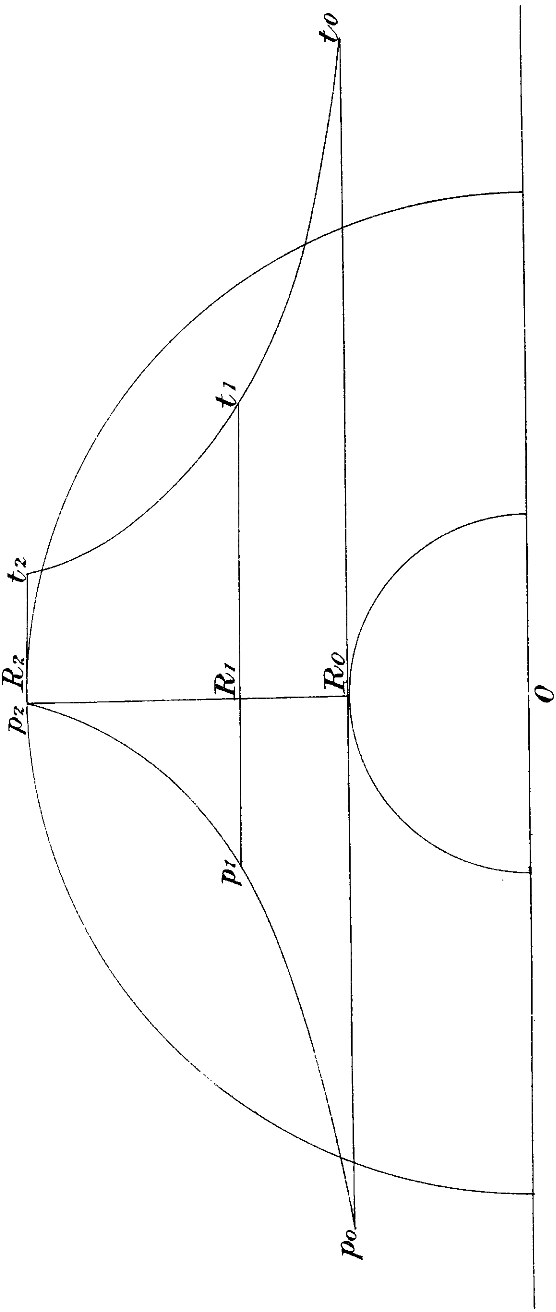


FIG. 2.B.

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Fig. 3A is drawn for the 6-inch gun, composed of A tube, breech-piece B, and jacket C, with

$$r_0 = 4, \qquad r_1 = 5{\cdot}6, \qquad r_2 = 8{\cdot}7, \qquad r_3 = 11{\cdot}8.$$

Performing the calculation by mean of the gunmaker's formula (17), beginning at the outside of the jacket and working inwards, subject to

(25) $t_2 = 18, \qquad t_1 = 18, \qquad t_0 = 15,$ tons/in.²

it will be found that

(26) $p_2 = 5{\cdot}3, \qquad p_1 = 15, \qquad p_0 = 24{\cdot}7,$,,

and then

(27) $t_3' = 12{\cdot}7, \qquad t_2' = 8{\cdot}3, \qquad t_1' = 5{\cdot}3,$,,

The state of stress shown in fig. 2A and fig. 3A is called the *firing stress*, as it is supposed to be set up when the gun is fired with the maximum allowable pressure.

To secure this state of stress when the gun is fired, the shrinkage of the hoops and jacket in the process of manufacture must impart a state of stress called the *initial stress*, or *stress of repose*, such that the addition of the stress due to the powder pressure, called the *powder stress*, shall set up the firing stress; or conversely, the deduction of the powder stress from the firing stress shall leave the stress of repose or initial stress.

To distinguish the *powder stress* and *firing stress* in the sequel, capital letters P and T will be used to designate *firing stresses*, the Greek letters ϕ and τ being used for *stresses of repose*.

The powder stress is calculated on the assumption that the gun is one homogeneous tube throughout, and initially devoid of stress; also that the internal pressure at the radius r_0 is the pressure p_0, calculated by the gunmaker's formula, and that the external pressure p_n, at the external radius r_n, is zero.

Thus the powder stress at any radius r is given by equations (12) and (13) in the form

(28) $$t = p_0 \frac{r^{-2} + r_n^{-2}}{r_0^{-2} - r_n^{-2}},$$

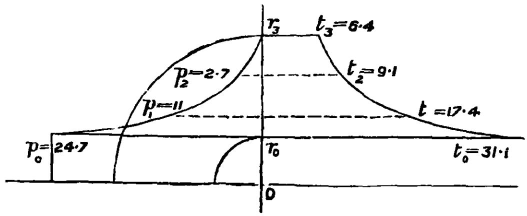
(29) $$p = p_0 \frac{r^{-2} - r_n^{-2}}{r_0^{-2} - r_n^{-2}},$$

Working with these formulas, we find for the powder stress in the 4·7-inch gun, shown in fig. 2B,

(30) $p_0 = 21{\cdot}04, \qquad p_1 = 6{\cdot}17, \qquad p_2 = 0$ tons/in.²

(31) $t_0 = 26{\cdot}25, \qquad t_1 = 11{\cdot}43, \qquad t_2 = 5{\cdot}26$,,

Fig. 3B.



The powder stress in the 6-inch gun shown in fig. 3B, is found by similar calculation from (12) and (13),

(32) $\quad p_0 = 24{\cdot}7, \quad p_1 = 11{\cdot}0, \quad p_2 = 2{\cdot}7, \quad p_3 = 0$ tons/in.²

(33) $\quad t_0 = 31{\cdot}1, \quad t_1 = 17{\cdot}5, \quad t_2 = 9{\cdot}1, \quad t_3 = 6{\cdot}3,$,,

Instead of working with (12) and (13), a return may be made to (8), and the values of a and b can be calculated from the assigned data; thence the value of p and t for any radius r can be calculated directly.

Now, distinguishing the corresponding values of the radial pressure and hoop tension in the *firing stress* by capital letters P and T, and in the *initial* stress by Greek letters ϕ and τ,

(34) $$P = p + \phi, \qquad T = t + \tau,$$

or

(35) $$\phi = P - p, \qquad \tau = T - t.$$

Thus in the 4·7-inch gun,

$P_0 = 21{\cdot}04,$	$T_0 = 15,$	tons/in.²
$P_1 = 9{\cdot}72,$	$T_1' = 3{\cdot}68, T_1 = 18$	,,
$P'_2 = 0{\cdot}00,$	$T_2' = 8{\cdot}28,$	,,
$p_0 = 21{\cdot}04,$	$t_0 = 26{\cdot}25,$	,,
$p' = 6{\cdot}17,$	$t_1 = 11{\cdot}43,$	,,
$p_0 = 0{\cdot}00,$	$t_2 = 5{\cdot}26,$	,,

$\phi_0 = 0$, $\tau_0 = -11{\cdot}25$ tons/in.²
$\phi_1 = 3{\cdot}55,$ $\tau_1' = -7{\cdot}74$,, } in the tube,

$\phi_1 = 3{\cdot}55,$ $\tau_1 = 6{\cdot}58$,,
$\phi_2 = 0$, $\tau_2' = 3{\cdot}03$,, } in the jacket,

giving the initial stress, as shown in fig. 2C; the negative values of τ_0 and τ_1' shows the extent to which the A tube is compressed circumferentially by the shrinkage of the external jacket.

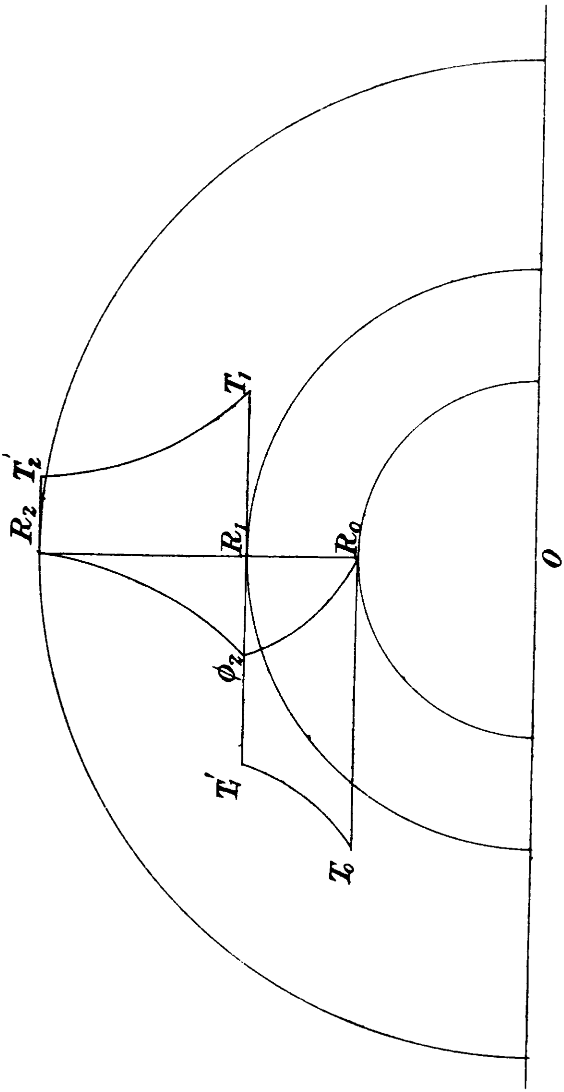


FIG. 2. C.

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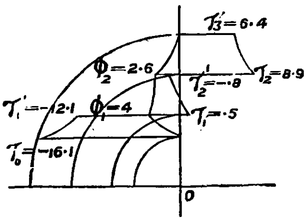
Fig. 3c.



In the 6-inch gun, the firing stress is given by

$P_0 = 24 \cdot 7,$	$T_\theta = 15 \cdot 0,$	tons/ins.²
$P_1' = 15 \cdot 0,$	$T_0' = 5 \cdot 3,$	,,
$P_1 = 15 \cdot 0,$	$T_1 = 18 \cdot 0,$	,,
$P_2' = 5 \cdot 3,$	$T_2' = 8 \cdot 3,$	,,
$P_2 = 5 \cdot 3,$	$T_2 = 18 \cdot 0,$	,,
$P_3 = 0 \cdot 0,$	$T^{81} = 12 \cdot 7,$	,,

and the powder stress is given by

$p_0 = 24 \cdot 7,$	$t_0 = 31 \cdot 1,$	,
$p_1 = 11 \cdot 0.$	$t_1 = 17 \cdot 5,$	,,
$p_2 = 2 \cdot 7,$	$t_2 = 9 \cdot 1,$	,,
$p_3 = 0 \cdot 0,$	$t_3 = 6 \cdot 3,$	,,

so that the initial stress is given by

$$\left.\begin{array}{ll} \phi_0 = 0\ , & \tau_0 = -\ 16 \cdot 1 \text{ tons/in.}^2 \\ \phi_1 = 4 \cdot 0, & \tau_1' = -\ 12 \cdot 1 \quad ,, \end{array}\right\} \text{in the tube}$$

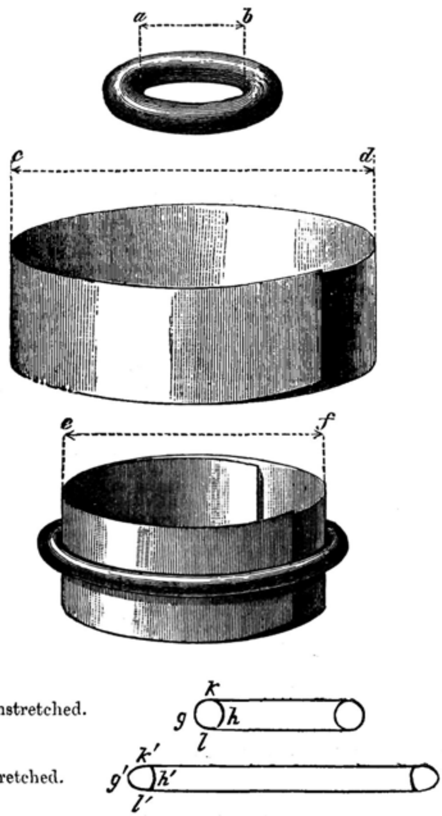
$$\left.\begin{array}{ll} \phi_1 = 4 \cdot 0, & \tau_1 = \quad 0 \cdot 5 \quad ,, \\ \phi_2 = 2 \cdot 6, & \tau_2' = -\ 0 \cdot 8 \quad ,, \end{array}\right\} \text{in the breech piece,}$$

$$\left.\begin{array}{ll} \phi_2 = 2 \cdot 6, & \tau_2 = \quad 8 \cdot 9 \quad ,, \\ \phi_3 = 0\ , & \tau_3' = \quad 6 \cdot 4 \quad ,, \end{array}\right\} \text{in the jacket,}$$

shown in fig. 3c.

A simple illustration of initial stress may be given by means of an india-rubber ring (fig. 4) of interior diameter *ab*, and a loose cardboard roll of greater diameter, *cd*. If the former is stretched, put over the latter, and left to itself, contraction of both will take place to some diameter *ef*, intermediate between *ab* and *cd*.

Fig. 4.



(*a*) Ring unstretched.

(*b*) Ring stretched.

A state of stress is thus produced between the ring and the roll, the one being larger and the other smaller than at first, and each having its elastic tendency to return to its own original dimensions resisted by the reaction of the other; a normal or radial pressure acts at the surface of contact, which causes a *lengthening* of the india-rubber ring circumferentially, indicating *hoop tension;* and this same normal pressure makes the cardboard roll *smaller* in circumference, indicating *hoop pressure.* The cardboard roll is stronger than before, to resist an interior normal pressure, while the ring is weaker, but still it may be strong enough for the tension which will come upon it; and the stress in the materials is more nearly equalised.

A similar method of construction is employed for fireworks or the light artillery of Gustavus Adolphus; the tubes of rockets and squibs are built up of layers of cardboard or brown paper, rolled together in a state of initial tension, while the guns of Gustavus Adolphus were composed of an interior copper tube, reinforced by strips of hide, wound tightly round the exterior.*

If fig. 4 represents a section of the india-rubber ring (*a*) in its unstretched state; (*b*) when it is expanded over the cardboard roll, we note that although the ring becomes larger in diameter when it is stretched, and slightly changes in volume, owing to its elasticity, it becomes thinner in section, both in the direction of the radius and of the axis of the roll, *i.e.*, *gh* contracts to *g'h'*, and *kl* contracts to *k'l'*.

As a matter of precaution no gun is allowed to be subjected to the full amount as calculated above, and which has been called the maximum allowable pressure, the charges being so arranged that the pressure shall not exceed a normal chamber pressure of about two-thirds of this P_0, so that under ordinary conditions the elastic limit of no part of the material may be reached or permanent extension take place. In the above example of the 6-inch gun, if the working pressure were limited to 17 tons, the gun would have a factor of safety of

$$24{\cdot}7 \div 17 = 1{\cdot}45.$$

This working pressure, or normal chamber pressure, is that pressure which should not be exceeded by the ordinary service charge, and in the case of cordite the temperature of the charge is fixed at 80° F. It might be called the specification pressure.

The actual pressure which a charge does produce, as ascertained by means of the crusher gauge and coppers, is frequently below the working pressure, and is dependent on such things as wear of the gun, temperature of charge, &c.

In the case of liners, no strength is accredited; for, being put in without shrinkage, they are taken as so much packing, and their effect as regards calculation of strength might be ignored but for the fact that they distribute the strain to a larger area. Of course, here, as in the case of shrinkage friction, with reference to longitudinal strength, any circumferential strength derived from the liner will be in addition to that calculated for.

Supposing the gun to have been designed for, and constructed originally with, a liner, then if P_0 represents the internal pressure on the liner, and P that transmitted to the interior of the A tube, and r_0 and r the respective radii, the formula is simply

$$(36) \qquad P_0 = P\frac{r}{r_0},$$

the liner acting as if cracked or segmental.

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* "Leather Guns," by Col. H. W. L. Hime, R.A., "Proceedings R.A. Institution," vol. 25.

GEOMETRICAL METHOD.

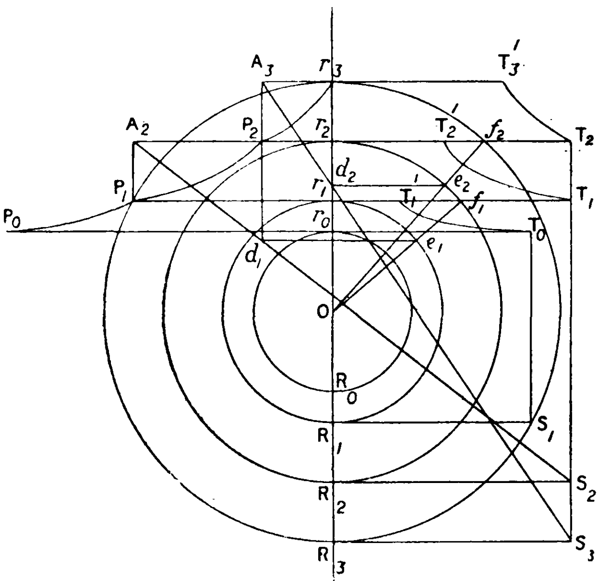
The stress in a gun can also be calculated by a geometrical method. For instance, taking the same 6-inch gun as before, in which

$$r_0 = 4,\ r_1 = 5\cdot6,\ r_2 = 8\cdot7, \text{ and } r_3 = 11\cdot8,$$

and knowing that T_0 is limited to 15 tons/in.² and T_1 and T_2 to 18 tons/in.², we can construct the firing stresses as follows:—

Fig. 5A.

Firing Stresses.



Draw circles with radii as given above for r_0, r_1, &c. (fig. 5A).

Fill in r_2 T_2, r_1 T_1, and r_0 T_0 to scale, representing tons/in.², and cutting the circles in f_2, f_1, and f_0.

Join Of_2, cutting the next inner circle in e_2. Draw e_2 d_2 parallel to r_2 T_2, also R_3 S_3 a tangent at R_3, meeting T_2 S_3 in S_3.

Join d_2 S_3 and produce it to meet the tangent at r_3 in A_3. Draw A_3 P_2 parallel to r_0 r_3 and meeting T_2 r_2 in P_2, then r_2 P_2 will represent the radial firing pressure at r_2.

In the same way we get f_1, e_1, and d_1, the last mentioned on A_3 P_2 produced. Then join d_1 with S_2, the point of intersection of T_1 S_2 and R_2 S_2, and produce it to meet r_2 P_2 produced in A_2, drop the perpendicular A_2 P_1, then r_1 P_1 represents the radial stress at r_1.

In the same way P_0 can be obtained.

The curves T_0 T_1', T_1 T_2', T_2 T_3' can then be completed as reflexions of the curves P_0 P_1, P_1 P_2, and P_2 P_3.

For the powder stresses we know the radii as before, also that $p_0 = 24{\cdot}7$ tons/in.^{2}, being equal to P_0.

Draw the circles, also tangents to them, at r_0, r_1, r_2, r_3, and R_3 (fig. 2B).

Join O with the point where the outer circle cuts the tangent at r_0. The line cuts the inner circle at *e*.

Draw *ed* parallel to $r_0\ p_0$.

The point p_0 being fixed, A can be obtained, join A*d* and produce it to meet $R_3\ S_3$ in S_3, cutting O*c* in C.

Then $R_3\ S_3 = r_0\ t_0$.

The centre C of the Barlow curves will be the mid-point of AS_3.

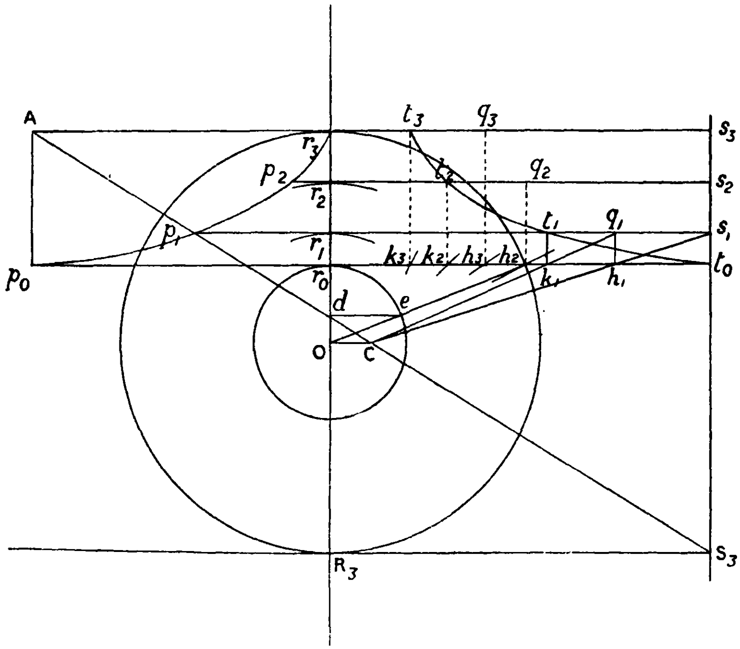
Join CS_1 cutting $r_0\ t_0$ in *h*, draw $h_1\ q_1$ vertically upwards, cutting $r_1\ s_1$ in q_1, join q_1 C, cutting $r_0\ t_0$ in k_1, draw $k_1\ t_1$ vertically upwards cutting $r_1\ s_1$ in t_1, this fixes the length $r_1\ t_1$.

In the same way t_2 and t_3 can be obtained as shown in the figure.

The curve $p_0\ p_1\ p_2$ can then be drawn in.

Fig. 5B.

Powder Stresses.



THE STRAIN OF THE GUNS.

So far we have dealt only with the *stress* in the metal, but when the gunmaker wishes to set up a given pressure of shrinkage between two cylinders, he has to determine, by calculation or experiment, the slight amount by which, when cold, the external radius of one cylinder must exceed the internal radius of the next cylinder which is shrunk on it.

The outer cylinder is expanded by heat and slipped on, in order that the given initial pressure may be set up on the cooling of the outer cylinder; and this, too, when other cylinders are shrunk on afterwards.

We must therefore determine the strain and deformation set up in a given cylinder due to given applied pressure, and thus we require the equations giving the strain due to given applied stress when the co-efficients of elasticity of the metal are known.

The mathematical steps leading to the solution of this problem will be given in Part II, p. 255; the general formula to be employed is

$$(37) \qquad {}_mS_{m+1} = (\tau_m - \tau_m') \frac{2r_m}{M}$$

Here any value from 0 upwards can be given to m so as to obtain ${}_mS_{m+1}$, which is either the contraction of the bore for $m = 0$, or the shrinkage between hoops, or the extension of the outside layer, in inches, at any radius of r_m inches; τ and τ' are the usual initial tensions, M the modulus of elasticity, taken usually as 12,500 tons /in^2 for gun steel.

A numerical example will show the use of the formula.

Calculating by formula (37) the shrinkages of the 6-inch gun, with the values of τ shown in fig. 3c, and commencing from the interior, we find that

$${}_0S_1 = \tau_0 \times 2r_0 \div M = 16{\cdot}1 \times 8 \div 12{,}500 = 0{\cdot}0103 \text{ inch,}$$

said practically to be 10-thousandths of an inch; this is the final contraction of the bore, or the amount by which its diameter must be turned larger at first, in order that its diameter may be 8 inches.

$${}_1S_2 = (\tau_1 - \tau_2') \times 2r_1 \div M = \{{\cdot}5 - (-12{\cdot}1)\} \times 11{\cdot}2 \div 12{,}500 = 0{\cdot}0112 \text{ inch,}$$

that is, the exterior of the A tube should be made 11-thousandths of an inch larger than the interior of the breech-piece.

$${}_2S_3 = (\tau_2 - \tau_2') \times 2r_2 \div M = 9{\cdot}7 \times 17{\cdot}4 \div 12{,}500 = 0{\cdot}0135 \text{ inch,}$$

or after the breech-piece has been shrunk on, its outside (now expanded) diameter should be 13-thousandths of an inch larger than the interior of the jacket.

$${}_3S_4 = \tau_3 \times 2r_3 \div M = 6{\cdot}3 \times 23{\cdot}6 \div 12{,}500 = 0{\cdot}012 \text{ inch,}$$

the elongation of the external diameter of the jacket up to its final diameter of 23·6 inches.

The values of τ_m and τ_m' are the *initial* stresses, and as the powder pressure p_m at r_m increases them by equal amounts to T_m and T_m', the firing stresses, their difference is unaltered, so that

$$(38) \qquad \tau_m - \tau_m' = T_m - T_m' ;$$

hence as long as we are considering the shrinkage between hoops, we can calculate it either from the firing, or initial stresses: for example, as above

$${}_1S_2 = (T_1 - T_1') \times 2r_1 \div M = (18 - 5{\cdot}3) \times 11{\cdot}2 \div 12{,}500$$
$$= 0{\cdot}0112 \text{ as before.}$$

With several layers of metal the addition of each part that is shrunk on, modifies the initial stresses previously existing.

FIG. 6—SHRINKAGE EXAGGERATED 50 TIMES.

Scale, $\frac{1}{8}$.

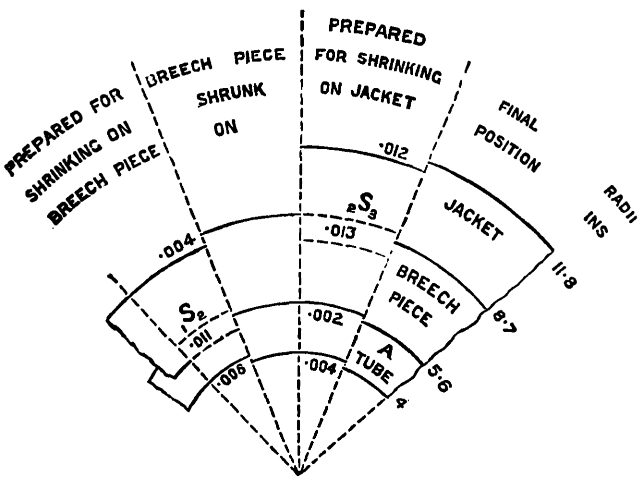


Fig. 6 shows the shrinkage (exaggerated for clearness) of the different parts, and the intermediate and final arrangements when a breech-piece and jacket are successively shrunk on, over the chamber portion of the A tube of a 6-inch B.L. gun.

WIRE GUN CONSTRUCTION.

An inspection of figs. 2A and 3A and of the serrated edge of the curve of circumferential tension, T_0, T_1, T_1', &c., shows that only the *inner* fibre of each layer of metal is doing its full share of resistance when the gun is fired.

Great economy of material would be effected if we could make all the circumferential fibres take up a full uniform tension (say of 18 tons per square inch) on firing; but to secure this condition only approximately, the number of layers of metal would have to be largely increased, and the cost, complication, and time of manufacture of a gun would be enormous.

But by adopting Mr. J. A. Longridge's plan of strengthening the tube by steel wire, wound round with appropriately varying tension, we are able to make the curve of circumferential firing tension $T_1T'_2$ a straight line for a given powder pressure (fig. 7A), and now all parts of the wire coil are equally strained under the interior pressure, and take an equal share in the resistance.

For full theoretical investigation of this subject, see Mr. Longridge's "Treatise on the Application of Wire to the Construction of Ordnance" (1884), and a paper of 1887, "Further Investigations regarding Wire Gun Construction," also a Work by Lieutenant G. Moch, "Les Canons à Fils d'Acier."

The following gives an illustration of the distribution of the firing, powder, and initial stresses in a wire gun, and shows how from given conditions the necessary calculations reparding them may be made, the method and formulas depending for the most part on what has already been explained:—

Taking the cross section of the gun across the powder chamber as composed of an A tube, a wire coil, and an outer jacket, then in the ideal state, the firing stresses will be represented in fig. 7A, where the curve of circumferential tension T_1T_2' in the wire coil becomes a straight line.

In the gun taken as an example, the inner tube is composed of two parts, an inner A tube and an A tube proper, but as there is no recognised shrinkage between these parts under the wire, they are treated in the calculations as one thickness of metal.

The jacket is required for the protection of the wire from damage and to provide the necessary longitudinal strength; it is fitted over the wire without any appreciable shrinkage.

When the gun is at rest the jacket will be free from stress, but when the gun is fired we may suppose the stress in it to be the powder stress only, on the assumption that the gun behaves as if homogeneous; then the curves t_3t_2 or $T_3'T_2$ of circumferential tension and r_3p_2 or r_3P_2 of radial pressure (the capital letters, as before, representing firing stresses) will be Barlow curves, the reflexions of each other.

The continuation of the Barlow curve r_3p_2 in fig. 7B down to p_0 will give graphically the powder pressure p_0, but now the curve of firing radial pressure in the wire and tube will be the broken curve $P_2P_1P_0$ (fig. 7A), of which P_1P_0 in the A tube is the portion of another Barlow curve, but of which P_2P_1 in the wire is a hyperbola and its equation is

(39) $$P + T = \frac{A}{r}$$

Fig. 7A.

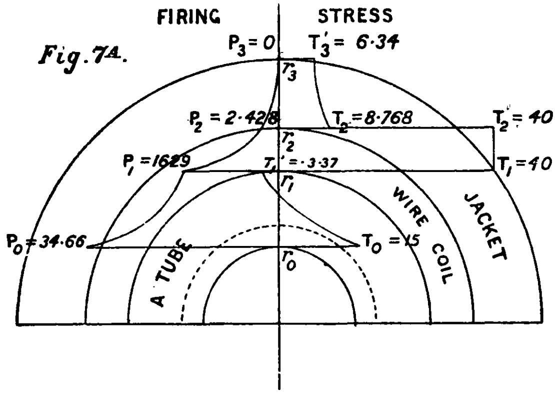
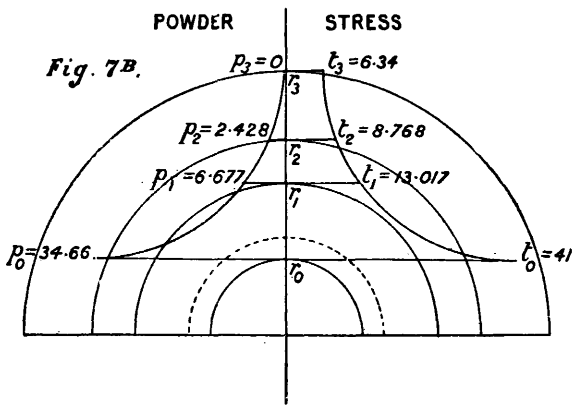


Fig. 7B.



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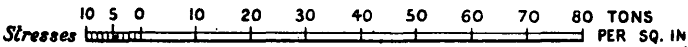
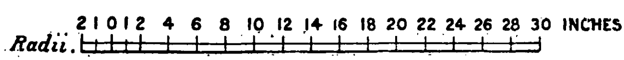




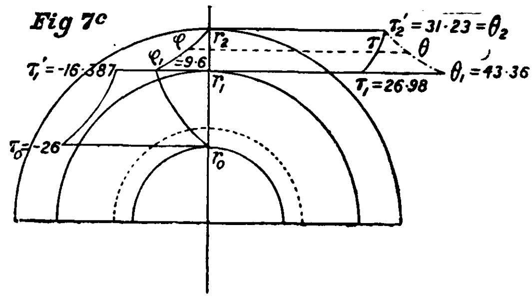
It will be noticed in fig. 7A that $T_0 = 15$. This is the maximum circumferential stress that is put on the inner edge of the A tube, and is therefore the same as in the ordinary steel construction.

The outer edge of the A tube is still in a slight state of compression on firing. We can see, therefore, from fig. 7A, that the chief stress on firing is thrown upon the wire.

If we now in the usual way deduct the powder stress from the firing stress, we shall obtain the initial state of stress in the gun, as shown in fig. 7C.

Fig. 7C.

INITIAL STRESS



The jacket being fitted without shrinkage, there will be no initial stresses in it. In other words,

$$P_3 = p_3,\ P_2,\ T_3' = t_3\ \ T_2 = t_2.$$

In the wire coil the state of initial circumferential tension will be obtained by subtracting the ordinates r_1t_1 . . . r_2t_2 of the Barlow curve (fig. 7B) from the ordinates of the straight line T_1T_2' (fig. 7A), whence we obtain the symmetrical Barlow curve $\tau_1\tau_2$ (fig. 7C) for the state of initial tension in the wire.

We see, therefore, that the wire is all in a state of extension, the outer layers more so than the inner ones, as they experience less compression from layers above.

The curve $r_2\phi_1$ of initial radial pressure in the wire coil, obtained by subtracting the ordinates of the Barlow curve p_2p_1 (fig. 7A) from the hyperbola P_2P_1, is now easily plotted, but is of a more complicated analytical character.

Finally we come to the state of initial stress in the A tube, obtained also by subtracting the powder from the firing stresses, and we obtain the curve of initial radial pressure $r_0\phi_1$, a Barlow curve, and its reflection, $\tau_0\tau_1'$, the curve of circumferential pressure or negative tension, in the tube.

For equilibrium the areas $r_1\tau_1\tau_2'r_2$ and $r_0\tau_0\tau_1'r_1$ must be equal.

We see from fig. 7C that the A tube is in a state of considerable compression, especially the inner edge, and the manufacturer has to be careful that this compression is not excessive.

So far we have only given a sketch of the principle of wire gun construction, but the gunmaker has to start with some data, usually T_0, the maximum circumferential stress to be allowed in the A tube on firing, and τ_0, the maximum circumferential pressure in the tube in repose.

Over the chamber T_0 is usually taken at 15, and τ_0 at about 26, negative tension. In the forward parts of the gun, where the thickness of the wire coil is considerably reduced, the value of τ_0 is usually lower, so as to avoid excessive tensions on the wire.

The gunmaker must also fix the diameter of the bore or chamber and the thickness of the metal in the various layers; in this, experience is the best guide. These points being settled, he can now find

P_0, the maximum allowable powder pressure in the bore, and also the corresponding circumferential tension in the wire coil in firing. Should this latter be excessive, say over 50 tons/in.², or P_0 be too small, then he can manipulate the radii until the desired result is attained, say a gun with a normal pressure of about 17 tons/in.², and a circumferential factor of safety of 2, making $P_0 = 34$, and the circumferential tension of the wire in firing, say about 40 tons/in².

We will now indicate how, with the above data, the necessary results can be obtained, and then work out an example.

As τ_0 and T_0 are given, we have t_0 directly from

$$t_0 = T_0 - \tau_0$$

(τ_0 of course is negative);

then
$$t_0 = p_0 \frac{r_n^2 + r_0^2}{r_n^2 - r_0^2}$$

gives p_0, where r_n is the radius to the outside of the gun, and r_0 of the bore.

Now $p_0 = P_0$, and since the curve P_0P_1 (fig. 7A) is a Barlow curve, we obtain P_1 from the gun maker formula (17)

$$P_{n-1} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2}(T_{n-1} + P_n) + P_n$$

by putting $n = 1$.

Now we know P_0, P_1, P_2, and P_3.

To obtain T we have the general equation to the hyperbola

$$P = \frac{A}{r} - T,$$

or
$$P_1 = \frac{A}{r_1} - T$$

$$P_2 = \frac{A}{r_2} - T.$$

Here T is the same as T_1 and T_2', since T_1T_2', (fig. 7A) is a straight line.

The above reduces to

$$P_1r_1 - P_2r_2 = T\ (r_2 - r_1),$$

which gives T.

In the later wire guns, such as the 12-inch VIII, the wire is wound on to the A tube after the inner A tube has been fitted into it, and so at first the A tube is not fully compressed, which gives an advantage in the first life of the gun, but on boring out for lining then the full compression takes place, and the advantage referred to is lost.

Similarly in the 6-inch Q F., Mark II, where the 1-B tube is shrunk on to the A tube under the wire, the metal under the wire coil should be treated as homogeneous, because in boring out for lining the A tube will be compressed, and the advantage of the shrinkage lost.

Numerical example of a wire gun (*see* figs. 7A, 7B, and 7C) the conditions being that

τ_0 shall not exceed -26,

T_0 „ 15.

Suppose $r_0 = 5{\cdot}25$, then $r_0^2 = 27{\cdot}57$.

$r_1 = 10{\cdot}3$, $r_1^2 = 106$.

$r_2 = 13{\cdot}66$, $r_2^2 = 186{\cdot}5$.

$r_3 = 18{\cdot}15$, $r_3^2 = 329{\cdot}5$.

$$t_0 = \tau_0 - T_0$$

$$= 15 + 26 = 41$$

Using the formula

$$p_0 = t_0 \frac{r_3^2 - r_0^2}{r_3^2 + r_0^2}$$

we have

$$p_0 = \frac{41 \times 301{\cdot}93}{357{\cdot}07}$$

$$= 34{\cdot}66.$$

Now to obtain the other powder stress as shown in fig. 7B,

$$p_1 = p_0 \frac{r_0^2 (r_3^2 - r_1^2)}{r_1^2 (r_3^2 - s_0^2)}$$

$$= 34{\cdot}66 \frac{27{\cdot}57 (329{\cdot}5 - 106)}{106 (329{\cdot}5 - 27{\cdot}57)}$$

$$= 34{\cdot}66 \frac{27{\cdot}77 \times 223{\cdot}5}{106 \times 301{\cdot}93}$$

$$= 6{\cdot}677.$$

$$p_2 = 34{\cdot}66 \frac{27{\cdot}57 \times 143}{186{\cdot}5 \times 301{\cdot}93}$$

$$= 2{\cdot}428.$$

The curve of radial powder pressure being now complete, and t_0 being known, the curve of circumferential stress can be drawn, as shown in fig. 7B.

Again, for the firing stress,

$$P_0 = p_0 = 34{\cdot}66.$$

To obtain P_1 we have

$$P_{n-1} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2} (T_{n-1} + P_n) + P_n,$$

or making $n = 1$,

$$P_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2} (T_0 + P_1) + P$$

$$34{\cdot}66 = \frac{78{\cdot}43}{133{\cdot}57} (15 + P_1) + P_1$$

from which

$$P_1 = 16{\cdot}29.$$

To obtain T we use the formula

$$P_1 r_1 - P_2 r_2 = T (r_2 - r_1).$$

$$16{\cdot}29 \times 10{\cdot}3 - 2{\cdot}428 \times 13{\cdot}66 = T\ 3{\cdot}36$$

$$T = \frac{134{\cdot}65}{3{\cdot}36}$$

$$= 40{\cdot}06.$$

Fig. 7A can now be completed, the stress in the jacket being the same as in fig 7B. $T_0 T_1'$ is a Barlow curve, and can be obtained in the usual way.

Therefore we see that in a gun of this construction the tension in the wire coil on firing is about 40 tons/in.^{2}, a very reasonable amount; also to strain the wire to this extent we must have a powder pressure of 34·66 tons/in.^{2}.

So that if the normal chamber pressure of the service charge is limited to 17 tons/in.^{2}, the circumferential factor of safety would be about 2, which is higher than that employed in most of the ordinary steel B.L. guns.

To obtain the initial stress (fig. 13) the powder must be subtracted from the firing stress. The jacket is not shown; not being shrunk on, there is no initial stress in it.

In the wire coil

$$\tau_1 = T_1 - t_1$$
$$= 40 - 13{\cdot}02$$
$$= 26{\cdot}98.$$
$$\tau_2' = T_2' - t_2$$
$$= 40 - 8{\cdot}768$$
$$= 31{\cdot}23.$$

In the A tube

$$\tau_0 = T_0 - t_0$$
$$= 15 - 41$$
$$= -26.$$
$$\tau_1' = T_1' - t_1$$
$$= -3{\cdot}37 - 13{\cdot}017$$
$$= -16{\cdot}387.$$

Lastly, there remains the importain practical detail to settle, viz., the tension with which the wire must be wound on to the tube, in order that when the coil is completed the curve of initial tension of the wire should be $\tau_1 \tau \tau_2'$, as already fixed. The curve of winding tension is shown in fig. 7c as $\theta_2 \theta \theta_1$.

Considering the very much simplified case of uniform modulus of elasticity, to determine θ for any radius r in the wire coil it is assumed that the winding tension θ of the wire is equal to the initial tension τ increased by the circumferential tension (of negative value and, therefore pressure) due to the initial radial pressure ϕ at the radius r acting on the tube and partly finished coil between the radii r_0 and r, and thus

$$\theta = \tau + \phi \frac{r^2 + r_0^2}{r^2 - r_0^2}.$$

In other words, it is assumed that the tension of repose τ is less than the winding tension θ by the amount due to the pressure ϕ, at a radius r and zero pressure at the radius r_0, treating the material as homogeneous.

$$\text{At } r_2,\ \phi_2 = 0;\ \text{therefore } \theta_2 = \tau_2' = 31{\cdot}23.$$

This is obviously the case, as the winding tension of the last layer of wire must be the same as the tension in repose.

In the example given we have

$$\begin{aligned}\theta_1 &= \tau_1 + \phi_1 \frac{r_1^2 + r_0^2}{r^2_1 - r_0^2} \\ &= 26{\cdot}98 + 9{\cdot}6\,\frac{106 + 27{\cdot}57}{106 - 27{\cdot}57} \\ &= 26{\cdot}98 + 16{\cdot}35 \\ &= 43{\cdot}33.\end{aligned}$$

This is the same thing as adding together the values for τ_1 and τ_1', treating τ_1' as positive. Thus

$$\begin{aligned}\theta_1 &= \tau_1 + \tau_1' \\ &= 26{\cdot}98 + 16{\cdot}387 \\ &= 43{\cdot}36.\end{aligned}$$

The reason of this is, of course, that 16·387 is the amount of circumferential tension due to the initial radial pressure of 9·6 at the radius 10·3.

Suppose now that we want to find out the strength of the gun with the inner A tube cracked through longitudinally on both sides. In this case we must find the value of P on the curve P_0P_1 (fig. 7A), at the junction of the inner A, and A tube.

Here we have a Barlow curve, P_0P_1, with $P_0 = 34{\cdot}66$ and $P_1 = 16{\cdot}29$, and the equation of the curve is

$$p = \frac{a}{r^2} - b,$$

so that

$$16{\cdot}29 = \frac{a}{106} - b,$$

$$34{\cdot}66 = \frac{a}{27{\cdot}58} - b,$$

from which to find a and b, the constants of the curve

$$a = 685, \qquad b = -9{\cdot}849.$$

so that

$$P = \frac{a}{r^2} - b,$$

where r is the radius to the outer edge of the inner A tube.

Now

$$r = 6{\cdot}804$$

$$P = 24{\cdot}639.$$

Now although the tensile strength of the inner A tube is *nil*, yet it serves to diminish the surface over which the powder pressure acts, and therefore P is the pressure transmitted to the interior of the A tube of radius r, and P_0 the pressure in the chamber of radius r_0, we have

$$P_0 = P\frac{r}{r_0},$$

where P_0 will again be the maximum allowable pressure in the chamber

$$P_0 = 24{\cdot}639\,\frac{6{\cdot}804}{5{\cdot}25}$$

$$= 31{\cdot}93.$$

To ascertain the state of stress in the various layers on firing the usual service charge, with a chamber pressure of 17 tons/in.², all that is necessary is to calculate a new set of stresses for the homogeneous gun with $p_0 = 17$, and add them to the initial stresses, which of course remain as before, and thus obtain the state of stress on firing.

Graphical Method.

The powder and firing stresses can also be obtained by a graphical geometrical procedure in a somewhat similar manner to that given previously (page 134). There will be a slight variation in the procedure consequent on the alteration of data.

In this case we are given t_0, and must first obtain p_0; the various steps can easily be followed in fig. 15.

Then, knowing ,P_0 and T_0, the firing stresses can be obtained as follows (fig. 8A): Bisect P_0T_0 at m_0. Draw m_0C' parallel to OP_3. Join A′C′, and produce it to cut the tangent at B in L.

Draw LI parallel to OP_3, cutting OA in I. Join A′I, cutting P_0T_0 in n_0. Draw n_0P_1 parallel to OP_3, cutting P_1r_1 in P_1, and A_2r_2 in A_2. This gives the point P_1 on the Barlow curve P_0P_1.

Again, P_2 is obtained from the powder stresses. Draw P_2n_1 parallel to OP_2, cutting P_1r_1 in n_1. Join A_2n_1, and produce it to cut OA in A. Draw AT_1T_2' parallel to OP_3. This gives the state of stress in the wire coil. As m_0 is the centre of the Barlow curve, T_1' can be obtained by measuring off $m_1T_1' = m_1P_1$.

The curve $T_3'T_2$ is obtained from the powder stresses.

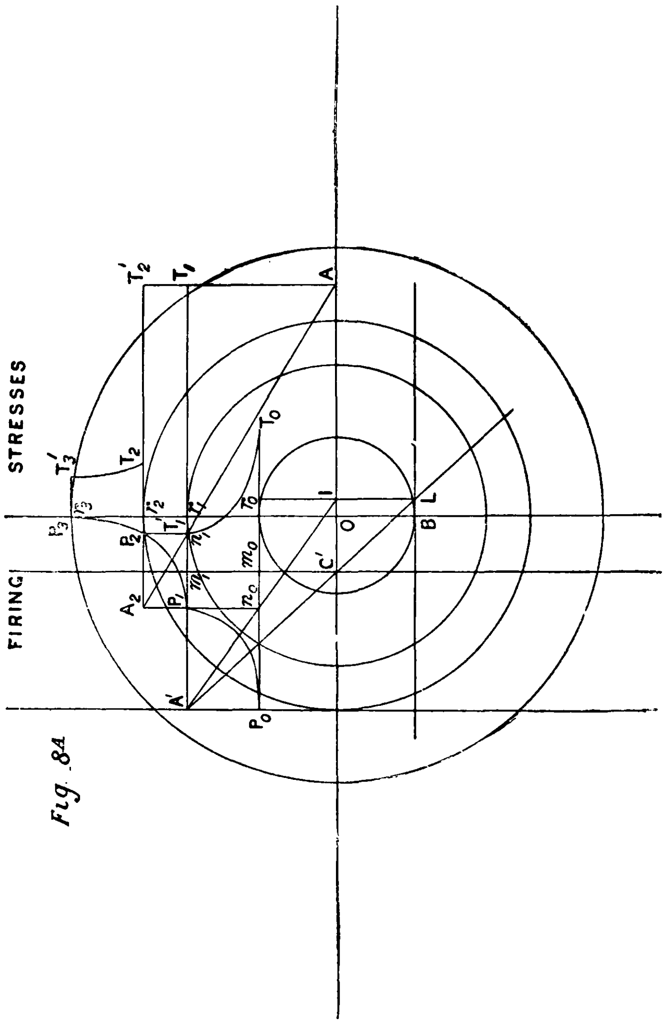


Fig. 84

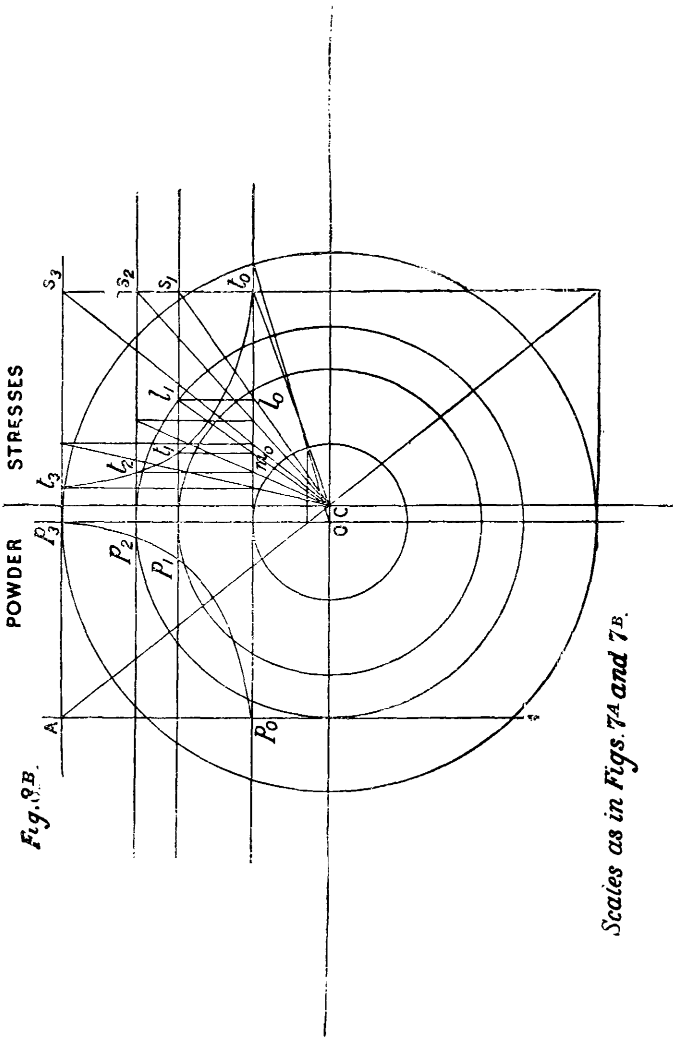


Fig. 9B.

Scales as in Figs. 7A and 7B.

The Longitudinal Tension in the Gun.

Practically it is usual to take the longitudinal tension as uniform across a cross section and as due to the powder pressure in the bore, treated as a closed vessel, closed at one end by the breech-screw, and at the other by the shot.

Thus supposing a breech-screw to gear into an A tube of which the internal radius is r_0 and the external radius of the gun is r_2, taking P_0 as the powder pressure, the average value R of the longitudinal tension will be found as follows:—The pressure multiplied by the sectional area of the chamber is resisted by a cross section of the metal subjected to longitudinal tension (according to the design of the gun); for equilibrium, these must be equal, therefore in this case

$$P_0 \pi r_0^2 = R\pi (r_2^2 - r_0^2)$$

$$R = P_0 \frac{r_0^2}{r_2^2 - r_0^2} \text{ tons/in.}^2.$$

In all steel guns of modern construction the breech-screw gears into the layer of metal above the A tube; in smaller guns direct; in heavier pieces of the most recent construction by means of a steel bush; the inner tube is thus relieved of longitudinal stress at the breech. For a gun consisting of a tube and jacket only, the formula would then become

$$R = P_0 \frac{r_0^2}{r_2^2 - r_1^2}.$$

Considering the longitudinal strength of the 6-inch B.L. gun,

$$P_0 = \frac{r_3^2 - r_1^2}{r_0^2} R;$$

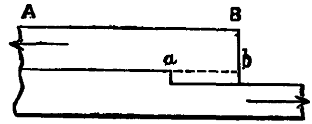
with the given numerical values and putting $R = R_1 = 18$ gives $P_0 = 121$, and dividing this by a normal pressure of 17 tons, we get a longitudinal factor of safety $= 7{\cdot}1$.

Practically the longitudinal strength is considered separately from the circumferential, and is specially provided for by shoulder, of which the *resistance to shearing* constitutes the longitudinal strength as calculated. No account is taken of frictional grip due to shrinkage, for it is considered as extremely probable that at the critical moment this becomes loosened by the elasticity of the different layers asserting itself more rapidly towards the interior as soon as the highest pressure has passed, while there is still a considerable longitudinal stress.

It is considered inadvisable to rely upon shrinkage in any way for longitudinal strength, and, consequently, any strength in this direction derived from the frictional grip will be in addition to the calculated strength.

The strain sustained by a shoulder is taken as a purely shearing one, and the strength of a shoulder is consequently dependent on its length; shearing strength, like resistance to tension, being directly proportional to the extent of surface where separation would take place. The strength is also here taken to be the same (in tons per square inch), which, if not strictly true, is rather in favour of the shearing strength.

The calculation, therefore, of length of shoulder for a given hoop is simple. For if AB is the supporting hoop, of internal and external



radii r_1, r_2 and length of shoulder ab ($= l$ say), it is only necessary to make the cylindrical area of ab = the annular sectional area of the hoop, or—

$$\pi(r_2^2 - r_1^2) = 2\pi r_1 l,$$

whence, $$l = \frac{r_2^2 - r_1^2}{2r_1}$$

The actual longitudinal strength of this arrangement would appear to be—

$$\pi(r_2^2 - r_1^2)\ \mathrm{T},$$

or $$2\pi r_1\ l\mathrm{T},$$

T being the resistance to rupture by tension or shearing in tons per square inch of material where separation would take place.

EXAMPLES.

1. What is the maximum allowable pressure in the chamber (diam. 8 inches) of a 6-inch B.L. steel gun having an A tube $1\frac{1}{2}$ inches thick, breech-piece and jacket 3 inches each?

(*Answer*, 24 tons/in.^{2}.)

2. What is the maximum allowable chamber pressure in a steel gun having a chamber diameter 4 inches, and thickness of metal in A tube and jacket 1 and 2 inches respectively?

(*Answer*, 17·6 tons/in.^{2}.)

3. Calculate the initial stress of repose in a 5-inch B.L. steel gun having chamber diameter 6 inches, tube 2 inches, and jacket 4 inches thick?

(*Answer*, $\tau_0 = -11{\cdot}25$, $\tau_1' = -7{\cdot}63$, $\tau_1 = 6{\cdot}87$, $\tau_2' = 3{\cdot}25$, $\phi_1 = 3{\cdot}62$.)

4. What will be stress on firing of the gun in problem 3 if a charge giving 16 tons/in.^{2} chamber pressure is used?

(*Answer*, $P_0 = 16$, $P_1 = 8{\cdot}1$, $T_0 = 8{\cdot}75$, $T_1 = 15{\cdot}35$, $T_1' = {\cdot}85$, $T_2 = 7{\cdot}25$, $p_1 = 4{\cdot}48$, $t_0 = 20$, $t_1 = 8{\cdot}48$, $t_2' = 4$.)

5. A gun of two layers of steel is to be designed to stand a maximum pressure in the chamber (diameter 5·8 inches) of 21 tons/in.^{2}, the tube being 2·1 inches thick; calculate what should be the external diameter of the gun.

(*Answer*, 17·372 inches.)

6. Calculate what the thickness of steel should be at two points on the chase of a light Q.F. gun of 3 inches calibre, where gas pressures of 5 and 4 tons/in.^{2} respectively are expected. A factor of safety of 2 must be allowed, and the surface of the bore must not be strained to more than 15 tons/in.^{2}.

(*Answer*, 1·854 inches and 1·219 inches.)

7. The wire coil of a 4·7-inch quick-firing gun exerts an initial radial pressure on the A tube of 7·1 tons/in.^{2}; calculate the state of initial hoop pressure of the latter, which is $1\frac{1}{4}$ inch thick, and has an internal diameter (at the centre of the cartridge chamber) of 5 inches.

(*Answer*, 18·46 tons/in.^{2} at the outside.)
25·56 „ „ inside.)

(8.) In a 12-pr. Q.F gun of 12 cwt. calculate:—

(*a*.) The maximum allowable pressure on firing at a point 54·35 inches from the front of the chamber.

(*b*.) The factor of safety at the point mentioned in (*a*):—

Diameter of bore, 3 inches.
Thickness of A tube, ·75 inch.
„ „ B „ ·95 „
Charge, 1 lb. 15 oz. cordite.
Cubic capacity of chamber, 125 in^3.

The table of pressure in an explosion vessel, page 104, to be used.

(*Answer*, about 1·98.)

(9.) In a 9·2 inch Mark V gun, calculate the factor of safety at a point 153·8 inches from the front of the chamber (just in front of the 1B tube), considering the gun as made of one thickness of metal at this point:—

Diameter of bore, 9·2 inches.
Thickness of metal, 4·4 inches.
Charge, 164 lb. powder.
Cubic capacity of chamber, 4,950 in.^{3}

Using the same table of pressure as in previous example.

(*Answer*, about 1·44.)

10. In the three following wire guns, compare the maximum allowable pressure in the chamber on firing, supposing that on the inner surface of the A tube the maximum allowable tension on firing is 15 tons/in.^{2}, and the maximum allowable compression at rest is 26 tons/in.^{2}.

	Diameter of chamber.	Thickness of A tube.	Thickness of wire coil.	Thickness of jacket.
a ..	16	4·44	4·56	4
b ..	17·5	4·88	4·80	3·87
c ..	18	4·335	4·58	3·085

(*Answer*, (*a*.) 30·61 tons/in.^{2}
(*b*.) 30·05 „
(*c*.) 28·28 „ .)

11. Find the winding-on tension of the first and last layers of wire in the case of the gun in question 10, *a*.

(*Answer*, 35·19; 22·68.)

12. Supposing the inner A tube (thickness 1·925 inch) of the gun in question 10*a* to be split, find the maximum allowable pressure in the chamber. (*Answer*, 28·1 tons/in.^{2}.)

13. Calculate the firing stress in the case of the gun given in question 10, *b*, supposing that the service charge gave a maximum pressure of 15 tons/in.^{2}.

(*Answer*—
$T_0 = -\ 5·552$ $\quad T_1' = -\ 9·095$
$T_1 = 27·145$ $\quad T_2' = 30·475$
$T_2 = 6·715$ $\quad T_3' = 5·448$.)

SECTION II.—THE RIFLING OF GUNS.

A gun is said to be rifled when the interior of the bore is cut into a number of spiral grooves, intended to engage in the projectile, and to give it a spin on leaving the muzzle.

In this manner the gun is enabled to fire an elongated projectile heavier than a spherical shot, and less influenced by the resistance of the air, so that the projectile ranges farther and hits harder and straighter; the spin imparted is sufficient to keep the shot moving point foremost, as otherwise the shot, if fired from a smooth bore, would soon set its axis across the line of motion, and the accuracy and range would be greatly diminished.

Small arms have been rifled from a very early date, as specimens preserved in Continental museums will prove; but, as the bullets employed were spherical, the only effect of the rifling was to increase the accuracy by distributing the resistance of the air equally over the foremost surface of the bullet.

The first suggestion of an elongated (egg-shaped) bullet appears in a paper by Benjamin Robins, "Of the Nature and Advantage of Rifled Barrel Pieces," read before the Royal Society on the 2nd July, 1747, but although Robins concludes by saying, as quoted also in Colonel Owen's "Modern Artillery," p. 175, "I shall, therefore, close this paper with predicting that whatever State shall thoroughly comprehend the nature and advantages of rifled barrel pieces, and, having facilitated and completed their construction, shall introduce into their armies their general use with a dexterity in the management of them; they will by this means acquire a superiority which will almost equal anything that has been done at any time by the particular excellence of any one kind of arms, and will, perhaps, fall but little short of the wonderful effects which histories relate to have been formerly produced by the first inventors of firearms;" it was not, however, till the Crimean War of 1854 that the elongated bullet was introduced with the French Minié rifle, and rifled field artillery did not come in till the Italian campaigns of 1860.

The requisite angle at which the grooves leave the muzzle is determined by the outside shape and proportions of the projectile, and by its interior density and distribution of material; but the grooves in passing from the breech to the muzzle may be made either—

(i) in a uniform twist,
or (ii) in an increasing twist.

In some small arms a *progressive* groove is employed, in which the depth of the groove varies, so that the bullet is gripped tighter, and the lead is more compressed as the bullet passes along the bore to the muzzle.

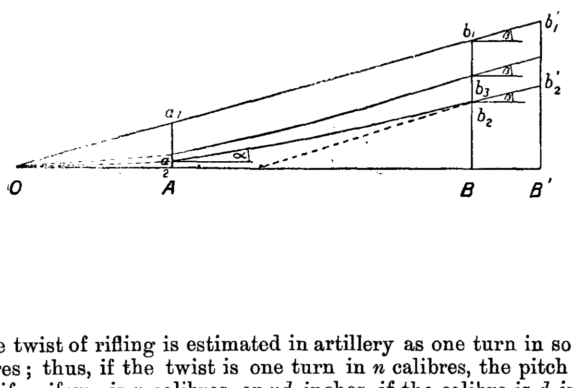
In the uniform twist the form of the grooves is a uniform spiral or helix, so that, if traced on a sheet of paper which is wrapped on the interior of the bore, the curve becomes a straight line when the paper is developed, or laid out flat.

This can be illustrated by a sheet of paper wrapped round a pencil, the angle of the rifling being the angle between the axis of the pencil and the edge of the paper.

In the gaining twist the developed curve is one which becomes more and more inclined to the line of axis of the bore in passing from the breech to the muzzle, where the inclination β to the axis must be the same as in the uniform twist of the gun is required to fire the same projectile.

The shape of the curve of the groove when developed is exhibited in the form and position of the rifling bar of a rifling machine, taking AB to represent the line of axis, and a_1b_1, *or* a_3b_3, the rifling bar or the developed curve of the groove.

Fig. 1.



The twist of rifling is estimated in artillery as one turn in so many calibres; thus, if the twist is one turn in n calibres, the pitch of the helix, if uniform is n calibres, or nd inches, if the calibre is d inches; and if β denotes the angle of rifling, that is the angle which the groove makes with the axis,

$$\tan \beta = \frac{\pi d}{nd} = \frac{\pi}{n}.$$

Thus, if $\beta = 7°$, $n = \pi \cot \beta = 25{\cdot}6$.

If the powder pressure and the frictional resistance in the bore are uniform, then the forces producing the rotation are uniform with a uniform twist.

But, as the powder pressure reaches a maximum after a short travel of the projectile, and afterwards rapidly diminishes towards the muzzle, the forces producing rotation on a uniform twist are apt to become extensive at the outset near the breech.

With a view of lessening this excessive stress and of producing more uniformity, the gaining twist was adopted.

The first increasing twist employed was parabolic in its developed form, starting from its vertex O parallel to the axis OAB, and finishing at the muzzle at b_2 at an inclination β, the same as that of the uniform twist, developed into the straight line Ob.

Then, from the property of the parabola, the tangent at b_2, which is parallel to Ob, will pass through the middle point of OB; also, b_2 will be the middle point of Bb_1, so that the projectile will now be turned through only half the angle turned through on the uniform twist, and this was formerly considered an advantage.

It was found, however, that this parabolic twist threw too much strain on the muzzle B, so an intermediate twist was adopted in the 80-ton gun, composed of a curve called the *semi-cubical parabola*, starting from O in the direction of OA, and finishing at b_3 in the direction Ob, such that Bb_3 = $\frac{2}{3}$ Bb, and the tangent at b_3, therefore, passes through the point of trisection of OB, nearest to O.

But now it is considered preferable, when a gaining twist is employed, to take the breech line at a point A instead of O, so that the rifling bar is a_2b_2, starting from a_2 at a certain angle a, with the axis of the bore at AB, or at a certain twist of one turn in m calibres, where m is given by—

$$\tan a = \frac{\pi}{m};$$

and to steady the shot at the muzzle, the muzzle line is moved from B to B′, and the part of the rifling bar from b_2 to b_2' is made straight and parallel to b_1b_1', so that the twist is uniform from B to B′.

Thus, the rifling starts at the breech line A at a twist of one turn in m calibres, and the twist increases up to one turn in n calibres at B, usually on a parabolic twist, although the bar a_2b_2 may be the arc of a circle; and from B to the muzzle B′ the twist is uniform, and one turn in n calibres.

Thus, for instance, the 6-inch Mark I gun is rifled for 65 inches with a twist increasing from one in 120 to one in 35, and continued for the remaining 59 inches to the muzzle on a uniform twist of one in 35.

But Sir Andrew Noble has found experimentally that the work lost by the friction of the grooves, and the mean rotating pressure is less with the uniform than with the gaining twist, although the maximum initial forces producing rotation are greater; this is shown by the accompanying Tables (I and II) of results of experiments with a 12-cm. (4·7-inch) gun, firing a 45-lb. projectile with a velocity of 2084 fs, published in the "Proceedings of the Royal Society," vol. 50.

Table I.

Travel of shot in the bore.	Total thrust on the base of the shot.	Velocity acquired.	Total thrust between the driving face of the grooves and the driving ring.	
			Uniform twist.	Increasing twist.
feet.	tons.	ft/sec.	tons.	tons.
0·5	254·7	548	19·9	7·9
1·0	264·0	849	20·7	9·7
1·5	245·0	1064	19·2	10·3
2·0	207·9	1224	16·3	10·5
2·5	175·7	1343	13·7	10·5
3·0	150·7	1437	11·8	10·4
4·0	115·2	1577	9·1	10·5
5·0	94·9	1680	7·4	10·8
6·0	80·6	1761	6·3	11·1
7·0	69·5	1828	5·4	11·4
8·0	60·0	1884	4·7	11·6
9·0	52·1	1931	4·1	11·8
10·0	44·8	1970	3·5	11·9
11·0	38·4	2004	3·0	12·0
12·0	32·9	2032	2·6	12·0
13·0	28·4	2056	2·2	12·1
14·0	24·3	2076	1·9	12·1
14·4	22·6	2084	1·8	12·1

Table II.

Travel of shot in bore in feet.	Total pressure on base of shot in tons.	Velocity, ft/sec.	Total pressure R between driving surface of groove and ring of projectile in tons.
0·5	254·7	548	7·9
1·0	264·0	849	9·7
1·5	245·0	1064	10·3
2.0	207·9	1224	10·5
2·5	175·7	1343	10·5
3·0	150·7	1437	10·4
4·0	115·2	1577	10·5
5·0	94·9	1680	10·8
6·0	80·6	1761	11·1
7·0	69·5	1829	11·4
8·0	60·0	1884	11·6
9·0	52·1	1931	11·8
10·0	44·8	1970	11·9
11·0	38·4	2004	12·0
12·0	32·9	2032	12·0
13·0	28·4	2056	12·1
14·0	24·3	2076	12·1
14·4	22·6	2084	12·1

If V denotes the forward axial velocity with which the shot leaves the muzzle, then the spin imparts to the points on the outside cylindrical surface a component velocity at right angles to the axis of magnitude—

$$V \tan \beta = \frac{\pi V}{n} \text{ f/s};$$

this is called the *linear velocity of rotation.*

The *angular velocity of rotation*, in radians/second, is obtained by dividing this by the radius of the shot in feet, a or $d \div 24$; it is, therefore—

$$\frac{\pi V}{nd} \text{ or } \frac{24\pi V}{nd};$$

and this again is converted into revolutions per second by dividing by 2π, since one revolution equals 2π radians; the shot, therefore, makes—

$$\frac{V}{2nd} \text{ or } \frac{12V}{nd} \text{ revs/sec.}$$

Thus, comparing the 6-inch gun and magazine rifle, in each of which $n = 30$; then for the same muzzle velocity, say 2000 f/s, the linear velocities of rotation will be the same, namely 209 f/s, but the rifle bullet will make 2640 revs/second, against 133 revs/second of the 6-inch projectile.

Formerly it was considered requisite for a projectile to possess a given linear velocity of rotation to ensure its stability in flight, and for this reason the twist of rifling in howitzers, firing with low velocities, was made very quick, even up to one in 12 calibres.

But, it is now found that the linear velocity of rotation should be a given fraction of the initial velocity, so that the same twist of rifling is suitable for high or low velocities, with a given projectile; but the determination of the appropriate twist from theoretical considerations is not a simple matter, and the twist must be settled by experiment to a great extent.

The investigation of the stability of an elongated projectile moving through the air in the direction of its axis with given angular velocity is very similar to that required for the stability of a top or gyrostat, spinning with its axis vertical, and the behaviour of the bodies have a close analogy. The annexed Table on p. 158 shows the result of such calculations.

When a top is spun, the motion of the axis is at first unsteady, but this unsteadiness soon disappears, and the top then spins upright, when it is said to go to sleep; after a time the friction of the point reduces the spin to such an extent that the vertical position becomes unstable, and the axis again begins to wobble; the axis inclines more and more from the upright position, until finally the top falls over on its side.

So, too, an elongated projectile fired from a rifled gun is at first rather unsteady from the first portion of its flight, but the friction of the air soon destroys the irregular gyrations, and the shot, if provided with sufficient spin, proceeds steadily in the direction of the axis.

Table of Rotation for Stability of Projectiles.

(Calculated from Professor Greenhill's formula by Major Cundill, R.A., and extended by Mr. A. G. Hadcock, R.A., Inspector of Ordnance Machinery, *vide Proc. R.A.I.*, vol. xi, No. 2, and vol. xiv, No. 3.)

Length of projectile in calibres.	Minimum twist at muzzle of gun requisite to give stability = 1 turn in n calibres.			
	Cast-iron common shell; cavity $=\frac{8}{27}$ths vol. of shell. (Density of cast iron 7·207.)	Palliser shell; cavity $=\frac{1}{8}$th vol. of shell. (Density of chilled iron 8·000.)	Solid steel bullet. (Density of steel 8·000.)	Solid lead and tin bullets of similar composition to M.-H. bullets. (Density of alloy 10·9.)
	n	n	n	n
2·0	63·87	71·08	72·21	84·29
2·1	59·84	66·59	67·66	78·98
2·2	56·31	62·67	63·67	74·32
2·3	53·19	59·19	60·14	70·20
2·4	50·41	56·10	57·00	66·53
2·5	47·91	53 32	54·17	63·24
2·6	45·65	50·81	51·62	60·26
2·7	43·61	48·53	49·30	57·55
2·8	41·74	46·45	47·19	55·09
2·9	40·02	44·54	45·25	52·72
3·0	38·45	42·79	43·47	50·74
3·1	36·99	41·16	41·82	48·82
3·2	35·64	39·66	40·30	47·04
3·3	34·39	38 27	38·84	45·38
3·4	33·22	36·97	37·56	43·84
3·5	32·13	35·75	36·33	42·40
3·6	31·11	34·62	35·17	41·05
3·7	30·15	33·55	34·09	39·79
3·8	29·25	32·55	33·07	38·61
3·9	28·40	31·61	32·11	37·48
4·0	27·60	30 72	31·21	36·43
4·1	26·85	29·88	30·36	35·43
4·2	26·13	29·08	29·55	34·49
4·3	25·45	28·33	28·78	33·59
4·4	24·81	27·61	28·05	32·74
4·5	24·20	26·93	27·36	31·94
4·6	23·65	26·32	26·74	31·21
4·7	23·06	25·66	26·08	30·44
4·8	22·53	25·08	25·48	29·74
4·9	22·03	24·51	24·91	29·07
5·0	21·56	23·98	24·36	28·44
5·1	21·08	23·46	23·84	27·83
5·2	20·64	22·97	23·34	27·24
5·3	20·22	22·50	22·86	26·68
5·4	19·81	22·05	22·40	26·14
5·5	19·42	21·61	21·96	25·63
5·6	19·04	21·19	21·53	25·13
5·7	18·68	20·79	21·12	24·66
5·8	18·33	20·40	20·73	24·20
5·9	18·00	20·03	20·35	23·75
6·0	17·67	19·67	19·98	23·33
7·0	14·99	16·68	16·95	19·78
8·0	13·02	14·48	14·72	17·18
9·0	11·50	12·80	13·00	15·18
10·0	10·31	11·47	11·65	13·60

If the spin of the projectile died away more rapidly than the forward velocity, the projectile, like the top, would again become unsteady.

But the forward retardation of the shot is much greater than the angular retardation, so that the shot moves as if on an increasing screw; and practically, if once steady, the shot will continue so throughout its trajectory, in consequence of this *overscrew*.

In high angle fire, however, the motion tends to become unsteady in the descending branch, in consequence of the great curvature of the trajectory.

Drift.

This is an effect observable with all rifled guns, by which the shot is deflected in its flight more or less from the vertical plane of fire; the deflection is to the right when the gun is rifled with a twist on a right-handed screw, to the left with a left-handed twist.

Thus it was found by Mr. Rigby, Superintendent R.S.A.F., Enfield, that with two barrels rifled with right- and left-handed twists, and laid parallel, the bullets struck on a target at 1000 yards on an average 15 inches farther apart than the muzzles, showing that the *drift* of the rifle bullet at this range is about $7\frac{1}{2}$ inches.

The drift increases rapidly with the elevation and range of the gun; thus it was found that the 9·2-inch fired at Shoeburyness with an elevation of 40° and a muzzle velocity of 2375 f/s, sent a shot weighing 380 lbs. to a range of 20,000 yards, and that the drift was about 1,000 yards to the right of the vertical plane of fire.

Disregarding theories and explanations, the established facts connected with drift are as follows:—With service projectiles having pointed heads and right-handed rotation, the drift is to the right; other things remaining unchanged, it is found that the greater the twist the greater the drift; the smooth and well-centred B.L. projectiles drift less than the M.L. shells, which are roughened by studs; at extreme ranges the drift always increases rapidly, and the projectile becomes unsteady in flight, owing to the greater curvature of the trajectory.

The minor effects of the resistance of the air and of the rotation of the projectile cause the axis of the latter to remain with the axis of the shot nearly tangential to the trajectory, but with the point of the projectile a little above and to the right of the vertical plane of fire; this is verified from the fact that the holes made in wooden targets are as nearly as possible circular, even when the angle of descent is considerable, and also from watching by eye the behaviour of projectiles fired with low velocities (500—600 f/s) at considerable elevations (50—70°).

Wind Deflection.

The deflection due to wind may be investigated at this stage, as it depends upon the principle just employed for the stability and drift of projectiles; the method is due to Captain F. Younghusband, R.N., late Superintendent Royal Gun Factories, and it has the advantage of explaining the observed differences of deflection of small arm bullets and artillery projectiles, the deflection of bullets being, as is well known, so much the greater.

Suppose the wind is blowing straight across the range with velocity W f/s; then the shot, on leaving the muzzle with velocity V f/s, will have a component sidelong motion up the wind and relatively to the air of W f/s, so that the resultant velocity relatively to the air makes an angle

$$\theta = \tan^{-1} (W/V)$$

with the line of fire.

The shot soon steadies itself to move axially in this direction, and it would therefore strike a target moving along with the wind, and having this direction at the instant of firing.

But at a range of R yards (3R feet) this target at this instant would be

$$3R \tan \theta = 3WR/V$$

feet to one side of the fixed point aimed at; and in the time of flight of the shot, t seconds, over the range R yards, the target moving with the wind would have drifted Wt feet, and therefore the bullet will be carried by the wind to the distance.

$$W \left(t - \frac{3R}{V}\right) \text{ feet.}$$

to one side of the point aimed at on the fixed target.

In other words, the wind deflection is

$$W\ (t - T) \text{ feet.}$$

where $T = 3R/V$, the time of flight over the range 3R feet, provided the initial velocity V is kept up all the way.

The time of flight t is found from the Range Table, or calculated by the intermediate of the remaining velocity v at the range of 3R feet by means of Bashforth's Tables from the formulæ

$$S_v = S_V - \frac{3R}{C},$$

$$t = C(T_V - T_v),$$

where C denotes the ballistic coefficient of the projectile.

As t is increased when C is decreased, we see that the deflection is greatest with small-arm bullets, and diminishes with the size and weight of the projectile.

Thus, for instance, at a range of 1000 yards, we find that with the same muzzle velocity, 2000 f/s, that

$t = 3$ secs. for the 0·303 bullet, weighing 215 grains, and
$t = 1{\cdot}6$ secs. for the 6-inch projectile weighing 100 lbs., while
$T = 3000 \div 2000 = 1{\cdot}5$ secs.

Therefore, with W = 50 f/s., the deflection is

$$50(t - 1{\cdot}5) \text{ feet,}$$

or 75 feet for the bullet, and 5 feet for the 6-inch projectile.

PART II.

CONTENTS.

CHAPTER I.

CONSTRUCTION OF BALLISTIC TABLES.

IN Part I, Chapter II, the practical importance of the Ballistic Tables has been illustrated by various examples, in which the solution was effected by the use of the tables; and now we proceed to examine the theory upon which the calculation of the tables is based, and the experimental data upon which the calculations are founded, repeating to some extent and amplifying the explanations of Chapter II, Part I.

The first requirement is the experimental determination of the *Resistance of the Air* to a projectile moving with a velocity within the limits of those found useful in artillery.

If it were the case that the resistance of the air varied according to some simple law, such as being proportional to the square or the cube of the velocity, we should be able to infer the resistance at all velocities from one single, well-determined value of the resistance at a standard velocity.

Thus, for instance, if we found by experiment that the resistance of the air to a 6-inch projectile, moving at a velocity of 2000 f/s, was 600 lbs.; and if we were sure that the resistance varied either as the square, or cube, or generally as the nth power of the velocity, then at any other velocity v f/s, the resistance R, in pounds, would be given by either

$$R_2 = 600\left(\frac{v}{2000}\right)^2, \text{ or } R_3 = 600\left(\frac{v}{2000}\right)^3, \text{ or } R_n = 600\left(\frac{v}{2000}\right)^n$$

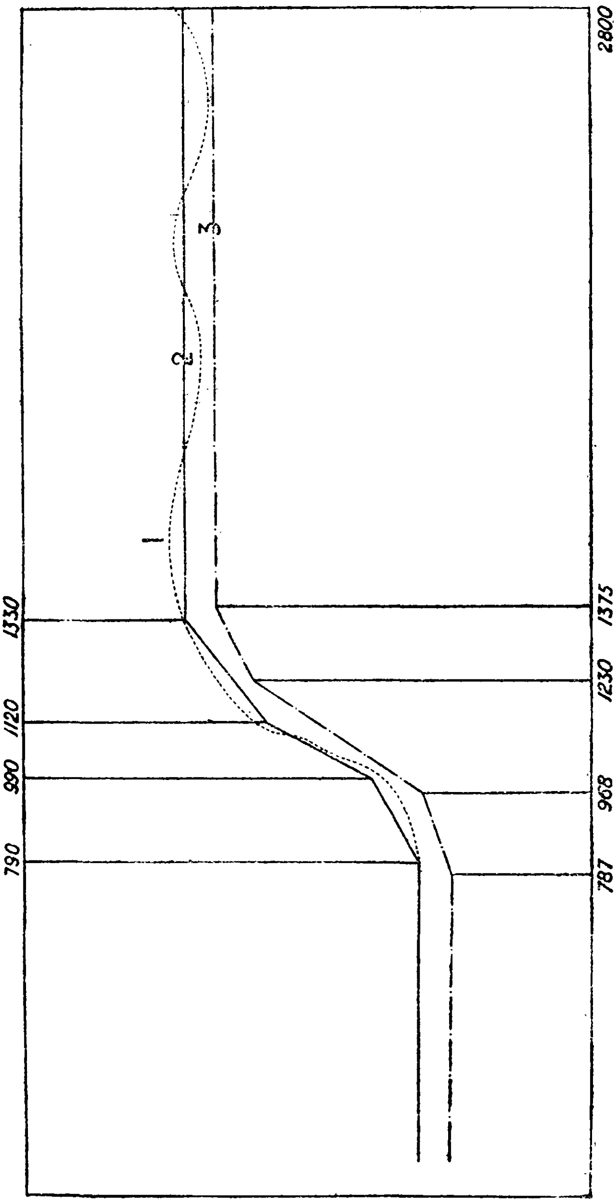
Thus, if $v = 1000$, $R_2 = 150$, $R_3 = 75$;

and if $v = 3000$. $R_2 = 1350$, $R_3 = 2025$.

But these simple mathematical laws, although occasionally useful for carrying on the ballistic tables provisionally by extrapolation beyond the limits of experimental knowledge, are not found to hold good over any extended range of the velocity within the limits of experiments hitherto carried out.

This is shown clearly in the accompanying diagram, drawn by Mr. Bashforth, in which the abscissa represents velocity, and the ordinate represents the quotient of the resistance of the air by the square of the velocity; the diagram shows that at a high or low velocity the quadratic law of resistance is a good approximation; and could be employed provisionally for calculating the resistance of the air, say, to a meteorite moving with ten times the velocity of a cannon ball.

The dotted line of curve (1) is derived from the results of the Bashforth experiments; curve (2) is drawn in accordance with the mathematical laws assumed by Colonel Siacci and Captain Ingalls; while curve (3) represents the results employed by Krupp and Mayevski.



1. Results of Bashforth experiments. (Revised account of the Bashforth Chronograph, p. 183.)
2. Law deduced from the above by Ingalls, p. 36, *Ballistics*.
3. Mayevski-Krupp Laws. (Ingalls, p. 29.)

It is better, then, to make no assumption of any mathematical law, but to determine by careful experiment the resistance of the air at a number of velocities, as explained in the next chapter, and to plot these resistances graphically (fig. 3, p. 10); and afterwards to calculate the Ballistic Tables by appropriate formulas from these experimental values.

The most important series of experiments carried out in this country are those of the Rev. F. Bashforth, B.D., the first Professor of Mathematics to the Advanced Class of Artillery Officers.

These experiments were conducted in 1865–1870 and in 1878–1879, and the results are tabulated in the *Reports on the Experiments made with the Bashforth Chronograph*, &c., 1865–1870; and *Final Report on Experiments with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Elongated Projectiles*, 1878–1880.

The projectiles employed in these experiments were of various weights and sizes, and were fired from guns of 3, 5, 6, 7, and 9 inches calibre; the external shape was nearly uniform for all, consisting of a cylindrical body with a flat or slightly rounded base, and provided with an ogival-pointed head, struck with a radius of 1½ diameters, as shown in fig. 1, Chapter II, Part I, p. 8.

A few projectiles with flat and variously shaped heads, shown in fig. 2, p. 9, were also fired, as well as spherical projectiles, so as to determine the variation of the resistance of the air with change of external shape.

As a first result of the experiments it was found that the resistance was proportional, at the same velocity, to the surface or to the square of the diameter.

(Newton, *Principia*, lib. ii, prop. xxxv, cor. 2, 3, 4, 5).

So also in naval architecture it is found that the resistance of similar vessels at the same speed is proportional, very nearly, to the *wetted surface.*

The resistance R can thus be split up into two factors, one of which is d^2, where d denotes the diameter of the shot in inches; and the other is the resistance of the air at the same velocity to a similar 1-inch projectile; this is denoted by p, so that

$$R = d^2 p,$$

and the value of p for velocities ranging from 100 to 2800 f/s is given in Table II, and plotted graphically in fig. 3, p. 10.

These values of p refer to a certain standard density of the air, of 534·22 grains per cubic foot, which is the density of dry air at sea-level, in the latitude of Greenwich, at a temperature of 62° F., and a barometric height of 30 inches.

It is further assumed that the resistance is proportional to the density of the air; so that if the density changes to δ grains per cubic foot, we must put

$$R = \tau d^2 p,$$

where

$$\tau = \frac{\delta}{534{\cdot}22};$$

and Table XI, calculated by Mr. Bashforth partially from the formula

$$\tau = \frac{460 + 32}{460 + F}\frac{b}{30},$$

derived from the laws of Boyle and Charles, gives the value of τ for different Fahrenheit temperatures F. and barometric heights b inches; this applies to dry air, so that a further correction is required from the hygrometrical tables given by the readings of the wet and dry bulb thermometers, as damp air is perceptibly lighter than dry air at the same temperature and pressure; the air is supposed to be two-thirds saturated, so that a pressure two-thirds of the pressure in inches of mercury of the aqueous vapour at the temperature F. is added. The effect of the vapour correction is to reduce the standard temperature from 62° to 60° F.

This factor τ is called the *coefficient of tenuity;* its effect becomes important in high-angle long-range fire, where the shot reaches the higher attenuated strata of the atmosphere; and where, as in the Jubilee Rounds, the barometer may sink to half its normal height or less at a height of 15,000 feet or more at the vertex of the trajectory; but in all carefully conducted experiments the value of τ should be calculated and allowed for from day to day.

On the other hand, $\tau = 800$ about, when shooting under water, the density of water being, in round numbers, 800 times that of ordinary air.

The resistance of the air is considerably reduced in modern projectiles by giving them a greater length and a sharper point; and a factor κ, called the *coefficient of shape*, is brought in to allow for this change.

For projectiles in which the ogival head is struck with a radius of 2 diameters, Mr. Bashforth puts $\kappa = 0{\cdot}975$; while, on the other hand, for flat-headed proof projectiles, κ is taken as 2, on the average.

For spherical shot κ is not constant, and a separate ballistic table (Table IX) is constructed; but $\kappa = 1{\cdot}7$, on the average.

Lastly, to allow for the superior centering of the projectile obtainable with breech-loading, Mr. Bashforth introduces a factor σ, called the *coefficient of steadiness.*

This steadiness may vary during the flight of the projectile, as the shot is often unsteady for some distance after leaving the muzzle, and finally steadies down afterwards, sometimes becoming unsteady again in high-angle howitzer fire.

For Zalinski projectiles, $\sigma = 8$ about.

Collecting all the coefficients, τ, κ, σ, we now put

$$R = nd^2p,$$

where

$$(1) \qquad n = \kappa\sigma\tau,$$

and n is called the *coefficient of reduction.*

Thus, by means of a well chosen value of n, determined by a few experiments, we can utilise the Bashforth experiments carried out with old-fashioned projectiles, pending further experiments with the most recent designs.

For instance, $n = 0{\cdot}8$ is taken a good average for the modern magazine rifle bullet.

(Rev. F. Bashforth, *Proceedings of the Royal Artillery Institution*, Vol. XIII, No. 10).

Suppose now that p has been determined experimentally and plotted for a standard projectile, fired under standard conditions in air of standard density, as explained in the next chapter.

We must first determine the time it takes for the velocity of a projectile, d inches in diameter and weighing w pounds, to fall from any initial velocity, V f/s, to any final velocity, v f/s.

If r denotes the retardation of the shot due to the resistance of R pounds, then, by Newton's Second Law of Motion,

"Change of Motion is proportional to the Impressed Force,"

$$(2) \qquad \frac{r}{g} = \frac{R}{w} = \frac{nd^2}{w}p.$$

If Δv denotes the loss of velocity in the small interval of time Δt,

$$\frac{\Delta v}{\Delta t} = \text{average retardation in the interval } \Delta t,$$

$$= r = \frac{R}{w}g = \frac{nd^2}{w}pg,$$

where p denotes the average value in the interval, and therefore

$$(3) \qquad \Delta t = \frac{w}{nd^2}\frac{v}{pg}.$$

The quantity $\frac{w}{nd^2}$ is called the *ballistic coefficient* of the projectile, and is denoted by the letter C; so that

$$\Delta t = C\frac{\Delta v}{pg};$$

or

$$\Delta t = C\Delta T,$$

where

$$(4) \qquad \Delta T = \frac{\Delta v}{pg};$$

so that ΔT is independent of the weight or size of the projectile.

Since p is tabulated as a function of v, the velocity v is taken as the argument of the table; and beginning with its lowest value 100 in Table III, v is made successively equal to 110, 120, 130, ... up to 2800, the highest value recorded in the experiments; so that Δv is constantly equal to 10.

The average value of p in an interval is taken as the arithmetic mean of the initial and final values of p in the interval; and then the successive values of ΔT are calculated from the formula (4), with $\Delta v = 10$, and tabulated in the column of differences under the head ΔT.

Afterwards these differences are summed by an arithmometer, similar to the one in the Royal Artillery Institution, and tabulated in the column under the head T.

The number T, sometimes denoted by $T(v)$, or T_v is called the *reduced time;* and

$$T(V) - T(v)$$

is the number of seconds which a projectile would take for its velocity to fall from V to v, if its ballistic coefficien C or $\frac{w}{nd^2}$ was unity, when acted upon by the resistance of the air only.

Thus, for instance, if the coefficient of reduction n is unity, then $C = 1$ for a 1-inch 1-pr., or 3-inch 9-pr.

Generally, for any projectile whose ballistic coefficient is C, if t denotes the number of seconds taken for the velocity to fall from any initial velocity V to any final velocity v, then

(5) $$t = C(T_V - T_v)$$

Next let Δs denote the number of feet traversed in the time Δt; then

$$\Delta s = v\Delta t,$$

where v denotes the mean velocity in the interval Δt.

Putting, as before,

$$\Delta t = C\Delta T,$$

and $$\Delta s = C\Delta S,$$

then

(6) $$\Delta S = v\Delta T;$$

whence ΔS can be calculated by multiplying ΔT by the corresponding mean velocity in the intervals, taken as the arithmetic mean in the interval,

105, 115, 125 .... .

These differences ΔS are entered in the corresponding column of the table; and their sum, obtained by the arithmometer, is entered in the column headed S.

The number S, variously denoted by $S(v)$, or S_v, is called the *reduced*, or *tabular* distance or range; and

$$S(V) - S(v)$$

is the number of feet which a standard projectile, for which $C = 1$, would go while the velocity fell from V to v under the influence of the resistance of the air, the attraction of gravity being left out of account.

Generally, for a projectile whose ballistic coefficient is C, the distance gone in feet while the velocity drops from V to v will be

(7) $$C\{S(V) - S(v)\}.$$

To save the trouble of proportional parts required when the velocity proceeds by increments of 10, Mr. Bashforth tabulates by interpolation the values of T and S for unit increments of f/s in the velocity, as given in his Tables III and IV.

It will be noticed that his tables are carried down to a velocity of 100 f/s; also that the initial values of T and S are not zero, but some arbitrary numbers, namely, 75·399 seconds and 1066 feet, probably originally 75 and 1000, before a recalculation.

The object of starting with some such numbers is to avoid the appearance of negative numbers in the tables, if it should be required to carry the tables on for still lower values of the velocity; or if it should be found requisite to revise the provisional experimental values of the resistance of the air at low velocities, and so recalculate the tables for these low velocities without disturbing the numbers for high velocities, at which the resistance of the air is known with greater accuracy.

But as in the practical use of the tables the formulas

$$t = C\{T(V) - T(v)\}$$
$$s = C\{S(V) - S(v)\}$$

only require *differences* of the tabular values of T and S, it is immaterial what numbers are employed as starting values.

These tables of Mr. Bashforth were published in his "Mathematical Theory of the Motion of Projectiles, 1872"; and they are universally employed in all our text books of gunnery.

In the first edition of the tables the tabulated values of T and S were shown increasing as the velocity diminished, to agree with the actual order; but as this arrangement had the disadvantage of requiring *negative proportional parts*, the arrangement was changed to that given here.

A third table (Table V), due to Mr. W. D. Niven, called the *degree* table, is useful for determining the change in direction of motion of a projectile while the velocity drops from any initial value V to any final value v.

To explain the theory of this table, let the tangent at the point of the trajectory, where the velocity is v, make an angle i radians with the horizon.

Then if di denotes the infinitesimal *decrement* of i in the infinitesimal increment of time dt, resolving normally in the trajectory,

$$v\frac{di}{dt} = g\cos i \tag{8}$$

This may be proved in the following manner: Suppose that in passing through the point P on the trajectory, where the inclination is i radians, the velocity drops from

$$v + \tfrac{1}{2}\Delta v \text{ to } v - \tfrac{1}{2}\Delta v \text{ f/s}$$

as the shot passes from Q to R, where the inclinations are

$$i + \tfrac{1}{2}\Delta i \text{ and } i - \tfrac{1}{2}\Delta i \text{ radians.}$$

Measure off lengths TU and TV from T, the point of intersection of the tangents at Q and R, to represent to scale the velocities at Q and R; then UV represents to the same scale the *change in velocity* in passing from Q to R.

Drawing UW vertical, and VW parallel to the tangent at P, so as to form the triangle UVW; then on the assumption that the average resistance of the air acts in the direction of the tangent at P, the triangle of velocities UVW shows that UW represents the change in velocity due to gravity, and WV the change due to the resistance of the air; so that if the shot takes Δt seconds to pass from Q to R, we may put

$$\text{UW} = g\Delta t,$$
$$\text{WV} = r\Delta t,$$

if r denotes the average retardation due to the resistance of the air.

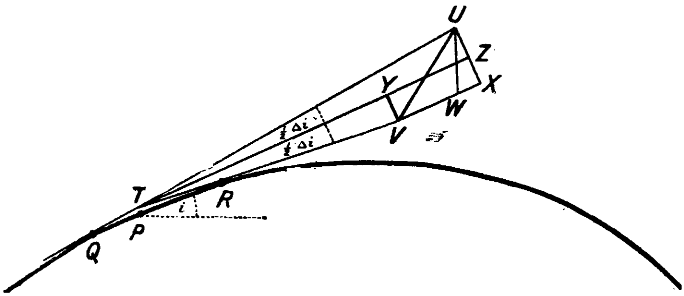
Drawing TYZ and VWX parallel to the tangent at P, and dropping the perpendiculars VY and UZX, then

$$\begin{aligned}\text{UX} &= g\,\Delta t\cos i.\\ &= \text{UZ} + \text{ZX}\\ &= (v + \tfrac{1}{2}\Delta v)\sin\tfrac{1}{2}\Delta i + (v - \tfrac{1}{2}\Delta v)\sin\tfrac{1}{2}\Delta\\ &= 2v\sin\tfrac{1}{2}\Delta i\end{aligned}$$

or $$g\cos i = 2v\,\frac{\sin\frac{1}{2}\Delta i}{\Delta t};$$

leading to equation (8) when Δt and Δi are indefinitely small.

Also $$XV = XW + WV = (g \sin i + r)\Delta t$$
$$= YZ = TZ - TY = \Delta v \cos \tfrac{1}{2}\Delta i,$$
$$g \sin i + r = \frac{\Delta v}{\Delta t} \cos \tfrac{1}{2}\Delta i.$$



In the limit

$$g \sin i + r = \frac{dv}{dt},$$

the equation obtained by resolving tangentially in the trajectory.

If the trajectory is sufficiently flat for cos i to be replaced by unity, equation (8) becomes

$$v \frac{di}{dt} = g,$$

or
$$\frac{di}{dt} = \frac{g}{v};$$

and therefore

(9) $$\frac{\Delta i}{\Delta t} = \frac{g}{v},$$

if v denotes the mean velocity during the small finite increment of time Δt, during which the direction of motion of the shot changes through Δi radians.

If we denote the inclination or change of direction in degrees by δ or $\Delta\delta$,

$$\frac{\delta}{180} = \frac{i}{\pi};$$

(10) $$\Delta\delta = \frac{180}{\pi}\Delta i = \frac{180g}{\pi}\frac{\Delta t}{v} = C\frac{180g}{\pi}\frac{\Delta T}{v}.$$

The differences

(11) $\dfrac{180g}{\pi}\dfrac{\Delta T}{v}$ are denoted by ΔD,

and are calculated from ΔT by dividing by the mean velocity v, and multiplying by $\frac{180g}{\pi}$; afterwards these differences are summed up by the arithmometer, and entered under the column of D.

As the approximation employed is unsuitable with low velocities and curved fire, it is useless to carry the table below a velocity of 500 or 400 f/s; and to avoid proportional parts, Table V has been interpolated with unit increments of f/s in the velocity.

For some purposes, as in Siacci's method, it is preferable to retain the circular measure, i radians; and now

$$\Delta i = Cg \frac{\Delta T}{v}$$

$$= C\Delta I,$$

where

(12) $$\Delta I = g \frac{\Delta T}{v};$$

and the differences ΔI are calculated, summed by the arithmometer, and entered in the column of I in Table VI.

Mr. Bashforth employs a similar function, which he denotes by R_v, and tabulates in his Table J. (*Supplement to a Treatise on the Motion of Projectiles*, 1881.) It will be found on comparison that

(13) $$\Delta R = \tfrac{1}{3}\Delta I.$$

Now, in applying these tables to a flat trajectory, if δ denotes the degrees of deviation in direction while the velocity of a shot, whose ballistic coefficient is C, falls from V to v, and if i denotes the radians in δ degrees,

(14) $$\delta = C\{D(V) - D(v)\}$$

(15) $$i = C\{I(V) - I(v)\},$$

or

(16) $$i = 3C\{R(V) - R(v)\}.$$

In an abridged Ballistic Table, the differences ΔT, ΔS, and ΔD were calculated by Mr. A. G. Hadcock, late R.A., from the formulas found above,

(4) $$\Delta T = \frac{\Delta v}{gp},$$

(6) $$\Delta S = v\Delta T,$$

(11) $$\Delta D = \frac{180g}{\pi} \frac{\Delta T}{v},$$

(12) $$\Delta I = g\frac{\Delta T}{v},$$

and the summation of the differences ΔT, ΔS, ΔD, and ΔI, for a constant difference $\Delta v = 10$ in v, to form the column T, S, D, and I was performed by using the arithmometer in the Royal Artillery Institution; and the results were verified by using the instrument for subtraction.

Mr. Bashforth's Tables III and IV for T and S, and Mr. Niven's Table V for D, were calculated by a more laborious process, explained elsewhere, in Chapter II, p. 183, for a constant difference $\Delta v = 10$ in v, the results per unit difference of velocity being interpolated; but it was found that the results of a gunnery problem obtained either by the use of these complete tables, or by the abridged Ballistic Table, differ inappreciably.

The following systematic scheme of calculation, worked out in detail for the interval of velocity 1000—1010, in which we may take the average velocity $v = 1005$, and the average $p = 2{\cdot}365$, will show the method of calculation of a Ballistic Table, in case this should be required for revised values of p, depending on new experiments :—

These are the numerical values tabulated in the abridged Table; but the value of ΔT from Bashforth's Table V in the interval of velocity 1000—1010 is $\Delta T = 0{\cdot}1315$, an increase of nearly 0·1°/₀, corresponding to a decrease of less than 0·1°/₀ in the value of p, which could be accounted for by an ascent of about 30 feet in the atmosphere.

Again, the arithmetic mean of the values of p for the velocities of 1000 and 1010 is $p = 2{\cdot}367$, an increase of nearly 0·2°/₀, corresponding to a difference of level of about 60 feet.

These slight discrepancies in the tables are met with principally at the low velocities, where the value of p is not known with great accuracy, and must be considered provisional; the discrepancies are of no practical importance, and they tend to disappear gradually as the velocity becomes higher.

For this reason the slide rule may replace the four-figure logarithms, with sufficient accuracy for practical purposes, in the computation of a new Ballistic Table.

		990—1000.	1000—1010.	1010—1020.
p		2·2954	2·3650	2·443
$\log p$			0·3738	
$\log \frac{\Delta v}{g}$			$\bar{1}$·4923	
$\log \frac{\Delta v}{gp} = \log \Delta T$			$\bar{1}$·1185	
ΔT		0·1354	0·1314	0·1272
T		24·2368	24·4722	24·6036
v		995	1005	1015
$\log v$			3·0022	
$\log v\Delta T = \log \Delta S$			2·1207	
ΔS		134·68	132·01	129·08
S		14693·34	14828·02	14960·03
$\log \frac{\Delta T}{v}$			$\bar{4}$·1163	
$\log \frac{180g}{\pi}$			3·2659	
$\log \frac{108g}{\pi} \frac{\Delta T}{v} = \log \Delta D$			$\bar{1}$·3822	
ΔD		0·2509	0·2411	0·2311
D		44·7993	45·0502	45·2913
$\log g$			1·5077	
$\log g\frac{\Delta T}{v} = \log \Delta I$			$\bar{3}$·6240	
ΔI		0·00438	0·00421	0·00403
I		0·78190	0·78628	0·79049

CHAPTER II.—THE RESISTANCE OF THE AIR.

UNTIL the time of Benjamin Robins, and of his invention of the Ballistic Pendulum (1740), the vaguest ideas prevailed as to the velocity of shot and the resistance of the air.

It was never realised that such an attenuated elastic medium could offer so enormous a resistance, in spite of Newton's caution (Ex Medii subtilitate resistantia projectilium celerrime motorum non multum diminuitur. *Philosophiæ Naturalis Principia Mathematica*, lib. ii, prop. xxxiii, cor. 5), so that artillerists were in the habit of neglecting this resistance, and of employing Galileo's parabolic theory for unresisted motion; and thereby the velocity of the shot was considerably under-estimated.

Thus, for instance, the velocity V required with an elevation of 9° to attain a range of 3500 yards is, according to this parabolic theory (Chapter II, § 4, Part I),

$$V = \sqrt{(gX \operatorname{cosec} 2\alpha)},$$

where $X = 10{,}500$, the range in feet, and $2\alpha = 18°$; so that we deduce

$$V = 1047 \text{ f/s}.$$

But it is found that the modern magazine rifle, with an initial velocity of 2000 f/s, can hardly attain a range of 3500 yards, whatever elevation is given; and the resistance of the air to the bullet at the outset is now estimated at about $1\frac{1}{4}$ lbs., or 40 times the weight of the bullet.

So also Robins found, in an experiment (*New Principles of Gunnery*, 1742, Chap. II, Prop. II) by firing at his ballistic pendulum at ranges of 25, 75, and 125 feet, that the mean velocities of impact were 1670, 1550, and 1425 f/s.

The musket employed was a 12 bore, so that the bullets weighed 12 to the pound; and the charge of powder was half the weight of the bullet.

Denoting by R the average resistance in pounds over the range of 100 feet, during which the velocity fell from $V = 1670$ to $v = 1425$,

$$R = \frac{w(V^2 - v^2)}{2g \times 100} = 10 \text{ lbs., about,}$$

or 120 times the weight of the bullet; this may be taken as the resistance of the air to a spherical bullet of this description, $\frac{3}{4}$ of an inch in diameter, moving with the velocity of 1550 f/s, at the mean range of 75 feet.

The conclusions of Robins naturally met with great opposition from the teachers of the ancient theory; thus, for instance, Professor Müller, in his *Treatise of Artillery*, *Supplement*, 1768, p. 110, proves that "the velocity from a 42-pr. can never amount to 914·7 f/s, and consequently much less in a smaller calibre."

But the experimental results, obtained by the modern method of shooting through electric screens, amply confirm Robins's results; and, according to Mr. Bashforth, these results of Robins, obtained from experiments with musket balls, are more accurate than those obtained 50 years later in Hutton's experiments with cannon balls and a larger ballistic pendulum.

The practical details of the construction and use of modern electro-ballistic apparatus are given in Chapter IV, § 2, Part I.

The experiments consist essentially in recording the instants of time,

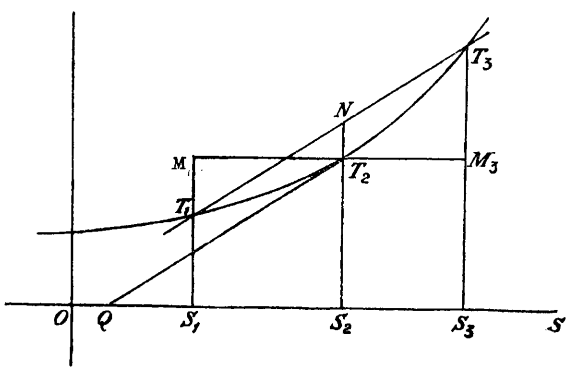
$$t_1, t_2, t_3 \text{ .... seconds,}$$

at which electric screens at distances

$$s_1, s_2, s_3 \text{ .... feet,}$$

measured from a fixed point, are cut by the passage of a shot flying nearly horizontally.

FIG. I.



Taking s and t as co-ordinates, a fair curve is drawn through the points

$$(s_1, t_1), (s_2, t_2), (s_3, t_3) \text{ ....}$$

to make sure that the instruments are in good working order (fig. 1); and now the problem is to determine the most appropriate analytical expression for this curve, in the form

$$t = f(s);$$

and thence to derive

$$\frac{dt}{ds} \text{ and } \frac{d^2t}{ds^2};$$

this problem may be solved in two or three different ways.

METHOD OF FINITE DIFFERENCES.

Mr. Bashforth employs the method of *Finite Differences;* in the notation of this subject, t_s or $f(s)$ denotes the value of t from a fixed point, say one of the screens to any distance s, to a given screen, for instance; and then t_{s+l} or $f(s + l)$ will denote the value of t to any extra distance $s + l$; say, to the next screen, l feet beyond; and generally, as required for the problem in hand, t_{s+nl} or $f(s + nl)$ will denote the time to the nth screen beyond the given screen, and t_{s-nl} or $f(s - nl)$ will denote the time to the nth screen in front of the given screen, the screens being spaced equally l feet apart.

Again, in the subject of Finite Differences, the symbol Δ is employed as a prefix (not as a factor) to denote the operation of differencing; and thus

$$t_{s+l} - t_s \text{ is denoted by } \Delta t_s;$$

or $$f(s + l) - f(s) \text{ is denoted by } \Delta f(s);$$

while $$\Delta t_{s+l} - \Delta t_s \text{ is denoted by } \Delta^2 t_s;$$

or $$\Delta f(s + l) - \Delta f(s) \text{ is denoted by } \Delta^2 f(s);$$

$$\Delta^2 t_{s+l} - \Delta^2 t_s \text{ is denoted by } \Delta^3 t_s;$$

and so on.

Thus, in fig. 1,

$$T_1M_1 = \Delta t_1, \quad M_3T_3 = \Delta t_2, \quad T_2N = \tfrac{1}{2}\Delta^2 t_1.$$

Then since

$$\Delta t_s = t_{s+l} - t_s,$$

therefore

$$\Delta^2 t_s = \Delta t_{s+l} - \Delta t_s$$
$$= t_{s+2l} - t_{s+l} - t_{s+l} + t_s$$
$$= t_{s+2l} - 2t_{s+l} - t_s,$$

and similarly,

$$\Delta^3 t_s = \Delta t_{s+2l} - 2\Delta t_{s+l} + \Delta t_s$$
$$= t_{s+3l} - 3t_{s+2l} + 3t_{s+l} - t_s;$$

and generally, by induction,

(1) $$\Delta^n t_s = t_{s+nl} - nt_{s+(n-1)l} + \frac{n(n-1)}{2} t_{s+(n-2)l} - \ldots\ldots$$

analogous to the Binomial Theorem.

Again—

$$t_{s+l} = t_s + \Delta t_s,$$
$$t_{s+2l} = t_{s+l} + \Delta t_{s+l},$$
$$= t_s + 2\Delta t_s + \Delta^2 t_s,$$
$$t_{s+3l} = t_s + 3\Delta t_s + 3\Delta^2 t_s + \Delta^3 t_s,$$

and generally, by induction,

$$(2) \qquad t_{s+nl} = t_s + n\Delta t_s + \frac{n(n-1)}{2!}\Delta^2 t_s + \ldots\ldots\ldots$$

again analogous to the Binomial Theorem.

But if t_{s+nl} or $f(s+nl)$ is expanded by Taylor's Theorem in ascending powers of nl, then

$$(3), \qquad t_{s+nl} = f(s) + nl\frac{df(s)}{ds} + \frac{n^2l^2}{2!}\frac{d^2f(s)}{ds^2} + \ldots\ldots\ldots$$

The general, $(r+1)$th, term in the series (2) can be written

$$\frac{n(n-1)\ldots(n-r+1)}{r!}\Delta^r t_s$$

$$= -(-1)^r n(1-n)\left(1-\frac{n}{2}\right)\ldots\left(1-\frac{n}{r-1}\right)\frac{\Delta^r t_s}{r}$$

$$= -(-1)^r\left\{n - n^2\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{r-1}\right)+\ldots\right\}\frac{\Delta^r t_s}{r}.$$

Collecting the coefficients of n and n^2 in (2),

$$(4) \quad t_{s+nl} = t_s + n\left\{\Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \ldots - (-1)^r\frac{\Delta^r t_s}{r} + \ldots\right\}$$

$$+ n^2\left\{\frac{\Delta^2 t}{2} - \frac{\Delta^3 t}{3}(1+\tfrac{1}{2}) + \frac{\Delta^4 t}{4}(1+\tfrac{1}{2}+\tfrac{1}{3})\right.$$

$$- \frac{\Delta^5 t}{5}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}) + \ldots$$

$$\left. + (-1)^{r-1}\frac{\Delta^r t_s}{r}\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{r-1}\right)+\ldots\right\} + \ldots;$$

so that, equating the coefficients of n and n^2 in these two different expressions for t_{s+nl}, given in (3) and (4),

$$(5) \quad l\frac{dt_s}{ds} = \Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \ldots - (-1)^r\frac{\Delta^r t_s}{r} + \ldots\ldots$$

$$(6) \quad l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \tfrac{11}{12}\Delta^4 t_s - \ldots$$

$$+ 2(-1)^r\frac{\Delta^r t_s}{r}\left(1+\tfrac{1}{2}+\tfrac{1}{3}+\ldots+\frac{1}{r-1}\right) + \ldots\ldots$$

If we had an unlimited number of screens, l feet apart, and their time records, we could find the successive differences of the records according to the following scheme on p. 176.

It will be noticed that the series of numbers

$$t_s,\ \Delta t_s,\ \Delta^2 t_s,\ \Delta^3 t_s,\ \Delta^4 t_s,\ \ldots,$$

run in a diagonal line slanting downwards, so that the preceding formulas (5) and (6) are suitable for employment at the initial screens of a series.

$t.$	$\Delta t.$	$\Delta^2 t.$	$\Delta^3 t.$	$\Delta^4 t.$	$\Delta^5 t.$	$\Delta^6 t.$	$\Delta^7 t.$	$\Delta^8 t.$
$t_s - 7l$								
	$\Delta t_s - 7l$							
$t_s - 6l$		$\Delta^2 t_s - 7l$						
	$\Delta t_s - 6l$		$\Delta^3 t_s - 7l$					
$t_s - 5l$		$\Delta^2 t_s - 6l$		$\Delta^4 t_s - 7l$				
	$\Delta t_s - 5l$		$\Delta^3 t_s - 6l$		$\Delta^5 t_s - 7l$			
$t_s - 4l$		$\Delta^2 t_s - 5l$		$\Delta^4 t_s - 6l$		$\Delta^6 t_s - 7l$		
	$\Delta t_s - 4l$		$\Delta^3 t_s - 5l$		$\Delta^5 t_s - 6l$		$\Delta^7 t_s - 7l$	
$t_s - 3l$		$\Delta^2 t_s - 4l$		$\Delta^4 t_s - 5l$		$\Delta^6 t_s - 6l$		$\Delta^8 t_s - 7l$
	$\Delta t_s - 3l$		$\Delta^3 t_s - 4l$		$\Delta^5 t_s - 5l$		$\Delta^7 t_s - 6l$	
$t_s - 2l$		$\Delta^2 t_s - 3l$		$\Delta^4 t_s - 4l$		$\Delta^6 t - 5l$		$\Delta^8 t_s - 6l$
	$\Delta t_s - 2l$		$\Delta^3 t_s - 3l$		$\Delta^5 t_s - 4l$		$\Delta^7 t_s - 5l$	
$t_s - l$		$\Delta^2 t_s - 2l$		$\Delta^4 t_s - 3l$		$\Delta^6 t_s - 4l$		$\Delta^8 t_s - 5l$
	$\Delta t_s - l$		$\Delta^3 t_s - 2l$		$\Delta^5 t_s - 3l$		$\Delta^7 t_s - 4l$	
t_s		$\Delta^2 t_s - l$		$\Delta^4 t_s - 2l$		$\Delta^6 t_s - 3l$		$\Delta^8 t_s - 4l$
	Δt_s		$\Delta^3 t_s - l$		$\Delta^5 t_s - 2l$		$\Delta^7 t_s - 3l$	
$t_s + l$		$\Delta^2 t_s$		$\Delta^4 t_s - l$		$\Delta^6 t_s - 2l$		$\Delta^8 t_s - 3l$
	$\Delta t_s + l$		$\Delta^3 t_s$		$\Delta^5 t_s - l$		$\Delta^7 t_s - 2l$	
$t_s + 2l$		$\Delta^2 t_s + l$		$\Delta^4 t_s$		$\Delta^6 t_s - l$		$\Delta^8 t_s - 2l$
	$\Delta t_s + 2l$		$\Delta^3 t_s + l$		$\Delta^5 t_s$		$\Delta^7 t_s - l$	
$t_s + 3l$		$\Delta^2 t_s + 2l$		$\Delta^4 t_s + l$		$\Delta^6 t_s$		$\Delta^8 t_s - l$
	$\Delta t_s + 3l$		$\Delta^3 t_s + 2l$		$\Delta^5 t_s + l$		$\Delta^7 t_s$	
$t_s + 4l$		$\Delta^2 t_s + 3l$		$\Delta^4 t_s + 2l$		$\Delta^6 t_s + l$		
	$\Delta t_s + 4l$		$\Delta^3 t_s + 3l$		$\Delta^5 t_s + 2l$			
$t_s + 5l$		$\Delta^2 t_s + 4l$		$\Delta^4 t_s + 3l$				
	$\Delta t_s + 5l$		$\Delta^2 t_s + 4l$					
$t_s + 6l$		$\Delta^2 t_s + 5l$						
	$\Delta t_s + 6l$							
$t_s + 7l$								

At the final screens the numbers end off in a diagonal line sloping upwards, containing the typical terms

$$t_s,\ \Delta t_{s-l},\ \Delta^2 t_{s-2l},\ \Delta^3 t_{s-3l},\ \Delta^4 t_{s-4l},\ \ldots .$$

But

$$t_{s-l} = t_s - \Delta t_{s-l}$$

$$t_{s-2l} = t_{s-l} - \Delta t_{s-2l}$$

$$= t_s - \Delta t_{s-l} - \Delta(t_{s-l} - \Delta t_{s-2l})$$

$$= t_s - 2\Delta t_{s-l} + \Delta^2 t_{s-2l},$$

and so on; so that generally

(7) $$t_{s-nl} = t_s - n\Delta t_{s-l} + \frac{n(n-1)}{2!}\Delta^2 t_{s-2l} - \ldots$$

$$= t_s - n(\Delta t_{s-l} + \tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{3}\Delta^3 t_{s-3l} + \ldots)$$

$$+ n^2(\tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{2}\Delta^3 t_{s-3l} + \tfrac{11}{24}\Delta^4 t_{s-4l} + \ldots);$$

and therefore, as before,

(8) $$l\frac{dt_s}{ds} = \Delta t_{s-l} + \tfrac{1}{2}\Delta^2 t_{s-2l} + \tfrac{1}{3}\Delta^3 t_{s-3l} + \ldots,$$

(9) $$l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-2l} + \Delta^3 t_{s-3l} + \tfrac{11}{12}\Delta^4 t_{s-4l} + \ldots,$$

the formulas appropriate at the final screens of a series.

But at the middle screens the numbers which run horizontally are typified by

$$t_s,\quad \frac{\Delta t_{s-l}}{\Delta t_s},\quad \Delta^2 t_{s-l},\quad \frac{\Delta^3 t_{s-2l}}{\Delta^3 t_{s-l}},\quad \Delta^4 t_{s-2l},\ \ldots .$$

The formulas required are now

(10) $$l\frac{dt_s}{ds} = \tfrac{1}{2}(\Delta t_{s-l} + \Delta t_s) - \frac{1}{3!}\tfrac{1}{2}(\Delta^3 t_{s-2l} + \Delta^3 t_{s-l})$$

$$+ \frac{1^2 . 2^2}{5!}\tfrac{1}{2}(\Delta^5 t_{s-3l} + \Delta^5 t_{s-2l}) \ldots$$

$$- (-1)^r \frac{1^2 . 2^2 . 3^2 \ldots (r-1)^2}{(2r-1)!}\tfrac{1}{2}(\Delta^{2r-1} t_{s-rl} + \Delta^{2r-1} t_{s-rl+l})$$

$$+ \ldots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(11) $$l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-l} - \frac{1}{3!}\frac{\Delta^4 t_{s-2l}}{2} + \frac{1^2 . 2^2}{5!}\frac{\Delta^6 t_{s-3l}}{3} - \ldots$$

$$- (-1)^r \frac{1^2 . 2^2 . 3^2 \ldots (r-1)^2}{(2r-1)!}\frac{\Delta^{2r} t_{s-rl}}{r} + \ldots\ldots$$

the first (10) involving odd differences, and the second (11) even differences only (De Morgan, *Differential and Integral Calculus*, p. 544).

This is proved, if equation (2) is replaced by an equivalent formula,

$$(12)\quad t_{s+nl} = t_s + n\Delta t_s + \frac{n(n-1)}{2\,!}\Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3\,!}\Delta^3 t_{s-l} + \dots$$

$$+ \frac{(n+r-1)\dots(n-r)}{(2n)\,!}\Delta^{2r} t_{s-rl}$$

$$+ \frac{(n+r)\dots(n-r)}{(2r+1)\,!}\Delta^{2r+1} t_{s-rl} + \dots$$

Putting

$$\Delta t_s = \Delta t_{s-l} + \Delta^2 t_{s-l},$$

and, generally,

$$\Delta^{2n+1} t_{s-nl} = \Delta^{2n+1} t_{s-(n+1)l} + \Delta^{2n+2} t_{s-(n+1)l},$$

formula (12) is equivalent to

$$(13)\quad t_{s+nl} = t_s + n\Delta t_{s-l} + \frac{n(n+1)}{2\,!}\Delta^2 t_{s-l} + \frac{(n+1)n(n-1)}{3\,!}\Delta^3 t_{s-2l} + \dots$$

$$+ \frac{(n-r+1)\dots(n+r)}{(2r)\,!}\Delta^{2r} t_{s-rl}$$

$$+ \frac{(n-r)\dots(n+r)}{(2r+1)\,!}\Delta^{2r+1} t_{s-(r+1)l} + \dots$$

Taking the half sum of (12) and (13),

$$(14)\quad t_{s+nl} = t_s + n\tfrac{1}{2}(\Delta t_{s-l} + \Delta t_s) + \frac{n^2}{2\,!}\Delta^2 t_{s-l}$$

$$+ \frac{(n+1)n(n-1)}{3\,!}\tfrac{1}{2}(\Delta^3 t_{s-2l} + \Delta^3 t_{s-l}) + \dots$$

$$+ \frac{n(n-r+1)(n-r+2)\dots(n+r-1)}{(2r)\,!}\Delta^{2r} t_{s-rl}$$

$$+ \frac{(n-r)\dots(n+r)}{(2r+1)\,!}\tfrac{1}{2}(\Delta^{2r+1} t_{s-(r+1)l} + \Delta^{2r+1} t_{s-rl}) + \dots$$

and equating the coefficients of n and n^2 in this equation and in (3) will lead to the two required formulas (10) and (11), already stated.

Having thus determined

$$l\frac{dt_s}{ds} \text{ and } l^2\frac{d^2 t_s}{ds^2}$$

by the successive differences of the screen records, the velocity v is the reciprocal of $\dfrac{dt}{ds}$; while the retardation r is given by

$$-r = \frac{d^2 s}{dt^2} = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt}.$$

$$= \frac{d}{ds}\left(\frac{1}{\frac{dt}{ds}}\right)\frac{ds}{dt} = \frac{-\frac{d^2t}{ds^2}\frac{ds}{dt}}{\left(\frac{dt}{ds}\right)^2} = -\frac{d^2t}{ds^2}\left(\frac{ds}{dt}\right)^3,$$

or

(15) $$r = \frac{d^2t}{ds^2}v^3.$$

Now if the shot weighs w lbs., and if R denotes the resistance of the air in pounds,

$$\frac{R}{w} = \frac{r}{g},$$

or

(16) $$R = \frac{w}{g}\frac{d^2t}{ds^2}v^3.$$

Hence the advantage of Mr. Bashforth's method of dividing the retardation r of the shot or the resistance R of the air into two factors, one of which is the cube of the velocity; for then the other factors are

$$\frac{d^2t}{ds^2} \text{ and } \frac{w}{g}\frac{d^2t}{ds^2},$$

which are given immediately by the differences of the screen records.

It is assumed, as the result of experiment, that the resistance of the air to similar projectiles is proportional to the cross section or the square of the diameter; so that if the projectile is d inches in diameter, then R can be divided into the factors nd^2p, where n is called the *coefficient of reduction* (p. 165), and p therefore denotes the resistance of the air in a normal state to a standard projectile one inch in diameter; and then

$$nd^2p = \frac{w}{g}\frac{d^2t}{ds^2}v^3,$$

or, denoting the *ballistic coefficient* $\frac{w}{nd^2}$ by C,

(17) $$p = \frac{C}{g}\frac{d^2t}{ds^2}v^3.$$

As the number $\frac{d^2t}{ds^2}$ is found to be a small decimal, beginning with seven or eight zeros when v is reckoned in units of feet per second, Mr. Bashforth finds it more convenient to reckon the velocity in thousands of f/s, and to write the last equation

(18) $$p = \frac{C}{g}\frac{d^2t}{ds^2} \times 10^9 \left(\frac{v}{1000}\right)^3$$

and

(19) $$r = -\frac{d^2s}{dt^2} = \frac{d^2t}{ds^2}v^3 = 10^9\frac{d^2t}{ds^2}\left(\frac{v}{1000}\right)^3$$

Since p is, on the above assumptions, the same function of v for all ordinary projectiles, therefore,

$$C\frac{d^2t}{ds^2} \times 10^9$$

is also the same coefficient for all projectiles; Mr. Bashforth denotes it by K_v, so that

$$p = \frac{K_v}{g}\left(\frac{v}{1000}\right)^3, \tag{20}$$

$$R = nd^2 \frac{K_v}{g}\left(\frac{v}{1000}\right)^3, \tag{21}$$

where

$$K_v = C\frac{d^2t}{ds^2} \times 10^9, \tag{22}$$

and the numerical values of K_v given in Table I, embody the results of Mr. Bashforth's series of experiments.

The coefficient K is seen to vary slowly for velocities from about 1090 to 1400 f/s, so that in this region of velocity we may assume that the resistance of the air varies as the cube of the velocity.

On this assumption

$$\frac{d^2t}{ds^2} = \text{a constant},$$

denoted by $2b$ by Mr. Bashforth; and integrating

$$\frac{d^2t}{ds^2} = 2b$$

twice with respect to s,

$$\frac{dt}{ds} = a + 2bs,$$

$$t = t_0 + as + bs^2; \tag{23}$$

so that the curve which is the *graph* of t is a parabola (fig. 1, p. 173).

Denoting by V the velocity where $s = 0$,

$$\frac{1}{V} = \left(\frac{dt}{ds}\right)_0 = a,$$

and denoting by U the average velocity over the distance s,

$$\frac{1}{U} = \frac{t - t_0}{s} = a + bs$$

$$= \tfrac{1}{2}a + \tfrac{1}{2}(a + 2bs)$$

$$= \tfrac{1}{2}\left(\frac{1}{V} + \frac{1}{v}\right), \tag{24}$$

if v denotes the final velocity at the end of the distance s.

Also $a + bs$ is the reciprocal of the velocity at the middle point of s, so that the average velocity over the distance s is the harmonic mean of the initial and final velocities, and it is the actual velocity at the middle point of the range.

Interpreted geometrically in fig. 1, the tangent QT_2 at the point T_2 on the parabola $T_1T_2T_3$ is parallel to the chord T_1T_3, if s_2 is midway between s_1 and s_3.

This is the rule employed in determining muzzle velocities at proof, where s represents the distance between the two screens, t_0 and t the initial and final chronograph records, and

$$\frac{s}{t - t_0} = U,$$

the average velocity between the screens, which is taken to be the actual velocity at a point midway between the screens.

At high velocities, say above 1330 f/s, or at low velocities, say below 790 f/s, it is found that the Newtonian Law of a resistance varying as the *square* of the velocity is more suitable for employment, as the values of $\frac{p}{v^2}$ or vK_v, are very nearly constant in these regions of velocity, as shown in the figure on p. 163.

In these cases Mr. Bashforth puts

$$R = nd^2 \frac{k_v}{g}\left(\frac{v}{1000}\right)^2,$$

so that

$$(24) \qquad k_v = K_v \frac{v}{1000},$$

and the numerical values of k_v will be found tabulated in his treatise on the *Bashforth Chronograph*, in Tables I and III given there, both for spherical and ogival-headed projectiles.

When the resistance R of the air is assumed to vary as the nth power of the velocity, a convenient form to express the relation is

$$(25) \qquad R = w\left(\frac{v}{\omega}\right)^n;$$

and then ω is called the *terminal velocity;* because $R = w$ when $v = \omega$, so that the resistance of the air balances the weight when the shot moves vertically downwards with this velocity, as in the vertical asymptote of a trajectory.

If we put

$$R = nd^2p, \text{ and } \frac{w}{nd^2} = C,$$

then

$$(26) \qquad p = C\left(\frac{v}{\omega}\right)^n,$$

so that $p = C$ when $v = \omega$, whence ω can be found for a given projectile from Table II.

Thus the terminal velocity of the 9·2-inch projectile, weighing 380 lb., is with $d = 9·15$, $n = 0·9$, $C = 5·044$, equal to 1140 f/s; and for the magazine rifle bullet, weighing 215 grains, in which $d = 0·303$ inch, and $n = 0·8$, so that $C = 0·4182$, the terminal velocity is 470 f/s. In very long range fire, the remaining velocity cannot exceed the terminal velocity.

If gravity is left out of account, as is permissible when the shot is flying horizontally, and if the projectile moves against this resistance R, the retardation r is given by

$$(27) \qquad \frac{r}{g} = \frac{R}{w} = \left(\frac{v}{\omega}\right)^n,$$

so that the equations of motion of the projectile are

$$(28) \qquad \frac{dv}{dt} = -r = -g\left(\frac{v}{\omega}\right)^n$$

$$(29) \qquad \frac{vdv}{ds} = -r = -g\left(\frac{v}{\omega}\right)^n$$

Then, inverting (29) and (30), and integrating with respect to v between any initial velocity V and final velocity v,

$$(30)\quad t = \int_v^V \left(\frac{w}{v}\right)^n \frac{dv}{g} = \frac{w}{g} \cdot \frac{1}{n-1}\left\{\left(\frac{w}{v}\right)^{n-1} - \left(\frac{w}{V}\right)^{n-1}\right\} \ldots$$

$$(31)\quad s = \int_v^V \frac{w^n}{v^{n-1}} \frac{dv}{g} = \frac{w^2}{g} \cdot \frac{1}{n-2}\left\{\left(\frac{w}{v}\right)^{n-2} - \left(\frac{w}{V}\right)^{n-2}\right\} \ldots.$$

from which Ballistic tables could be constructed on these theoretical assumptions.

But exceptional cases occur when $n = 1$ and 2.

When $n = 1$,

$$\frac{dv}{dt} = -g\frac{v}{w}, \text{ and } \frac{dv}{ds} = -\frac{g}{w};$$

so that

$$(32)\quad t = \frac{w}{g}\int_v^V \frac{dv}{v} = \frac{w}{g}\log\frac{V}{v},$$

$$(33)\quad s = \frac{w}{g}\int_v^V dv = \frac{w}{g}(V - v).$$

When $n = 2$,

$$\frac{dv}{dt} = -g\frac{v^2}{w^2}, \text{ and } \frac{dv}{ds} = -\frac{gv}{w^2};$$

$$(34)\quad t = \frac{w^2}{g}\int_v^V \frac{dv}{v^2} = \frac{w}{g}\left(\frac{w}{v} - \frac{w}{V}\right).$$

$$(35)\quad s = \frac{w^2}{g}\int_v^V \frac{dv}{v} = \frac{w^2}{g}\log\frac{V}{v}.$$

Mr. Bashforth took $n = 3$ in the construction of his Ballistic tables; and here

$$(36)\quad t = \frac{w}{2g}\left\{\left(\frac{w}{v}\right)^2 - \left(\frac{w}{V}\right)^2\right\}$$

$$(37)\quad s = \frac{w^2}{g}\left(\frac{w}{v} - \frac{w}{V}\right).$$

Introducing Bashforth's K in place of the terminal velocity w, by means of the relation

$$(38)\quad r = \frac{nd^2}{w}\,K\left(\frac{v}{1000}\right)^3 = g\left(\frac{v}{w}\right)^3,$$

so that

$$(39)\quad \left(\frac{w}{1000}\right)^3 = \frac{w}{cd^2}\,\frac{g}{K},$$

then

$$(40)\quad \frac{nd^2}{w}\,t = \frac{500}{K}\left\{\left(\frac{1000}{v}\right)^2 - \left(\frac{1000}{V}\right)^2\right\}.$$

$$(41)\quad \frac{nd^2}{w}\,s = \frac{(1000)^2}{K}\left(\frac{1000}{v} - \frac{1000}{V}\right),$$

formulas by means of which Mr. Bashforth calculates his tables for

$$\frac{nd^2}{w}t \text{ and } \frac{nd^2}{w}s, \text{ or } \frac{t}{C} \text{ and } \frac{s}{C},$$

in our notation, for differences of 10 f/s is the velocity, taking K as constant and equal to its mean value in the interval; the values of $\frac{t}{C}$ and $\frac{s}{C}$ for unit difference of f/s were then interpolated by proportional parts.

Thus, in the interval from $v = 1000$ to $V = 1010$, we may take the average value of $K = 75$, according to Bashforth's former table of values of K, *Motion of Projectiles*, 1873; and now

$$\log \frac{1000}{V} = \bar{1}\cdot9957, \quad \frac{1000}{V} = 0\cdot990$$

$$\frac{1000}{v} - \frac{1000}{V} = 0\cdot0099$$

$$\log 1000^2 \left(\frac{1000}{v} - \frac{1000}{V}\right) = 3\cdot9957$$

$$\log K = 1\cdot8751$$

$$\log \frac{s}{C} = 2\cdot1206$$

$$\frac{s}{C} = 132\cdot0.$$

Also

$$\frac{t}{C} = \tfrac{1}{2}\left(\frac{1000}{v} + \frac{1000}{V}\right)\frac{s}{1000C}$$

$$\tfrac{1}{2}\left(\frac{1000}{v} + \frac{1000}{V}\right) = 0\cdot99505$$

$$\log \tfrac{1}{2}\left(\frac{1000}{v} + \frac{1000}{V}\right) = \bar{1}\cdot9978$$

$$\log \frac{s}{1000C} = \bar{1}\cdot1206$$

$$\log \frac{t}{C} = \bar{1}\cdot1185$$

$$\frac{t}{C} = 0\cdot1314;$$

and these values may be compared with the corresponding values given in Tables V and VI.

Numerical illustrations taken from the *Reports on Experiments made with the Bashforth Chronograph, to determine the Resistance of the Air to the Motion of Projectiles*, 1865–1870 (London, 1870) or from *A Revised Account of the Experiments made with the Bashforth Chronograph* (Cambridge, 1890), will make the preceding theory more clear.

Take Round 1, 7th October, 1867, in which a solid shot, weighing 12 lb., was fired from a 3-inch gun, with a charge of 2 lb. of powder; the instants of time at which the 10 screens, 150 feet apart, were cut by the shot, are recorded in the following table, where the time differences are also given.

It will be noticed that the second differences are very nearly constant, and on the average equal to 0·0021, or 0·0022, and that the higher differences are illusory; this is because the chronograph does not record smaller intervals of time than the ten-thousandth of a second, recorded in the fourth place of decimals; and this figure is therefore subject to a correction, which may reach to nearly $\pm$ 5 in the fifth place.

If the chronograph could record to this fifth place, the third differences, $\Delta^3 t$, would become nearly constant, and the higher differences would be illusory, and so on.

A fifth figure can, however, be introduced, so as to smooth the irregularities in the second differences; and this fifth figure, as given in Bashforth's *Chronograph*, 1890, p. 33 has been introduced in the following records.

ROUND 1.

Number of screen.	t.	Δt.	$\Delta^2 t$.	$\Delta^3 t$.
1	0·00000			
		0·12457		
2	0·12457		0·00211	
		0·12668		
3	0·25125		0·00212	
		0·12880		
4	0·38005		0·00213	
		0·13093		
5	0·51098		0·00214	
		0·13307		0·00001
6	0·64405		0·00215	
		0·13522		
7	0·77927		0·00216	
		0·13738		
8	0·91665		0·00217	
		0·13955		
	1·05620		0·00218	
		0·14173		
10	1·19793			

Taking the formula (5) with $l = 150$, and denoting by v_m the velocity at the mth screen, then v_m can be found according to the following scheme of calculation, worked out in detail for the first screen; the remaining columns can be filled in as an exercise.

	Screens.		
$m =$	1	2	3
$\Delta t =$	0·12457		
$\frac{1}{2}\Delta^2 t =$	0·00105		
$\frac{l}{v_m} =$	0·12352		
$\log \frac{l}{v_m} =$	$\bar{1}$·0916		
$\log l =$	2·1761		
$\log v_m =$	3·0855		
$v_m =$	1217	1194	1174

At the last screen, the tenth, from formula (8),

$$\frac{l}{v_{10}} = \Delta t_9 + \tfrac{1}{2}\Delta^2 t_8 + \tfrac{1}{3}\Delta^3 t_7 \quad . \quad . \quad . \quad .$$

$$= 0{\cdot}14173$$

$$+ 0{\cdot}00104$$

$$= 0{\cdot}14277$$

$$\log \frac{l}{v_{10}} = \bar{1}{\cdot}1547$$

$$\log l = 2{\cdot}1761$$

$$\log v_{10} = 3{\cdot}0214$$

$$v_{10} = 1051 \text{ f/s}.$$

At the ninth screen, from formula (10),

$$\frac{l}{v_9} = \tfrac{1}{2}(\Delta t_8 + \Delta t_9)$$

$$= 0{\cdot}14085$$

$$\log \frac{l}{v_9} = \bar{1}{\cdot}1488$$

$$\log l = 2{\cdot}1761$$

$$\log v_9 = 3{\cdot}0273$$

$$v_9 = 1065.$$

The same formula (10) can also be employed for all the other screens except the first and last, and it will be found to lead practically to the same results.

In Report III, Table II, on p. 33 of *Reports, &c.*, 1865—1870, will be found tabulated the velocity of the shot at distances of 150, 300, ... feet from the gun, that is, midway between the screens, as the muzzle of the gun was 75 feet from the first screen; these velocities may be taken as the average between the screens, and calculated from the formula

$$v_{m+\frac{1}{2}} = \frac{l}{\Delta t_m},$$

$v_{m+\frac{1}{2}}$ denoting the velocity half way between the mth and $(m+1)$th screens.

Again, on the average,

$$l^2 \frac{d^2t}{ds^2} = \Delta^2 t = 0{\cdot}0021$$

$$\log l^2 \frac{d^2t}{ds^2} = \bar{3}{\cdot}3222$$

$$\log l^2 = 4{\cdot}3522$$

$$\log \frac{d^2t}{ds^2} = \bar{8}{\cdot}9700$$

$$\log \frac{d^2t}{ds^2} 10^9 = 1{\cdot}9700.$$

The projectile was 2·92 inches in diameter, and weighed 12 lb.; and at 3 P.M., on the 7th October, 1867, the barometer reading was 29·62 inches, the wet and dry bulb thermometer reading 48 and 52° F., and taking Mr. Bashforth's reduction of the value of τ for this and other rounds from p. 51, § 74 of his book, *The Bashforth Chronograph*, 1890,

$$\tau = 1{\cdot}002.$$

The projectile was of standard shape, so that we put $\kappa = 1$, and the ballistic coefficient

$$C = \frac{w}{\kappa\tau d^2},$$

when $$w = 12,\ d = 2{\cdot}92,\ \kappa\tau = 1{\cdot}002:$$

$$\log d = 0{\cdot}4654$$

$$\log d^2 = 0{\cdot}9308$$

$$\log \kappa\tau = 0{\cdot}0008$$

$$\text{og } \kappa\tau d^2 = 0{\cdot}9316$$

$$\log w = 1{\cdot}0792$$

$$\log C = 0{\cdot}1476.$$

Therefore, from (21),

$$\begin{aligned}\log K_v &= 0{\cdot}1476 \\ & + 1{\cdot}9700 \\ &= 2{\cdot}1176 \\ K_v &= 131{\cdot}1.\end{aligned}$$

Contrasting this with the average value,

$$K_v = 109{\cdot}6,$$

for a velocity, $v = 1200$ (Table IV), shows that this Round 1 must have been rather unsteady, as the coefficient of steadiness σ required to reduce it to normal conditions would be

$$\sigma = 13 \div 109{\cdot}6 = 1{\cdot}2.$$

As another example, take Round 479, fired on March 12th, 1879: the instants of time at which the screens, 150 feet apart, were cut by the shot are recorded in the following Table, taken from the *Final Report* on Experiments made with the Bashforth Chronograph, 1878–80, page 14; the fifth figure has been added, as given in Bashforth's *Chronograph*, p. 41. Take $d =$ $w = 50$, $\tau = 1{\cdot}014$.

ROUND 479.

Number of screen.	t.	Δt.	$\Delta^2 t$.	$\Delta^3 t$.
1	0·00000			
		0·06659		
2	0·06659		0·00109	
		0·06768		
3	0·13427		0·00110	
		0·06878		
4	0·20305		0·00109	
		0·06987		
5	0·27292		0·00109	
		0·07096		
6	0·34388		0·00109	
		0·07205		
7	0·41593		0·00109	
		0·07314		
8	0·48907		0·00110	
		0·07424		
9	0·56331		0·00110	
		0·07534		
10	0·63865		0·00110	
		0·07644		
11	0·71509		0·00110	
		0·07754		
12	0·79263			

A third example is given of the reduction of round 463, in which the slide rule has been used for the calculations. It will generally be found that the calculated values of K rarely agree with those printed by Mr. Bashforth, and even Mr. Bashforth's own values of K, as printed in the *Report on Experiments made with the Bashforth Chronograph*, 1865–1870 (London, 1870), do not always agree with those given in his *Revised Account of the Experiments made with the Bashforth Chronograph* (Cambridge, 1890).

To sift out the cause of these discrepancies we must start with the different values of K, and work back to the corresponding values of

$$l^2 \frac{d^2t}{ds^2},$$

and now it will be found that the discrepancies depend on different estimates of the fifth decimal in the screen records, or on one hundred-thousandth of a second, equivalent to a displacement of the shot of about 0·02 of a foot, or say a quarter of an inch, which is far beyond the accuracy of measurement of the electric screens employed in the experiments.

6-Inch B.L. Gun.

Round 463. March 7th, 1879. $w = 70$, $d = 6$, $T = 1·042$, $\kappa = 1$, $C = 1·866$, $l = 150$.

	t.	Δt.	$\Delta^2 t$.	$\Delta^3 t$.	Screen.	1	2	3	4	5	6	7	8	9	10	11	12
1	0·00000				$l\frac{dt}{ds}$	0·07679	0·07770	0·07860	0·07954	0·08048	0·08142	0·08236	0·08330	0·08426	0·08522	0·08620	0·08718
		0·07724															
2	7724		0·00091		ι	1953·4	1930·6	1908·4	1885·8	1863·8	1842·3	1821·3	1800·6	1780·2	1760·1	1740·2	1720·5
		7815		0·00001													
3	15539		92		$l^2\frac{d^2t}{ds^2}$	0·00090	92 0·00090	0·00094	0·00094	0·00094	0·00094	0·00094	0·00095	0·00096	0·00097	0·00098	0·00099
		7907		2													
4	23446		94		$K = C\frac{d^2t}{ds^2}\cdot 10^9$	74·6	76·3 74·6	77·9	77·9	77·9	77·9	77·9	78·8	78·8	80·4	81·3	82·1
		8001		0													
5	31447		94		$r = \frac{d^2t}{ds^2}v^3$	298·2	294·3 287·8	290·4	280·3	270·5	261·3	252·4	246·4	238·2	239·1	229·4	224·0
		8095		0													
6	39542		94		$\frac{R}{w} = \frac{r}{g}$	92·6	91·5 89·4	90·3	87·1	84·0	81·1	78·5	76·6	74·0	74·4	71·4	69·6
		8189		0													
7	47731		94		$R = w\frac{r}{g}$	648·5	640·5 625·5	632·3	610·0	588·0	568·0	549·5	536·0	518·0	520·5	499·5	487·3
		8283		1													
8	56014		95		$p = \frac{R}{Td^2}$	17·28	17·10 16·66	16·85	16·24	15·67	15·13	14·63	14·27	13·80	13·86	13·60	12·96
		8378		1													
9	64392		96		$\bar{p}$	16·19	15·81	15·47	15·17	14·88	14·59	14·29	14·01	13·74	13·47	13·19	12·91
		8474		1													
10	72866		97		$\sigma = \frac{p}{\bar{p}}$	1·068	1·081 1·064	2·088	1·070	1·053	1·037	1·024	1·018	1·004	1·029	1·029	1·002
		8571		1													
11	81437		98														
		8669															
12	90106																

Systematic Scheme of the Calculation.

Round 479.

Number of screen.	1.	2.	
$\frac{l}{v}$ or al	0·06605		
$\log al$	$\bar{2}$·8199		
$\log l$	2·1761		
$\log v$	3·3562		
v	2271		
$\Delta^2 t_s$ or $l^2 \frac{d^2t}{ds^2}$ or $2bl^2$	0·00109		
$\log l^2 \frac{d^2t}{ds^2}$	$\bar{3}$·0374		
$\log l^2$	4·3522		
$\log \frac{d^2t}{ds^2}$	$\bar{8}$·6852		
$\log 10^9 \frac{d^2t}{ds^2}$	1·6852		
log C	0·1367		
log K	1·8219		
K	66·36		
$\log g$	1·5077		
$\log \frac{K}{g}$	0·3142		
$\log \left(\frac{v}{1000}\right)^3$	1·0686		
$\log p$	1·3828		
p	24·14		
$\log cd^2$	1·5635		
$\log$ (R or cd^2p)	2·9451		
R	881·2		

The other columns corresponding to the remaining screens can be filled in as an exercise. The results for K and p agree closely with those given in Tables III and IV, thus showing that Round 479 was of average steadiness, or $\sigma = 1$.

The method of Finite Differences is a powerful one for revealing any irregularities in the records or any error in transcribing them; it also enables us to detect the calculated interpolated values in what professes to be a series of genuine observations, and to determine the formula employed in the calculation.

Thus, for example, from the series of numbers,

4, 11, 22, 37, 56, 79, 106, ...

by writing them as screen records and forming the differences,

n.	t.	Δt.	$\Delta^2 t$.	$\Delta^3 t$.
1	4			
		7		
2	11		4	
		11		0
3	22		4	
		15		0
4	37		4	
		19		0
5	56		4	
		23		0
6	79		4	
		27		
7	106			

we see that the second differences are constant; so that if t_n denotes the n-th term of the series

$$\Delta^2 t_n = 4$$

$$\Delta t_n = 4(n-1) + 7$$

$$t_n = 2(n-1)(n-2) + 7(n-1) + 4;$$

the formula by which the given series of numbers can be calculated by putting $n = 1, 2, 3, 4, ...$, and by which the series can be extended if necessary.

Take, for instance, the following series of numbers, from the Hythe *Text-Book of Musketry*, giving the elevation in minutes and decimals of a minute for every 100 yards of range for the magazine rifle, and form the successive differences.

We deduce that this array of figures can be calculated from

$$\Delta^3\alpha = 0{\cdot}049,$$

$$\Delta^2\alpha = 0{\cdot}049\,(n) + 0{\cdot}84851,$$

$$\Delta\alpha = 0{\cdot}049\,\frac{n(n-1)}{2} + 0{\cdot}84851\,(n) + 4{\cdot}4039233,$$

$$\alpha = 0{\cdot}049\,\frac{n(n-1)(n-2)}{6} + 0{\cdot}84851\,\frac{n(n-1)}{2} + 4{\cdot}4039233\,(n),$$

where α is the elevation in minutes for a range of n hundreds of yards.

Thus, putting $n = 35$, we find $\alpha = 980' = 16^\circ\ 20'$, the elevation given by this formula for a range of 3500 yards; but practically this elevation gives a very much smaller range.

Range in yards.	Elevation in minutes.	Differences. First.	Second.	Third.	Fourth.
000	0·0000000	4·4039233			
100	4·4039233	5·2524333	0·84851		
200	9 6563566	6·1499433	0·89751	0·049	
300	15·8062999	7·0964533	0·94651	0·049	
400	22·9027532	8·0919633	0·99551	0·049	
500	30·9947165	9·1364733	1·04451	0·049	
600	40·1311898	10·2299833	1·09351	0·049	
700	50·3611731	11·3724933	1·14251	0·049	
800	61·7336664	12·5640033	1·19151	0·049	
900	74·2976697	13·8045133	1·24051	0·049	
1000	88·1021830	15·0940233	1·28951	0·049	0
1100	103·1962063	16·4325333	1·33851	0·049	
1200	119·6287396	17·8200433	1·38751	0·049	
1300	137·4487829	19·2565533	1·43651	0·049	
1400	156·7053362	20·7420633	1·48551	0·049	
1500	177·4473995	22·2765733	1·53451	0·049	
1600	199·7239728	23·8600833	1·58351	0·049	
1700	223·5840561	25·4925933	1·63251	0·049	
1800	249·0766494	27·1741033	1·68151	0·049	
1900	276·2507527	28·9046133	1·73051	0·049	
2000	305·1553660	30·6841233	1·77951	0·049	
2100	335·8394893	32·5126333	1·82851	0·049	
2200	368·3521226	34·3901433	1·87751	0·049	
2300	402 7422659	36·3166533	1·92651	0·049	
2400	439·0589192	38·2921633	1·97551	0·049	
2500	477·3510825				

With chronographic records recording to four places only of the decimal of a second, the third and higher differences become illusory; but the following fictitious numerical illustrations of the preceding formulas has been concocted on the basis of Round 479, as the imaginary result of ideal screens and an ideal chronograph, reading to seven decimal figures, to show a more general application of the theory.

Screen.	t.	∇.	Δ^2.	Δ^3.	Δ^4.
1	0·0000000	0·0665924			
2	0·0665924	676934	0·0011010	0·0000025	
3	0·1342858	687969	11035	36	
4	0·2030827	699040	11071	47	
5	0·2729867	710158	11118	58	
6	0·3440025	721334	11176	69	0·0000011
7	0·4161359	732581	11247	80	
8	0·4893940	743908	11327	91	
9	0·5637848	755326	11418	102	
10	0·6393174	766846	11520	113	
11	0·7160020	778479	11633		
12	0·7938499				

At the fifth screen, using formula (10),

$$\frac{dt}{ds} = \tfrac{1}{2}(\Delta_4 + \Delta_5) - \tfrac{1}{12}(\Delta_3^3 \Delta_4^3)$$

$$\frac{l}{v_5} = \frac{0 \cdot 1409198}{2} - \frac{0 \cdot 0000105}{12}$$

$$= 0 \cdot 070459900$$

$$- 0 \cdot 000000875$$

$$= 0 \cdot 070459795$$

Using formula (11),

$$l^2 \frac{d^2t}{ds^2} = \Delta^2_3 - \tfrac{1}{12}\Delta_2^4 = 0 \cdot 0011118 - \frac{0 \cdot 0000011}{12}$$

$$= 0 \cdot 0011118$$

$$0 \cdot 00000009$$

$$= 0 \cdot 00111171$$

At the sixth screen,

$$l\frac{dt}{ds} = \frac{0 \cdot 1431492}{2} - \frac{0 \cdot 0000127}{12}$$

$$= 0 \cdot 07157460$$

$$- 0 \cdot 00000106$$

$$= 0 \cdot 07157354$$

$$l^2 \frac{d^2t}{ds^2} = 0 \cdot 00111760$$

$$- 0 \cdot 00000009$$

$$= 0 \cdot 00111751$$

At the first screen, using formulas (5) and (6),

$$l\frac{dt}{ds} = \Delta t - \tfrac{1}{2}\Delta^2 t + \tfrac{1}{3}\Delta^3 - \tfrac{1}{4}\Delta^4 \; \ldots\ldots$$

$$= 0 \cdot 066592400 - 0 \cdot 000550500$$

$$+ 0 \cdot 000000833 - 0 \cdot 000000275$$

$$= 0 \cdot 066593233$$

$$- 0 \cdot 000550775$$

$$= 0 \cdot 066042458$$

$$l^2 \frac{d^2t}{ds^2} = \Delta^2 t - \Delta^3 t + \tfrac{11}{12}\Delta^4 t$$

$$= 0 \cdot 0011010 - 0 \cdot 0000025$$

$$+ 0 \cdot 0000010$$

$$= 0 \cdot 0011020$$

$$- 0 \cdot 0000025$$

$$= 0 \cdot 0010995$$

At the last screen, using formulas (8) and (9),

$$l\frac{dt}{ds} = \Delta t + \tfrac{1}{2}\Delta^2 t + \tfrac{1}{3}\Delta^3 t + \tfrac{1}{4}\Delta^4 t$$

$$\begin{aligned} &= 0.077847900 \\ &\ 0{\cdot}000581650 \\ &\ 0{\cdot}000003767 \\ &\ 0{\cdot}000000275 \\ &= 0{\cdot}078433592 \end{aligned}$$

$$l^2\frac{d^2t}{ds^2} = \Delta^2 t + \Delta^3 t + \tfrac{11}{12}\Delta^4 t + \ldots$$

$$\begin{aligned} &= 0{\cdot}0011633 \\ &\ 0{\cdot}0000113 \\ &\ 0{\cdot}0000010 \\ &= 0{\cdot}0011756 \end{aligned}$$

Take $l = 150,\ d = 6,\ \tau = 1{\cdot}014,\ w = 50.$

$$\begin{aligned} \log d &= 0{\cdot}7781513 \\ \log d^2 &= 1{\cdot}5563025 \\ \log \tau &= 0{\cdot}0073210 \\ \log \tau d^2 &= 1{\cdot}5636235 \\ \log w &= 1{\cdot}6989700 \\ \log C &= 0{\cdot}1353465 \\ \log l^2 &= 4{\cdot}3521825 \\ \log \frac{C \times 10^9}{l^2} &= 4{\cdot}7831640. \end{aligned}$$

Collecting the results in the annexed scheme

Screen.	1.	5.	6.	12.
$l\frac{dt}{ds}$	0·066042458	0·070459795	0·07157354	0·078433592
$\log l\frac{dt}{ds}$	$\bar{2}$·8198232	$\bar{2}$·8479414	$\bar{2}$·8547525	$\bar{2}$·8942518
$\log l$	2·1760913	2·1760913	2·1760913	2·1760913
$\log v$	3·3562681	3·3281499	3·3213388	3·2818395
v	2271·27	2128·77	2095·75	1913·55
$l^2\frac{d^2t}{ds^2}$	0·0010995	0·00111171	0·00111751	0·0011756
$\log l^2\frac{d^2t}{ds^2}$	$\bar{3}$·0411952	$\bar{3}$·0459915	$\bar{3}$·0482514	$\bar{3}$·0702596
$\log \frac{C}{l^2} \times 10^9$	4·7831640	4·7831640	4·7831640	4·7831640
$\log K$	1·8243592	1·8291555	1·8314154	1·8534236
K	66·735	67·477	67·829	71·355

METHOD OF INTERPOLATION.

In consequence of the slight discrepancies with Bashforth's numerical reductions, it is advisable to employ an alternative method, as a check, the Method of Interpolation.

When two screen records only, t_1 and t_2, are taken, as with the Boulengé chronograph at the proof butts, the average velocity between the screens is all that can be determined.

Three records, t_1, t_2, t_3, are the fewest which will enable the resistance of the air to be determined.

We suppose the screens l feet apart, and take the origin from which s is measured at the middle screen; and now the simplest *graph* of the time t is a parabola (fig. 1, p. 173), whose equation is

(42) $$t = t_2 + as + bs^2,$$

where a and b are determined from the conditions that

$$t = t_1, \text{ when } s = -l;$$
$$t = t_3, \text{ when } s = +l.$$

Then $$t_1 = t_2 - al + bl^2,$$
$$t_3 = t_2 + al + bl^2;$$

and therefore

(43) $$al = \tfrac{1}{2}(t_3 - t_1).$$

(44) $$2bl^2 = t_1 - 2t_2 + t_3,$$

and this is what was denoted by $\Delta^2 t_1$.

Now, at the middle screen, where $s = 0$,

$$\frac{1}{v_2} = \frac{dt}{ds} = a, \text{ and } \frac{d^2t}{ds^2} = 2b;$$

so that

(45) $$v_2 = \frac{2l}{t_3 - t_1},$$

the average velocity from the first to the third screen; this is interpreted geometrically on the parabola by the fact that the chord joining the tops of the ordinates representing t_1 and t_3 is parallel to the tangent at the top of t_2, midway between.

Again, if R_2 lbs. denotes the resistance of the air at the middle screen to the shot, of weight w lb.,

(46) $$R_2 = \frac{w}{g}\frac{d^2t}{ds^2}v_2^3 = \frac{w}{g}2bv_2^3$$
$$= \frac{w}{g}\frac{t_1 - 2t_2 + t_3}{l^2}v_2^3.$$

These formulas are equivalent to those employed above in (10) and (11), when the second difference $\Delta^2 t$ is taken as constant; except that they cannot be employed for the first or last screen; but in that case we must take the velocities in Harmonic Progression, as in (24), so that, with twelve screens,

$$\frac{1}{v_1} = \frac{2}{v_2} - \frac{1}{v_3}$$
$$\frac{1}{v_{12}} = \frac{2}{v_{11}} - \frac{1}{v_{10}}.$$

In this way we find, in Round 479 (p. 189),

$$v_2 = \frac{300}{0{\cdot}1343} = 2234$$

$$v_3 = \frac{300}{0{\cdot}1365} = 2198$$

$$\frac{1}{v_1} = \frac{0{\cdot}2686 - 0{\cdot}1365}{300} = 0{\cdot}0004403$$

$$v_1 = 2271 \text{ f/s},$$

as before; and

$$v_{10} = \frac{300}{0{\cdot}1518} = 1976$$

$$v_{11} = \frac{300}{0{\cdot}1540} = 1948$$

$$\frac{1}{v_{12}} = \frac{0{\cdot}3080 - 0{\cdot}1518}{300} = 0{\cdot}0005207$$

$$v_{12} = 1921 \text{ f/s};$$

agreeing practically with the results in Bashforth's Table II, *Final Report*, &c., p. 18.

If five screen records are taken, the fair curve as the graph of t passing through the tops of the time ordinates,

$$t_1,\ t_2,\ t_3,\ t_4,\ t_5,$$

can be drawn, given by

(47) $$t = t_3 + as + bs^2 + cs^3 + es^4,$$

if the origin is taken at the middle screen.

The constants a, b, c, e are determined by the equations

$$t_1 = t_3 - 2al + 4bl^2 - 8cl^3 + 16el^4$$
$$t_2 = t_3 - al + bl^2 - cl^3 + el^4,$$
$$t_4 = t_3 + al + bl^2 + cl^3 + el^4;$$
$$t_5 = t_3 + 2al + 4bl^2 - 8cl^3 + 16el^4.$$

Then

$$t_4 - t_2 = 2al + 2cl^3$$
$$t_5 - t_1 = 4al + 16cl^3,$$

so that

(48) $$al = \tfrac{2}{3}(t_4 - t_2) - \tfrac{1}{12}(t_5 - t_1).$$

Again,

$$t_2 - 2t_3 + t_4 = 2bl^2 + 2el^4,$$
$$t_1 - 2t_3 + t_5 = 8bl^2 + 32el^4;$$

so that

(49) $$2bl^2 = \tfrac{4}{3}(t_2 - 2t_3 + t_4) - \tfrac{1}{12}(t_1 - 2t_3 + t_5).$$

If we confine our attention to the velocity and retardation at the middle screen, the third, then

(50) $$\frac{1}{v_3} = \left(\frac{dt}{ds}\right)_0 = a$$

(51) $$r_3 = \left(\frac{d^2t}{ds^2}\right)_0 v_3^3 = 2bv_3^3.$$

so that a and $2b$ alone are required, and c and e need not be determined.

Generally, in the interpolation method, it is convenient to consider the given screen as the middle one of an odd number of screens; and now, if there are $2n + 1$ screen records,

$$t_{m-n},\ t_{m-m+1},\ \ldots,\ t_{m-1},\ t_m,\ t_{m+1},\ \ldots,\ t_{m+n-1},\ t_{m+n},$$

the simplest representation of the graph of t by a curve passing through the tops of the time ordinates at the screens, l feet apart, is given by an equation of the form

$$t = \frac{(nl+s)\{(n-1)l+s\}\ldots\ldots(l+s)s(l-s)\ldots\ldots(n-1)l-s\}(nl-s)}{2n\,!\ l^{2n}}$$

$$\left\{ C_0 \frac{t_m}{s} + C_1\left(\frac{t_{m+1}}{l-s} - \frac{t_{m-1}}{l+s}\right) + C_2\left(\frac{t_{m+2}}{2l-s} - \frac{t_{m-2}}{2l+s}\right)\right.$$

$$\left. + \ldots\ldots + C_n\left(\frac{t_{m+n}}{rl-s} - \frac{t_{m-n}}{rl+}\right) + \ldots\ldots + C_n\left(\frac{t_{m+n}}{nl-s} - \frac{t_{m-n}}{nl+s}\right)\right\}$$

(52) —(*Principia*, lib. iii, lemma v).

For putting $s = rl$ in this expression, then

$$t = \frac{(n+r)(n-1+r)\ldots\ldots(r+1)r(r-1).2.1.*.1.2\ldots\ldots(n-r)}{2n\,!}$$
$$(-1)^{r-1}C_r t_{n+r}$$

(53) $$= \frac{(n+r)\,!(n-r)\,!}{2n\,!}(-1)^{n-1}C_r t_{n+r},$$

so that we must choose C_r such that

(54) $$C_r = (-1)^{r-1}\frac{2n\,!}{(n+r)\,!\,(n-r\,!)},$$

or $(-1)^{n-1}C_r$ is the coefficient of x^{n+r}, or x^{n-r} in the binomial expansion of $(1+x)^{2n}$.

A similar result is obtained by putting $s = -rl$, $t = t_{m-r}$; also

(55) $$C_0 = \frac{2n\,!}{(n\,!)^2}.$$

Expanding t in ascending powers of s, in the form

(56) $$t = t_m + as + bs^2 + \ldots\ldots$$

we find, on collecting the coefficients of s and s^2,

$$al = \sum_{r=1}^{r=n} \frac{n\,!\ n\,!}{r\ .\ n\,!}\ C_r(t_{m+r} - t_{m-r})$$

$$= \Sigma\ \frac{(-1)^{r-1}}{r}\ \frac{n\,!}{(n+r)\,!}\ \frac{n\,!}{(n-r)\,!}(t_{m+r} - t_{m-r})$$

$$= \Sigma\ \frac{(-1)^{r-1}}{r}\ \frac{n(n-1)\ldots\ldots(n-r+1)}{(n+1)(n+2)\ldots\ldots(n+r)}(t_{m+r} - t_{m-r})$$

57)

$$bl^2 = \sum_{r=1}^{r=n} \frac{n\,!\ n\,!}{r^2\,.\,2n\,!}\ C_r(t_{m+r} - 2t_m + t_{m-r})$$

$$= \Sigma\ \frac{(-1)^{r-1}}{r^2}\ \frac{n\,!}{(n+r)\,!}\ \frac{n\,!}{(n-r)\,!}\ (t_{m+r} - 2t_m + t_{m-r})$$

$$= \Sigma\ \frac{(-1)^{r-1}}{r^2}\ \frac{n(n-1)\ldots\ldots(n-r+1)}{(n+1)(n+2)\ldots\ldots(n+r)}(t_{m+r} - 2t_m + t_{m-r})$$

(58).

It is convenient to employ the notation of

(59) $D_r t_m$ for $t_{m+r} - t_{m-r}$,

and

(60) $D_r^2 t_m$ for $t_{m+r} - 2t_m + t_{m-r}$;

thus, for seven screens,

(61) $al = \frac{3}{4}D_1 t_m - \frac{3}{20}D_2 t_m + \frac{1}{60}D_3 t_m$,

(62) $bl^2 = \frac{3}{4}D_1^2 t_m - \frac{3}{40}D_2^2 t_m + \frac{1}{180}D_3^2 t_m$.

Let us apply these interpolation formulas and notation to Round 473, Final Report, No. VIII, Table I, p. 14; and let us suppose also that the chronograph records could only be read to three places of decimals, or to thousandths of a second; and consider the middle screen, No. 6.

Then $t_6 = 0{\cdot}362$;

$$
\begin{array}{rlrl}
t_5 &= 0{\cdot}287, & t_7 &= 0{\cdot}438; \\
D_1 t_6 &= 0{\cdot}151, & D_1^2 t_6 &= 0{\cdot}001, \\
t_4 &= 0{\cdot}213, & t_8 &= 0{\cdot}515, \\
D_2 t_6 &= 0{\cdot}302, & D_2^2 t_6 &= 0{\cdot}004. \\
t_3 &= 0{\cdot}141, & t_9 &= 0{\cdot}594, \\
D_3 t_6 &= 0{\cdot}453, & D_3^2 t_6 &= 0{\cdot}011, \\
t_2 &= 0{\cdot}070, & t_{10} &= 0{\cdot}673; \\
D_7 t_6 &= 0{\cdot}603, & D_4^2 t_6 &= 0{\cdot}019. \\
t_1 &= 0{\cdot}000, & t_{11} &= 0{\cdot}754; \\
D_5 t_6 &= 0{\cdot}754, & D_5^2 t_6 &= 0{\cdot}030.
\end{array}
$$

Then using three screen records

$$al = \tfrac{1}{2}D_1 t_6 = 0{\cdot}0755,$$
$$bl^2 = \tfrac{1}{2}D_1^2 t_6 = 0{\cdot}0005.$$

Using five screens

$$al = \tfrac{2}{3}D_1 t_6 - \tfrac{1}{12}D_2 t_6 = 0{\cdot}0755,$$
$$bl^2 = \tfrac{2}{3}D_1^2 t_6 - \tfrac{1}{24}D_2^2 t_6 = 0{\cdot}0005.$$

With seven screen records

$$al = \tfrac{3}{4}D_1 t_6 - \tfrac{3}{20}D_2 t_6 + \tfrac{1}{60}D_3 t_6 = 0{\cdot}0755,$$
$$bl^2 = \tfrac{3}{4}D_1^2 t_6 - \tfrac{3}{40}D_2^2 t_6 + \tfrac{1}{180}D_3^2 t_6 = 0{\cdot}00051.$$

With nine screen records

$$al = \tfrac{4}{3}D_1 - \frac{1.4.3}{2.5.6}D_2 + \frac{1.4.3.2}{3.5.6.7}D_3 - \frac{1.4.3.2.1}{4.5.6.7.8}D_4 = 0{\cdot}75503,$$

$$bl^2 = \tfrac{4}{5}D_1^2 - \frac{1.4.3}{2^2.5.6}D_2^2 + \frac{1.4.3.2}{3^2.5.6.7}D_3^2 - \frac{1.4.3.2.1}{4^2.5.6.7.8}D_1^2 = 0{\cdot}000512;$$

and similarly with the whole eleven screen records.

The formulas agree in giving very concordant results for al and bl^2; and now

$$\log al = \bar{2}{\cdot}8779$$
$$\log l = 2{\cdot}1761$$
$$\log v_6 = 3{\cdot}2982,\ v_6 = 1987.$$

Round 473 was fired on March 11th, 1879, when the reading of the barometer was 30·25 inches, and of the wet and dry bulb thermometers was 42° F. and 45° F.; so that from Table XI we can put

$$\tau = 1.$$

The shot weighed 50 lb., and was 6 inches in diameter, so that

$$\log w = 1·6990$$
$$\log d^2 = 1·5563$$
$$\log C = 0·1427$$

Taking the value

$$bl^2 = 0·000512$$
$$\log 2bl^2 = \bar{3}·0103$$
$$\log l^2 = 4·3522$$
$$\log \frac{d^2t}{ds^2} 10^9 = 1·6581$$
$$\log C = 0·1427$$
$$\log K = 1·8008,$$
$$K = 63·22.$$

Mr. Bashforth's average value of K at this velocity is 69·0, so that this shot, Round 473, must have been steadier than the average, its coefficient of steadiness being

$$\sigma = \frac{63·22}{69·00} = 0·916.$$

On the other hand, in Bashforth's *Final Report*, p. 30, we find against Round 473, at this velocity,

$$K = 70·7;$$

so that, working backwards, we find

$$\Delta^2 t = 0·00114$$
$$bl^2 = 0·00057$$

must have been the numbers adopted in the calculation, showing a discrepancy of one ten-thousandth of a second, equivalent to a displacement of about 0·2 of a foot.

It will be noticed, on reference to Report VIII, Table V, that the average value of K from a series of rounds finally adopted by Mr. Bashforth is the mean of numbers which differ considerably among each other, sometimes to 50 °/₀, especially at low velocities.

These discrepancies must not be laid to the fault of the chronograph, but, on the contrary, they are revealed as differences in steadiness between successive rounds, and in the manner in which the shot broke the thread in passing a screen.

When chronographs come to be constructed which will read to the fifth or higher places of decimals of a second, these discrepancies will be rendered more manifest, even with the increased steadiness of breech-loading projectiles.

But a great advantage of the increased accuracy of reading consists in the possibility of bringing the electric screens closer together; thus a chronograph reading to the fifth decimal will give the same accuracy of determination in b or $\frac{d^2t}{ds^2}$, when the screens are brought to a distance which is one $\sqrt{10}$th, or about one-third of the present distance of 150 feet; since the quantity bl^2 can now be read with ten times the former accuracy.

These improvements in the chronograph will be especially valuable in the determination of the resistance of the air at low velocities, where Mr. Bashforth found it necessary to halve the distance between the screens, and to place them 75 feet apart, in consequence of the greater curvature of the trajectory at low velocities.

If the screens, instead of being equidistant, were placed at distances

$$s_1, s_2, s_3, \ldots, s_n,$$

from a fixed origin, and if

$$t_1, t_2, t_3, \ldots, t_n,$$

denoted the coresponding time records, then, according to Lagrange's Interpolation Formula, the simplest algebraical expression for t may be written

$$t = \frac{*\,(s - s_2)(s - s_3)\,\ldots\,(s - s_n)}{*(s_1 - s_2)(s_1 - s_3)\,\ldots\,(s_1 - s_n)} t_1$$

$$+ \frac{(s - s_1)*(s - s_3)\,\ldots\,(s - s_n)}{(s_2 - s_1)*(s_2 - s_3)\,\ldots\,(s_2 - s_n)} t_2$$

$$+ \ldots\ldots\ldots$$

$$+ \frac{(s - s_1)(s - s_2)\,\ldots\,*\,\ldots\,(s - s_n)}{(s_r - s_1)(s_r - s_2)\,\ldots\,*\,\ldots\,(s_r - s_n)} t_n$$

$$+ \ldots\ldots\ldots$$

$$+ \frac{(s - s_1)(s - s_2)\,\ldots\,(s - s_{n-1})\,*}{(s_n - s_1)(s_n + s_2)\,\ldots\,(s_n - s_{n-1})\,*} t_n,$$

a formula which agrees in giving

$$t = t_1, \text{ when } s = s_1;$$

$$t = t_2, \text{ when } s = s_2;$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$t = t_r, \text{ when } s = s_r;$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$t = t_n, \text{ when } s = s_n;$$

the asterisk * showing the position of the omitted vanishing factors.

CHAPTER III.—THE UNRESISTED MOTION OF A PROJECTILE.

ALTHOUGH, as has been shown in Chapters I and II, Part II, the attraction of gravity is a force which is usually small in comparison with the resistance of the air in ordinary problems of direct fire, and may therefore be left out of account in a first approximation to the solution of these problems; still, on the other hand, in high angle fire with low velocities these conditions are reversed; and it is the resistance of the air which becomes comparatively unimportant, and which may be disregarded in comparison with the attraction of gravity.

On this assumption we obtain a fair approximation to the trajectory in high angle fire at short ranges, as for instance with howitzer and mortar fire.

The theory of the unresisted motion of a projectile in a parabolic trajectory, inaugurated by Galileo in 1638, is therefore still of practical importance, and we proceed to develop it in the same manner as that to be employed in resisted motion, in the next chapter.

Supposing R the resistance of the air, and therefore also r the retardation it produces to be zero, equations (1) and (2) of Chapter IV, Part II (p. 215), become

(1) $$\frac{d^2x}{dt^2} = 0,$$

(2) $$\frac{d^2y}{dt^2} = -g.$$

Integrating these equations with respect to t, supposing the shot projected from the origin O with velocity V at an elevation α,

(3) $$\frac{dx}{dt} = \text{a constant} \qquad = V\cos\alpha,$$

(4) $$\frac{dy}{dt} = \text{a constant} - gt = V\sin\alpha - gt.$$

Integrating these equations (3) and (4) again with respect to t,

(5) $$x = Vt\cos\alpha,$$

(6) $$y = Vt\sin\alpha - \tfrac{1}{2}gt^2,$$

no constants of integration being required if the time of flight, t, is reckoned from the instant the shot leaves the point of projection O; these are the equations employed in Chapter II, § 4, Part I.

(7) From (5) $$t = \frac{x}{V\cos\alpha},$$

(8) and, substituting this value of t in (6),

$$y = x\tan\alpha - \frac{gx^2}{2V^2\cos^2\alpha}.$$

Treating this equation as a quadratic in x, by writing it

$$x^2 - \frac{2V^2}{g} x \sin\alpha \cos\alpha = -\frac{2V^2}{g} y \cos^2\alpha,$$

and completing the square in x, then

$$(9) \qquad \left(x - \frac{V^2}{g}\sin\alpha\cos\alpha\right)^2 = -\frac{2V^2}{g}\cos^2\alpha\left(y - \frac{V^2}{2g}\sin^2\alpha\right)$$

which is of the form

$$(10) \qquad (x - h)^2 = -2l(y - k)$$

the equation of a parabola (fig. 1) whose axis is vertical, and vertex at the highest point (h, k), where

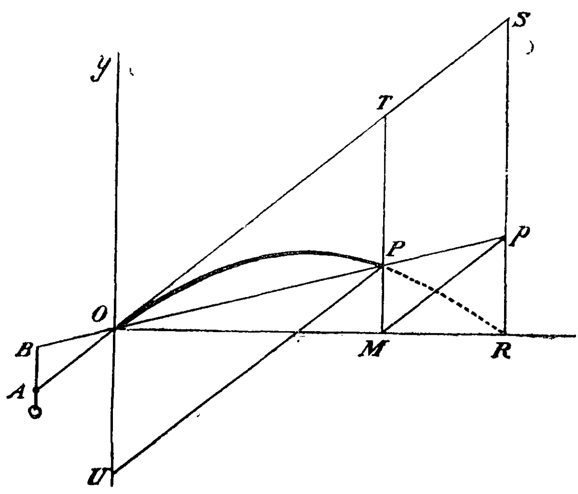
$$(11) \qquad h = \frac{V^2}{2g}\sin 2\alpha, \qquad k = \frac{V^2}{2g}\sin \alpha;$$

also the latus rectum is

$$(12) \qquad 2l = \frac{2V^2\cos^2\alpha}{g};$$

the trajectory is therefore a parabola, as first pointed out by Galileo, in 1638.

FIG. 1.



The co-ordinates of the focus F (fig. 6) are,

$$(13) \qquad h = \frac{V^2}{2g}\sin 2\alpha, \; k - \tfrac{1}{2}l = -\frac{V^2}{2g}\cos 2\alpha;$$

and the height of the directrix HK is (fig. 6)

$$(14) \qquad \text{OH} = k + \tfrac{1}{2}l = \frac{V^2}{2g}.$$

Denoting by v the velocity at any point (x, y) of the parabolic trajectory,

$$(15) \qquad v^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$
$$= V^2 \cos^2 \alpha + (V \sin \alpha - gt)^2$$
$$= V^2 - 2g(Vt \sin \alpha - \tfrac{1}{2}gt)^2$$
$$= V^2 - 2gy$$
$$= 2g(OH - MP) = 2g.PK,$$

(fig. 6), so that the velocity v is that which would be due to falling freely from the level of the directrix, the depth PK below the directrix being called the *head* or *impetus* of the velocity v.

Denoting by X the range, and T the time of flight over a horizontal line Ox through O, obtained by putting $y = 0$ in (6) and (8), then

$$(16) \qquad T = \frac{2V \sin \alpha}{g},$$

$$(17) \qquad X = \frac{2V^2 \sin \alpha \cos \alpha}{g} = \frac{V^2 \sin 2\alpha}{g}.$$

Thus for a given value of V, the range X is a maximum when

$$\sin 2\alpha = 1, \text{ or } \alpha = 45°.$$

Generally

$$(18) \qquad \sin 2\alpha = \frac{gX}{V^2},$$

giving the elevation α required for a range X; or

$$(19) \qquad V^2 = gX \operatorname{cosec} 2\alpha,$$

giving the initial velocity V required for a range X with elevation α, as in Chapter II, p. 172.

Thus if r denotes the distance between the front and back sight, and e the elevation of the back sight required for a horizontal range X, and if A denotes the maximum horizontal range,

$$\sin 2\alpha = \frac{X}{A}, \text{ where } \tan \alpha = \frac{e}{r}, \text{ so that,}$$

$$\frac{e}{r} = \frac{\sqrt{(A + X)} - \sqrt{(A - X)}}{\sqrt{(A + X)} + \sqrt{(A - X)}} = \frac{X}{A + \sqrt{(A^2 - X^2)}},$$

for which a geometrical construction can be devised.

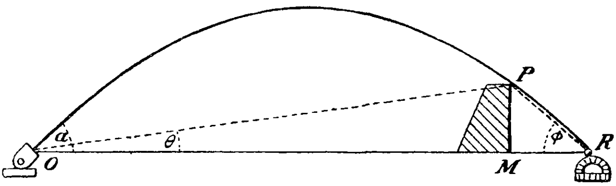
Since $y = 0$, when $x = 0$ or X, equation (8) may now be written

$$(20) \qquad y = x \tan \alpha \left(1 - \frac{x}{X}\right),$$

$$(21) \qquad \tan \alpha = \frac{y}{x\left(1 - \frac{x}{X}\right)} = \frac{y}{x} + \frac{y}{X - x}$$
$$= \tan \theta + \tan \phi,$$

if θ and ϕ are the angular elevations of the point P, as seen from O and R, the beginning and end of the range; this theorem is useful in determining the elevation required with a given range X, so as to

FIG. 2.



clear an obstacle, a wall or rampart, of height y at a distance x from O (fig. 2).

Denoting, as before, the whole time of flight over the range on a horizontal plane through O by T, and the time of flight from O to P by t; denoting also the time of flight from P down to the ground again by t', then

$$t + t' = \mathrm{T},$$

and
$$\mathrm{V} \sin \alpha = \tfrac{1}{2}g\mathrm{T};$$

so that (6) may be written

(22)
$$y = \tfrac{1}{2}g\mathrm{T}t - \tfrac{1}{2}gt^2$$
$$= \tfrac{1}{2}gt(\mathrm{T} - t) = \tfrac{1}{2}\,gtt';$$

Colonel Sladen's formula, of p. 50, useful in plotting approximately points on a trajectory in direct fire, even when the resistance of the air is taken into account, but where the vertical component of the resistance is insensible.

At the vertex A, $t = t' = \frac{1}{2}\mathrm{T}$, and the height of the vertex

(23)
$$k = \tfrac{1}{8}g\mathrm{T}^2 = 4\mathrm{T}^2 = (2\mathrm{T})^2,$$

taking $g = 32$; hence the practical rule :—

"The square of twice the time of flight in seconds is the height of the vertex of the trajectory in feet."

Thus if the time of flight is 5 seconds, the height of the vertex is 100 feet; if

$\mathrm{T} = 0{\cdot}1$ sec., $k = \frac{1}{25}$ foot, less than $\frac{1}{2}$ inch;

$\mathrm{T} = 60$ secs, $k = 14{,}400$ feet.

When firing up a slope Ox, at an inclination of β to the horizon, the equations of motion are

(24)
$$\frac{d^2x}{dt^2} = -\,g \sin \beta;$$

(25)
$$\frac{d^2y}{dt^2} = -\,g \cos \beta;$$

and, integrating twice,

(26)
$$\frac{dx}{dt} = \mathrm{V} \cos \alpha - gt \sin \beta;$$

(27)
$$\frac{dy}{dt} = \mathrm{V} \sin \alpha - gt \cos \beta;$$

(28)
$$x = \mathrm{V}t \cos \alpha - \tfrac{1}{2}gt^2 \sin \beta;$$

(29)
$$y = \mathrm{V}t \sin \alpha - \tfrac{1}{2}gt^2 \cos \beta;$$

and α now denotes the *tangent* elevation of the gun, the *quadrant* elevation being $\alpha + \beta$.

Then, with the preceding notation,

$$T = \frac{2V \sin \alpha}{g \cos \beta};$$

$$X = \frac{2V^2 \sin \alpha \cos \alpha}{g \cos \beta} - \frac{2V^2 \sin^2 \alpha \sin \beta}{g \cos^2 \beta}$$

$$= 4a \frac{\sin \alpha \cos (\alpha + \beta)}{\cos^2 \beta}$$

$$= 2a \frac{\sin (2\alpha + \beta) - \sin \beta}{\cos^2 \beta},$$

if a denotes $\frac{1}{2}V^2/g$, the *head* or *impetus* of the velocity V

Thus for given V or a, and a given slope β, the range X is a maximum when

$$\sin (2\alpha + \beta) = 1,$$

$$2\alpha + \beta = 90°,$$

$$\alpha = 45° - \tfrac{1}{2}\beta,$$

a direction which bisects the angle between the slope and the vertical.

Also, as before,

(30) $$y = \tfrac{1}{2}gtt' \cos \beta,$$

so that the distance from the slope Ox, measured vertically, is still

$$\tfrac{1}{2}gtt';$$

as in Sladen's formula.

Geometrical Investigation of the Parabolic Trajectory.

Many problems of parabolic motion are best solved by a geometrical construction, in accordance with the principles investigated here, and in Chapter II, Part I.

Suppose the body is projected from O in the direction OT with velocity V f/s (fig. 1), then, in the absence of gravity and resistance, the body will be found after t seconds at T, where

$$OT = Vt \text{ (feet)}.$$

But in the same time t seconds a body, if let fall from O, will have reached a point U vertically below O, such that

$$OU = \tfrac{1}{2}gt^2 \text{ (feet)}.$$

Galileo asserted that the body, if projected from O in the direction OT with velocity V, will under the influence of gravity be found after t seconds at P, vertically below T, such that

$$OT = Vt, \; TP = \tfrac{1}{2}gt^2;$$

and the elimination of t leads to the invariable relation for all points on the trajectory OP—

(31) $$\frac{OT^2}{TP} = \frac{V^2t^2}{\frac{1}{2}gt^2} = \frac{2V^2}{g} = 4OH,$$

if OH is measured vertically upwards from O to a height (fig. 6)

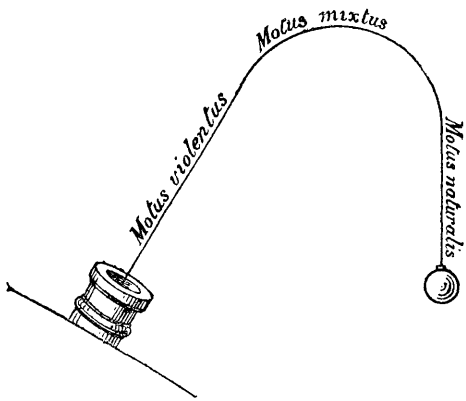
$$(32) \qquad \text{OH} = \frac{\frac{1}{2}V^2}{g};$$

this is the vertical height the body would reach if projected upwards with velocity V, or the vertical depth the body would have to fall to acquire the velocity V; and OH is called the *impetus* or *head* of the velocity V.

According to ancient writers on gunnery, OT was called the *motus violentus*, and TP the *motus naturalis*, while the trajectory OP formed by their combination was called the *motus mixtus;* and Galileo was the first to show that the *motus mixtus* may be supposed resolved into the *motus violentus* and the *motus naturalis*, considered either as simultaneous, or successive, in time.

Ancient diagrams of trajectories show them as composed of a middle portion of *motus mixtus*, with an initial *motus violentus*, supposed to be the so-called *point blank* range, and a final portion of *motus naturalis*, as in fig. 3.

FIG. 3.



The fallacy of the *motus violentus*, during which the motion is taken as exactly rectilinear and *point blank*, was refuted by Tartaglia, in 1554, but the idea still survives (". . . . as easy as a cannon will shoot point blank twelve score," *i.e.*, 240 yards—*The Merry Wives of Windsor*), even to the present day; as it is not easy to detect the curvature of the path at the beginning when the velocity is great.

So also the *motus naturalis* is the motion to which a projectile in a resisting medium gradually approximates, as it tends to coincidence with a vertical asymptote.

Thus to some extent the old theory of the trajectory is a better representation of the motion in a resisting medium than the exact parabolic theory, on the supposition of no resistance.

A jet of water or mercury forms a permanent picture of the trajectory, something intermediate to the parabola and the figure 3.

The above relation for an unresisted trajectory—

$$(33) \qquad \text{OT}^2 = 4\text{OH} \cdot \text{TP},$$

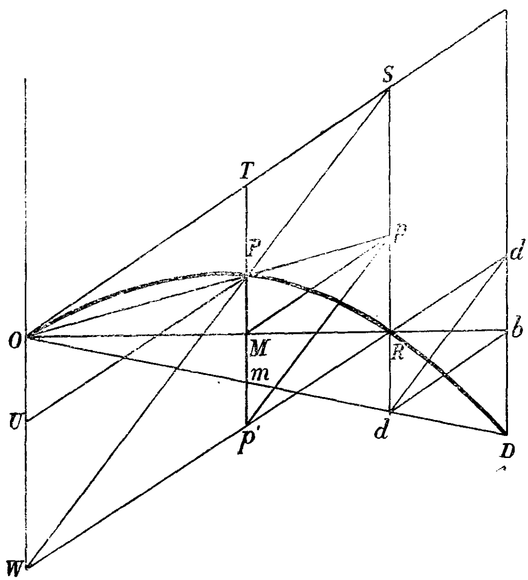
$$\text{or} \quad \text{PU}^2 = 4\text{HO} \cdot \text{OU},$$

defines a *parabola*, according to a fundamental property of the curve.

Take, for instance, the question so often raised of the alteration of elevation required when shooting up or down a slope; and consider it when the resistance of the air is left out of account.

If, instead of measuring the range from the muzzle O, we are allowed to measure it from the point U vertically below O at a depth $\frac{1}{2}gt^2$, equal to the distance a body would fall in the time of flight, t seconds; then for the given range UP the requisite elevation of the gun for any slope of UP will be given by means of a swinging back-sight AB (as used with mountain batteries), pivoted at A so as to hang vertically (fig. 1).

Fig. 4.



If OT is horizontal, we obtain the relation

$$y^2 = 4ax,$$

connecting OH $= a$, OT $= y$, and TP $= x$, from which relation the curve derives its name *parabola*, from having been employed by Greek geometers for the graphic comparison (παραβολή) of squares and square roots.

If OR is the range on a horizontal (or inclined) plane through O and the vertical RS cuts OT in S, then

$$\frac{\text{TP}}{\text{SR}} = \frac{\text{OT}^2}{\text{OS}^2} = \frac{\text{OM}^2}{\text{OR}^2}$$

But if OP produced cuts SR in p,

$$\frac{\text{TP}}{\text{S}p} = \frac{\text{OT}}{\text{OS}},$$

so that

$$\frac{\text{S}p}{\text{SR}} = \frac{\text{OT}}{\text{OS}} = \frac{\text{OM}}{\text{OR}} \qquad (34)$$

and therefore Mp is parallel to OT.

This gives a method of geometrical construction of the parabola, given the range OR and the direction of projection OT.

Thus, if it is required to find where the shot will strike the descending slope OD, produce the vertical line SR to cut OD in d, and draw db parallel to OT, cutting OR produced in b; then the vertical line through b will cut OD in the required point D.

(Blondel, *L'art de jetter les Bombes*, 1699.)

Since M moves along OR with constant velocity, therefore p descends SR also with constant velocity; this explains why when we watch the shot P from O and refer it to the position p in SR, it appears to descend with constant velocity; and conversely, as seen from R, the ball will appear to rise with constant velocity, as is noticeable in catching a cricket ball.

If t' is the time of flight from P to R,

$$\frac{t'}{t} = \frac{\text{MR}}{\text{OM}} = \frac{\text{R}p}{p\text{S}} = \frac{\text{MP}}{\text{PT}}:$$

and
$$\text{PT} = \tfrac{1}{2}gt^2,$$

therefore,

(35)
$$\text{MP} = \tfrac{1}{2}gtt',$$

as in Colonel Sladen's formula.

More generally, if t'' is the time of flight from P to D, and if PM meets OD in m,

(36)
$$m\text{P} = \tfrac{1}{2}gtt''.$$

Also if T denotes the whole time of flight from O to D,

$$\frac{t}{\text{O}m} = \frac{t''}{m\text{D}} = \frac{\text{T}}{\text{OD}},$$

so that

(37)
$$m\text{P} = \tfrac{1}{2}g\text{T}^2\frac{\text{O}m.m\text{D}}{\text{OD}^2},$$

so that T varies as $\sqrt{(m\text{P})}$, if m is a fixed point on the line OD.

A jet of water or mercury and a stream of bullets from a Maxim gun will form an apparently continuous parabola in the air like an inverted catenary, and it would stand as an arch in the air if suddenly arrested and solidified.

The horizontal component of the velocity being uniform, equidistant vertical planes will cut off equal quantities of matter, or the horizontal distribution of weight is uniform; forming the jet into a chain, and inverting it, the jet will serve as the chain of a suspension bridge, in which the weight is supposed concentrated in a uniform roadway.

The height of the C.G. of the jet OPR is the average height of the ordinates, or two-thirds the height of the vertex of the jet, since the parabolic area OPR is two-thirds of the circumscribing rectangle.

This shows that the average height of a projectile in a parabolic trajectory is two-thirds of the height of the vertex; Captain James M. Ingalls, U.S.A., has pointed out the practical use of this result in allowing for the tenuity of the air at great altitudes in a long trajectory, as showing that a good approximation is obtained to the average density of the air traversed by the projectile at a height in the atmosphere of two-thirds of the estimated height of the vertex.

Thus, in a range of 12 miles, with an estimated height of vertex of 3 miles, assume as a first approximation the mean density of the air as the density 2 miles high; this is about 0·68 of the density at the ground.

A jet of water or stream of bullets may be directed vertically upwards, and the C.G. of the jet will be at two-thirds of the height; conversely, the C.G. of a waterfall height h will be a depth $\frac{1}{3}h$ below the crest.

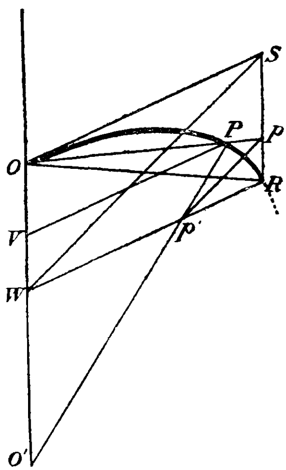
If the parallelogram OSRW is completed, and if TP produced meets RW in p', then

$$\text{(38)} \qquad \frac{Sp}{SR} = \frac{OM}{OR} = \frac{Wp'}{WR},$$

so that pp' is parallel to the diagonal SW (fig. 4).

Conversely, to find where the trajectory cuts Op or Od, draw pp' or dd' parallel to SW, cutting RW in p' or d'; then the vertical line through p' or d' will cut Op or Od in P or D, the required points.

FIG. 5.



This second method of drawing the trajectory is equally applicable to a hyperbolic or elliptic trajectory, except that the lines through p' or d' instead of being vertical or parallel to the axis of the parabola, must be drawn through O′, the other end of the diameter of the hyperbola or ellipse through O (fig. 5);

$$\text{for} \qquad \frac{PV}{OV} = \frac{OS}{Sp} = \frac{RW}{Sp},$$

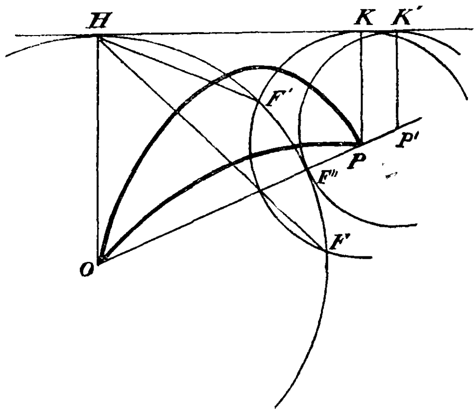
$$\frac{PV}{O'V} = \frac{p'W}{O'W} = \frac{RW}{O'W} \cdot \frac{Sp}{OW},$$

$$\text{(39)} \qquad \text{so that} \qquad \frac{PV^2}{O'V \,.\, OV} = \frac{RW^2}{O'W \,.\, OW},$$

the property of a point P on the hyperbola or ellipse OPR, touching OS, and having the diameter OO′.

To determine the directions of projection from a point O, with given velocity due to the head OH, so as to strike a point P, describe a circle with centre P and radius PK, touching the horizontal line HK through H in K; then if this circle cuts the circle with centre O and radius OH in F and F′, the required directions are perpendicular to HF and HF′, or bisect the angles HOF and HOF′; and these directions, therefore, are equally inclined to the bisector of the angle POH, which is the direction of projection for maximum range on the plane OP; and if the circles do not intersect, the point P is out of range.

FIG. 6.



These results follow because

(40) $$FO = OH, \qquad FP = PK;$$
$$F'O = OH, \qquad F'P = PK;$$

so that F and F′ are the foci of the parabolas that can be drawn passing through O and P, and having the common directrix HK.

The lower parabola with the smaller angle of projection is that required for *direct fire*, and the upper parabola for *high angle* or *mortar* fire.

When the points F and F′ coalesce in F″ the circles touch, and the point P is out of the range attainable from O in the direction OP, when it is beyond P′, where

$$OP' = OH + P'K' = P'K'',$$

and

(41) $$K''K' = OH,$$

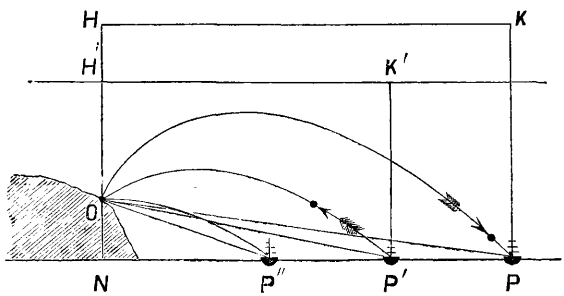
so that the locus of points just within range is the parabola whose focus is O and vertex H; and the space inside the paraboloid generated by the revolution of this parabola about its axis is the space which can be covered from O with the given velocity of projection, points outside this paraboloid being out of range from O.

Suppose, for instance, that OP is the trace of an inclined plane through O, this plane will cut the paraboloid in an ellipse with focus at O, and this ellipse will be the area covered on the inclined plane OP by a gun at O.

The section of the paraboloid made by a vertical plane PK will be a parabola; this will be, for instance, the area covered on a vertical wall PK by a fire engine at O, supposing OH is the greatest height to which the engine can send the jet; and to attain the boundary of the area, the jet must be aimed at points on the wall lying on the horizontal straight line at a height 2OH, twice the *impetus* or *head* of the velocity.

These geometrical considerations can be applied to the problem of a ship P and a fort O engaging, the fort being at an elevation above sea level of h feet; the ship and fort are supposed armed with the same guns, and the resistance of the air is left out of account.

FIG. 7.



The ship will come under the fire of the fort at a point P, where

$$OP = OH + PK = a + a + h = 2a + h.$$

But the ship will not be able to return the fire until the range is OP′, where

$$OP' = OH' + P'K' = a - h + a = 2a - h;$$

and the zone from P to P′ is called the *helpless zone*.

[*Hurrah for the Life of a Sailor*, by Admiral Kennedy :—p. 181, "All this time we could make no reply, as the forts of Sebastopol, from their elevated position, could reach us before we got their range."]

To batter the fort most effectually, the ship must come in closer to P″, so as to make O the vertex of the trajectory of its projectiles; and now

$$NP''^2 = 4OH' \,.\, ON = 4(a - h)h,$$

$$OP'' = \sqrt{(4ah - 3h^2)}.$$

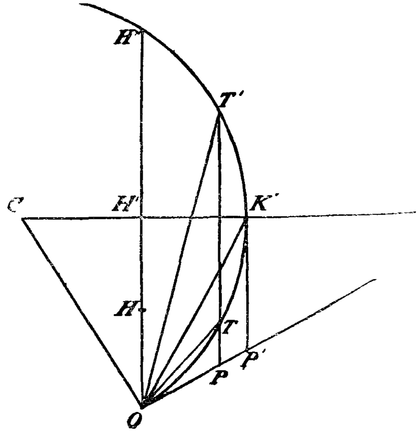
The various directions of projection are also easily inferred from the preceding principles.

Another method of determining the directions of projection required, with given impetus OH from O, to strike a point P depends upon the original relation

$$OT^2 = 4OH \,.\, TP.$$

Draw the horizontal line H′K′ at double the height of H above O (fig. 8); draw OC perpendicular to OP meeting H′K′ in C, and with centre C and radius CO describe a circle cutting the vertical through O again in H″, and the vertical through P in T and T′.

FIG. 8.



Then, from the similar triangle OPT, OTH″,

$$\frac{TP}{OT} = \frac{OT}{OH''} = \frac{OT}{4OH},$$

or

$$OT^2 = 4OH \,.\, TP,$$

so that OT is one direction of projection, the other being OT′.

The times of flight will be the times of falling freely under gravity through TP and T′P.

The point P will be out of range when it is beyond P′, when K′P′ is the vertical tangent of the circle, and where the points T and T′ coalesce in K′.

The direction of projection OK′ which gives the maximum range OP′ is thus directed at a point K′ vertically above P′ at a height above O equal to twice the impetus OH, and OK′ bisects the angle P′OH′; also

$$OP' = PK,$$

so that the locus of P′ is the parabola HP′, with focus O and direction H′K′, as before.

Also the time of flight from O to P′ is equal to the time of falling freely under gravity through a vertical height K′P′ = OP′.

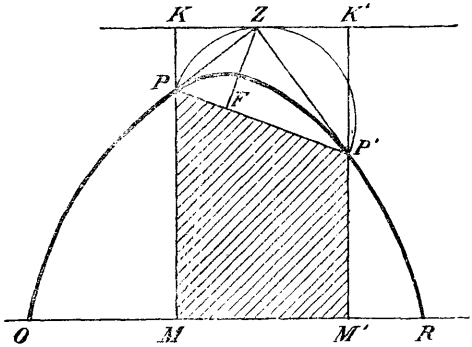
The problem of determining the maximum range with given velocity is obviously the same as that of determining the minimum velocity for given range.

Suppose, for instance, it is required to determine the best position to take up on the ground, so as to drive a ball over a given obstacle MPP′M′, with the least exertion or velocity (fig. 9).

Assuming any horizontal straight line KK′ as the directrix of the trajectory, the circles drawn with centres P and P′, touching KK′, will intersect in points F, F′, the foci of the possible parabolic trajectories.

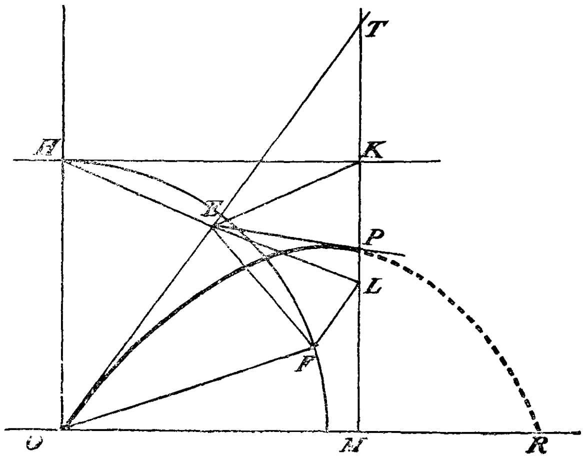
The height of KK′ will be least, and therefore the velocity at P or P′ will be least, when F and F′ coincide; and then the focus F of the unique parabolic trajectory will lie in PP′; and KK′ will be the tangent at the highest point Z of the circle on PP′ as diameter, and ZF will be perpendicular to PP′

FIG. 9.



Now with centre F and radius MK describe a circle, cutting the ground in O and R, then OPP'R will be the requisite trajectory of minimum velocity.

FIG. 10.



Bisect OT in E and join EP (fig. 10); then, since

$$\frac{\text{ET}}{\text{TP}} = \frac{\frac{1}{2}\text{V}}{\frac{1}{2}gt^2} = \frac{\text{V}}{gt},$$

and gt is the velocity which has been *poured into* the body by gravity, we may take ETP as the triangle of velocities, and EP will therefore be the direction of motion at P, or, in other words, the tangent at P; and if V, v denote the velocities at O and P,

$$\frac{\text{V}}{v} = \frac{\text{ET}}{\text{EP}} = \frac{\text{OE}}{\text{EP}}.$$

Produce HE to meet PM in L; then TL = HO; and

$$\text{TE}^2 = \tfrac{1}{4}\text{OT}^2 = \text{HO} \,.\, \text{TP} = \text{TL} \,.\, \text{TP},$$

so that the circle described round the triangle EPL touches ET.

Also $FE = EH = EK = EL$,

and $FEO = HEO = LET$,

so that FL is parallel to OT, and F thus lies on the circle round EPL.

Therefore $EPF = ELF = EFL = EPK$,

so that $FP = PK$,

another demonstration of the fundamental property of the trajectory.

The angle $OFE = OHE = ELP = EFP$,

so that the tangents at O and P subtend equal angles at the focus F, and intersect in a point E, midway between OH and PK; and since

$$OEF = EFL = ELF = EPK,$$

the triangles OEF, EFP are similar; these are well known geometrical properties of the parabola.

Thus $$\frac{FO}{FP} = \frac{OE^2}{EP^2} = \frac{V^2}{v^2};$$

and since $FO = OH = \frac{1}{2}V^2/g$,

therefore $FP = PK = \frac{1}{2}v^2/g$,

so that PK is the *impetus* or *head* of the velocity v at P; and the velocity v at any point P is therefore the velocity which would be acquired in falling freely from the level of the directrix.

Also FK is perpendicular to EP the tangent at P, as FH is perpendicular to OT, the tangent at O.

The sides of the triangle FHK are perpendicular to the sides of the triangle ETP, and the two triangles are therefore similar; so that if v denotes the velocity at the point P,

$$FH : FK : HK = ET : EP : TP = V : v : gt;$$

so that FK is perpendicular, and proportional to the velocity at P.

The directrix HK may thus be taken as the *hodograph* of the trajectory; it possesses the property that the velocity at O represented by FH is changed into the velocity at P, represented by FK, by the vector addition of the velocity represented by HK, which is the velocity communicated by gravity in the time t of passing from O to P.

If this velocity was added by means of a single blow instead of the incessant action of gravity, the velocity would have to be communicated at the point E, on the line midway between OH and PK, and the magnitude should be such as to make the body assume the direction of motion EP perpendicular to FK; and then,

since $FE = EH = EK$,

it follows that $FP = PK$,

a third demonstration.

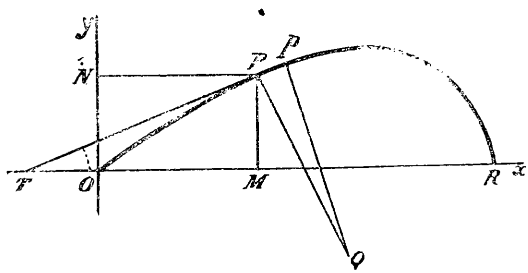
This last demonstration is useful, as it can be applied immediately to the case where the range is so great that the variations in the direction and magnitude of gravity must be taken into account; for instance with bodies projected from a volcano; this is worked out in treatises on Dynamics.

CHAPTER IV.—HIGH ANGLE FIRE.

WHEN the curvature of the trajectory becomes considerable, as in High Angle and Curved Fire, the methods of Chapter II, Part I, for Direct Fire, require modification; we proceed then to consider the equations of motion of a projectile in a resisting medium, when projected with given velocity in a given direction; and to show how these equations, where otherwise intractable, can be slightly modified so as to give tangible practical results.

The motion is referred to two coordinate axes, Ox and Oy, drawn horizontally and vertically in the plane of fire through O, the muzzle of the gun; the resistance of the air is taken to act in the opposite direction to the motion of the centre of gravity of the projectile, so that there is no cause tending to draw the shot out of its original plane of fire, and to cause drift or deviation: this subsidiary effect must be considered separately.

FIG. 1.



Let x, y denote (in feet) the coordinates of the C.G. of the shot P after a time of flight of t seconds; and let θ denote the angle (in *radians* of circular measure) which the tangent TP of the trajectory makes with the horizontal Ox; then

$$\tan\theta = \frac{dy}{dx};$$

also $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are the horizontal and vertical components of the velocity at P.

We denote by V the initial velocity at O, in f/s, and by v the velocity at P, after any time t seconds; so that if the length of the arc OP is s feet

$$v = \frac{ds}{dt};$$

also

$$\cos\theta = \frac{dx}{ds}, \sin\theta = \frac{dy}{ds}$$

The component horizontal and vertical accelerations of the shot P are

$$\frac{d^2x}{dt^2} \text{ and } \frac{d^2y}{dt^2};$$

so that, if g denotes the acceleration of gravity, and r the retardation due to the resistance of the air, the equations of motion may be written

$$(1) \qquad \frac{d^2x}{dt^2} = -r\cos\theta \qquad = -r\frac{dx}{ds}$$

$$(2) \qquad \frac{d^2y}{dt^2} = -r\sin\theta - g = -r\frac{dy}{ds} - g,$$

reducing to the equation of unresisted motion of the preceding chapter, when $r=0$.

Eliminating r,

$$(3) \qquad \frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2} = -g\frac{dx}{dt}.$$

But if $\tan\theta$ or $\frac{dy}{dx}$ is denoted by p, then

$$\frac{dp}{dt} = \frac{d}{dt}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right) = \frac{\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2}}{\left(\frac{dx}{dt}\right)^2};$$

so that

$$\frac{dx}{dt}\frac{d^2y}{dt^2} - \frac{dy}{dt}\frac{d^2x}{dt^2} = \frac{dp}{dt}\left(\frac{dx}{dt}\right)^2,$$

and equation (3) may be written

$$(4) \qquad \frac{dp}{dt}\frac{dx}{dt} = -g,$$

This equation could be obtained immediately by resolving normally in the trajectory, when, as on p. 168,

$$(5) \qquad v\frac{d\theta}{dt} = -g\cos\theta.$$

Denoting $\frac{dx}{dt}$, the horizontal component of the velocity, by q, then with p as independent variable,

$$(6) \qquad \frac{dt}{dp} = -\frac{q}{g}$$

$$(7) \qquad \frac{dx}{dp} = -\frac{q^2}{g}$$

$$(8) \qquad \frac{dy}{dp} = -\frac{pq^2}{g}$$

Before we can integrate these equations, we must determine q as a function of p.

Now from (1) and (4),

$$\frac{dq}{dt} = -r\frac{dx}{ds} = -\frac{rq}{v},$$

$$\frac{dp}{dt} = -\frac{g}{q};$$

so that, by division,

(9) $$\frac{dq}{dp} = \frac{rq^2}{gv}$$

an equation which will determine theoretically the relation between q and p, when r is a given function of v, since

(10) $$v = q\sqrt{(1+p^2)}$$

But as these equations are very intractable, even on the simplest assumptions of laws of the resistance of the air, it is usual nowadays to employ the methods invented by Mr. W. D. Niven (Director of Studies at the R.N. College, Greenwich, formerly Professor of Mathematics to the Advanced Class of Artillery Officers) and by Major F. Siacci, of the Italian Artillery, methods which we proceed to describe.

Keeping to the previous notation, let us denote the gp of p. 166 by $F(v)$, so that $F(v) \div g$ is the resistance of the air in *pounds* to a 1-inch projectile moving with velocity v f/s under standard conditions; thus, in Bashforth's notation (p. 180),

(11) $$F(v) = K\left(\frac{v}{1000}\right)^3$$

Then

$$\frac{r}{g} = \frac{R}{w} = \frac{nd^2p}{w} = \frac{nd^2F(v)}{wg},$$

so that, putting $\frac{w}{nd^2} = C$, the ballistic coefficient,

(12) $$r = \frac{F(v)}{C}$$

Now, since $v = q\sec\theta$, equation (1) may be written

$$\frac{dq}{dt} = -\frac{F(q\sec\theta)}{C}\cos\theta,$$

(13) $$\frac{dt}{dq} = -C\frac{\sec\theta}{F(q\sec\theta)}$$

and then

(14) $$\frac{dx}{dq} = -C\frac{q\sec\theta}{F(q\sec\ )}$$

(15) $$\frac{dy}{dq} = -C\frac{q\sec\theta\,t\ \,n\,\theta}{F(q\sec\theta)}$$

so that q is now the independent variable; and integrating these equations, supposing Q the initial value of q, making Q the upper imit so as to cancel the negative sign,

(16) $$t = C\int_q^Q \frac{\sec\theta\, dq}{F(q\sec\theta)}$$

(17) $$x = C\int_q^Q \frac{q\sec\theta\, dq}{F(q\sec\theta)}$$

(18) $$y = C\int_q^Q \frac{q\sec\theta\tan\theta\, dq}{F(q\sec\theta)}$$

Again, from equation (4),

(19) $$\frac{d\tan\theta}{dt} = -\frac{g}{q},$$

so that

(20) $$\frac{d\theta}{dt} = -\frac{g\cos^2\theta}{q};$$

and multiplying by equation (13),

(21) $$\frac{d\tan\theta}{dq} = C\frac{g\sec\theta}{qF(q\sec\theta)},$$

(22) $$\frac{d\theta}{dq} = C\frac{g}{q\sec\theta(Fq\sec\theta)};$$

and integrating, denoting the initial value of θ by ϕ, when $q = Q$,

(23) $$\tan\phi - \tan\theta = C\int_q^Q \frac{g\sec\theta\, dq}{qF(q\sec\theta)}$$

(24) $$\phi - \theta = C\int_q^Q \frac{gdq}{q\sec\theta F(q\sec\theta)}$$

Now the integrations required in equations (16) (17), (18), (23), (24) are quite intractable, as the relation connecting θ and q, obtained from (23) or (24) is unknown, in the absence of any simple mathematical form of the function $F(v)$.

But, as originally pointed out by Euler, these difficulties can be turned if we notice that in the ordinary trajectories in practice the quantities θ, $\cos\theta$, and $\sec\theta$ vary so slowly that they may be replaced by their *mean* values η, $\cos\eta$, and $\sec\eta$; especially if in the calculations the trajectory, when considerable, is divided up into arcs of small curvature (the *curvature* of an arc is defined as the angle between the tangents or normals at the ends of the arc).

Replacing then in equation (16) the variable angle θ by some mean value η, the formula for t becomes

$$t = C\int_q^Q \frac{\sec\eta\, dq}{F(q\sec\eta)}$$

and introducing Siacci's *pseudo*-velocities u and U, defined by

(25) $$U = Q\sec\eta = V\cos\phi\sec\eta$$

(26) $$u = q\sec\eta = v\cos\theta\sec\eta$$

(27) $$t = C\int_u \frac{du}{F(u)}$$

Similarly, equations (17), (18), (23), (24) become modified into

$$(28) \qquad x = C\int_q^Q \frac{q \sec \eta dq}{F(q \sec \eta)}$$

$$= C \cos \eta \int_u^U \frac{udu}{F(u)}$$

$$(29) \qquad y = C \sin \eta \int_u^U \frac{udu}{F(u)}$$

$$(30) \qquad \tan \phi - \tan \theta = C \sec \eta \int_u^U \frac{gdu}{uF(u)}$$

$$(31) \qquad \phi - \theta = C \cos \eta \int_u^U \frac{gdu}{uF(u)}$$

According to the notation employed in Chapter II, Part I, and Chapter I, Part II, for problems of direct fire, these integrals are the same as those which gave the functions T, S, and I, with the pseudo-velocity u as the argument, instead of the real velocity v, for

$$(32) \qquad \int_u^U \frac{du}{F(u)} = \int_u^U \frac{du}{gp} = T(U) - T(u),$$

$$(33) \qquad \int_u^U \frac{udu}{F(u)} = S(U) - S(u),$$

$$(34) \qquad \int_u^U \frac{gdu}{uF(u)} = I(U) - I(u),$$

while Niven's $D(u)$ is connected with $I(u)$ by the relation

$$(35) \qquad D(u) = \frac{180}{\pi} I(u).$$

Therefore

$$(36) \qquad t = C\{T(U) - T(u)\}$$

$$(37) \qquad x = C \cos \eta\{S(U) - S(u)\}$$

$$(38) \qquad y = C \sin \eta\{S(U) - S(u)\}$$

$$(39) \qquad \tan\phi - \tan\theta = C \sec \eta\{I(U) - I(u)\}$$

$$(40) \qquad \phi - \theta = C \cos \eta\{I(U) - I(u)\}$$

while, expressed in degrees,

$$(41) \qquad \phi^0 - \theta^0 = C \cos\eta \{D(U) - D(u)\},$$

It will be noticed that η cannot be exactly the same mean angle in all these equations: thus it is obviously different in equations (39) and (40); but, considering that we are dealing with arcs of small curvature, the discrepancies due to using the same η throughout will be insensible.

Equations (36), (37), (38), (39), (40) are now in the form employed by General Mayevski, who slightly modified Siacci's original equations by the introduction of Euler's mean angle η; and in the numerical applications we can employ Bashforth's tables for T and S, and Niven's table for D.

We must now explain the meaning of Siacci's *altitude function*, which is denoted by A(u).

Taking equation (39), and replacing tan θ by $\frac{dy}{dx}$,

$$\tan\phi - \frac{dy}{dx} = \mathrm{C}\sec\eta\,\{\mathrm{I(U)} - \mathrm{I}(u)\},$$

and integrating with respect to x over the arc considered,

(42) $$x\tan\phi - y = \mathrm{C}\sec\eta\,\{x\mathrm{I(U)} - \int_0^x \mathrm{I}(u)dx\}$$
$$= \mathrm{C}x\sec\eta\,\mathrm{I(U)} - \mathrm{C}^2\int_u^{\mathrm{U}}\frac{u\mathrm{I}(u)du}{\mathrm{F}(u)},$$

since, from equation (28),

$$\frac{dx}{du} = -\,\mathrm{C}\cos\eta\,\frac{u}{\mathrm{F}(u)}.$$

In Siacci's notation,

(43) $$\int_u^{\mathrm{U}}\frac{u\mathrm{I}(u)du}{\mathrm{F}(u)} = \mathrm{A(U)} - \mathrm{A}(u),$$

where A(u) is called the Altitude Function.

The calculation of the altitude function A was carried out, by Mr. Hadcock, from the

(44) $$\Delta\mathrm{A} = \frac{u\mathrm{I}(u)}{\mathrm{F}(u)}\,\Delta_u = \mathrm{I}(u)\Delta\mathrm{S}$$

taking the mean value of I(u) in any interval.

Thus in continuation of the calculations of the Abridged Table on p. 14,

	990—1000	1000—1010	1010—1020
I	...	0·78839	...
log I	...	$\bar{1}$·89674	...
log ΔS	...	2·12061	...
log ΔA	...	2·01735	...
ΔA	105·60	104·08	102·30
A	5230·14	5335·74	5439·82

Then, dividing by x,

$$\tan\phi - \frac{y}{x} = \mathrm{C}\sec\eta\,\mathrm{I(U)} - \mathrm{C}^2\frac{\mathrm{A(U)} - \mathrm{A}(u)}{x},$$

or, since (39) $x = \mathrm{C}\cos\eta\,\{\mathrm{S(U)} - \mathrm{S}(u)\}$,

(45) $$\frac{y}{x} = \tan\phi - \mathrm{C}\sec\eta\left\{\mathrm{I(U)} - \frac{\mathrm{A(U)} - \mathrm{A}(u)}{\mathrm{S(U)} - \mathrm{S}(u)}\right\}$$

and thus Mayevski's modified form of Siacci's equation is established.

If we assume that the mean angle η in equations (37) and (38) is the same, then by division we obtain simply

(46) $$\frac{y}{x} \quad \tan \eta,$$

the equation employed by Niven in his calculation of trajectories (*Proceedings of the Royal Society*, 1877).

This equation is useful as a first approximation, and is a check upon the calculation by Siacci's altitude function in equation (45).

Very much depends then on a suitable choice of η, the mean inclination in an arc from ϕ to θ, and the most appropriate value of η will not necessarily be the same in all the formulas.

It is the great advantage of Siacci's method that the mean angle η enters only in the form of cos η or sec η, slowly varying quantities for moderate values of η as in practice, so that η need not be determined with great accuracy, as required in Niven's method.

Thus, for instance, according to Niven's calculations (*Proceedings of the Royal Society*, 1877), the best value to employ in (36) is

(47) $$\eta = \tfrac{1}{2}(\phi + \theta) + \tfrac{1}{6}\frac{Q - q}{Q + q}(\phi - \theta),$$

and in (37) is

(48) $$\eta = \tfrac{1}{2}(\phi + \theta) + \tfrac{1}{3}\frac{Q - q}{Q + q}(\phi - \theta);$$

and it is this second value of η which must be employed in equation (38).

According to Didion ("Traité de Balistique," p. 119), the mean angle η in (37) is obtained by supposing the arc from ϕ to θ a portion of a parabola with a vertical axis, and that

(49) $$\sec \eta = \frac{s}{x} = \frac{\int_\theta^\phi \frac{ds}{d\theta}\, d\theta}{\int^\phi \frac{dx}{d\theta}\, d\theta}.$$

Then, if the latus rectum of the parabola is $2l$,

$$\frac{dx}{d\theta} = l \sec^2 \theta;$$

nd $$\frac{ds}{d\theta} = l \sec^3 \theta,$$

so that (50) $$\sec \eta = \frac{\int_\theta^\phi \sec^3 \theta d\theta}{\int_\theta^\phi \sec^2 \theta d\theta} = \frac{i(\phi) - i(\theta)}{\tan \phi - \tan \theta},$$

where $$i(\phi) = \int^\phi \sec^3 \theta d\theta$$

$$= \tfrac{1}{2} \tan \phi \sec \phi + \tfrac{1}{2} \log (\sec \phi + \tan \phi).$$

a function tabulated in Table VIII; and otherwise useful in the calculation of a trajectory when the quadratic law of resistance is assumed.

But if η is another mean angle for the determination of $\frac{y}{x}$, then

$$\tan\eta = \frac{y}{x} = \frac{\int_\theta^\phi \frac{dy}{d\theta}d\theta}{\int_\theta^\phi \frac{dx}{d\theta}d\theta}$$

$$= \frac{\int_\theta^\phi \tan\theta\sec^2\theta d\theta}{\int_\theta^\phi \sec^2\theta d\theta}$$

$$= \frac{\frac{1}{2}(\tan^2\phi - \tan^2\theta)}{\tan\phi - \tan\theta}$$

(51) $$= \tfrac{1}{2}(\tan\phi + \tan\theta);$$

so that Niven's formula (48) is, according to Didion's method, best replaced by

(52) $$\frac{y}{x} = \tfrac{1}{2}(\tan\phi + \tan\theta),$$

equivalent to taking the mean direction as given by the chord of the parabolic arc, having the same initial and final direction.

It will be noticed, however, that the right hand side of these equations contains $\cos\eta$ or $\sin\eta$, the value of which depends on ϕ, which we are seeking to determine; also that U and u, the initial and final pseudo-velocities, depend upon ϕ and θ.

Suppose now that X denotes the range in feet on a horizontal plane obtained with initial velocity V and elevation ϕ, and suppose that v denotes the striking velocity and β the angle of descent, then from equation (37)

(53) $$X = C\cos\eta\{S(U) - S(u)\}$$

where

(54) $$U = V\cos\phi\sec\eta,\ u = v\cos\beta\sec\eta;$$

so that u is determined from

(55) $$S(u) = S(U) - \frac{X}{C}\sec\eta.$$

Also putting $y = 0$ in equation (45),

(56) $$\tan\phi = C\sec\eta\left\{I(U) - \frac{A(U) - A(u)}{S(U) - S(u)}\right\}$$

thus determining ϕ, the requisite angle of elevation; and then putting $\theta = -\beta$ in equation (39),

$$\tan\beta = -\tan\phi + C\sec\eta\{I(U) - I(u)\}$$

(57) $$= C\sec\eta\left\{\frac{A(U) - A(u)}{S(U) - S(u)} - I(u)\right\}$$

determining the angle of descent β.

According to Siacci (*Ballistica*, Chapter V), sec η is replaced by $\sec^2 \phi$ or $\sec^2 \beta$, so that equations (56) and (57) become

$$(58) \qquad \sin 2\phi = 2C\left\{ I(U) - \frac{A(U) - A(u)}{S(U) - S(u)} \right\}$$

$$(59) \qquad \sin 2\beta = 2C\left\{ \frac{A(U) - A(u)}{S(U) - S(u)} - I(u) \right\}$$

where U and u may be replaced by V and v in Direct Fire.

In the problems on Direct Fire the pseudo-velocities U and u are replaced by the real velocities V and v, and it is then also permissible to replace cos η or sec η by unity, so that

$$(60) \qquad S(v) = S(V) - \frac{X}{C}$$

$$(61) \qquad \tan \phi = C\left\{ I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right\}$$

$$(62) \qquad \tan \theta = C\left\{ \frac{A(V) - A(v)}{S(V) - S(v)} - I(v) \right\}$$

$\tan \phi$ and $\frac{1}{2} \sin 2\phi$ being practically the same.

Denoting by u_0 the value of u at the vertex of the trajectory, where $\theta = 0$, then according to equation (37)

$$(63) \qquad \tan \phi = C \sec \eta \, \{I(U) - I(u_0)\}$$

$$(64) \qquad \tan \beta = C \sec \eta \, \{I(u_0) - I(u)\}$$

so that, from equation

$$(65) \qquad \frac{A(U) - A(u)}{S(U) - S(u)} = I(u_0)$$

This function $I(u_0)$ is thus a function given by a table of double entry for the arguments U and u, and to save numerical labour these tables have been drawn up by Captain Braccialini Scipione, of the Italian Artillery, in his *Problemi del Tiro*, Roma, 1883.

Scipione's tables have been adapted to British units by Mr. A. G. Hadcock in Table VI, giving by double entry the value of the function

$$(66) \qquad a = 2\left[I(V) - \frac{A(V) - A(v)}{S(V) - S(v)} \right]$$

in terms of the initial velocity V, and

$$(67) \qquad \frac{X}{C} = \frac{1}{3}\,[S(V) - S(v)],$$

the *reduced* range is *yards*; and now in Direct Fire the requisite elevation ϕ for a range of X yards with initial velocity V for a gun whose ballistic coefficient is C, is given by

$$(68) \qquad \sin 2\phi = Ca \text{ or } \tan \phi = \tfrac{1}{2}\, Ca.$$

These equations are also useful when the height of burst of a shell and the direction of its motion is required at any point of the trajectory.

At any intermediate range of x yards the height of y in yards is given by

(69) $$\frac{y}{x} = \tan\phi - \tfrac{1}{2}\,\mathrm{C}a',$$

where a' refers to the reduced range $\frac{x}{\mathrm{C}}$, and then the angle θ with the horizon at which the shot is moving is given by

(70) $$\tan\theta = \tan\phi - \mathrm{C}\,[\mathrm{I(V)} - \mathrm{I}(v)],$$

When firing at high angles of elevation with high muzzle velocities, as in the "Jubilee rounds," fired in 1888 at Shoeburyness from the 9·2-inch wire gun, at elevations ranging from 18° to 45°, with muzzle velocity 2375 f/s, the calculation of the arcs of the trajectory requires great care in the determination of the mean angle η at the beginning and end of the trajectory, where the inclination is considerable.

The middle highest part of the trajectory, however, is similar to the trajectory of ordinary direct fire, except that the coefficient of tenuity τ is considerably reduced in consequence of the altitude of the vertex, probably from 15,000 to 18,000 feet, where τ is reduced to nearly half its value at the ground.

In this region the inclination θ is so small and changes so slowly that

$$\sin\theta\,\frac{d\theta}{dt},$$

the product of the two small quantities $\sin\theta$ and $\frac{d\theta}{dt}$, is insensible.

Then, since $$q = v\cos\theta,$$

and $$\frac{dq}{dt} = \frac{dv}{dt}\cos\theta - v\sin\theta\,\frac{d\theta}{dt},$$

we may put

(71) $$\frac{dq}{dt} = \frac{dv}{dt}\cos\theta$$

while $$\frac{dq}{dt} = \frac{\mathrm{F}(v)}{\mathrm{C}}\cos\theta\,;$$

so that equation (13) becomes

$$\frac{dt}{dv} = -\,\mathrm{C}\,\frac{1}{\mathrm{F}(v)},$$

or $$t = \mathrm{C}\int_v^{\mathrm{V}} \frac{dv}{\mathrm{F}(v)}$$

(72) $$= \mathrm{C}\{\mathrm{T(V)} - \mathrm{T}(v)\}$$

Similarly equation (17) becomes

$$x = \mathrm{C}\int_v^{\mathrm{V}} \frac{v\cos\theta\,dv}{\mathrm{F}(v)}$$

(73) $$= \mathrm{C}\cos\eta\,\{\mathrm{S(V)} - \mathrm{S}(v)\}$$

and equations (39), (40), (41) and (45) become

(74) $$\tan\phi - \tan\theta = C\sec\eta\{I(V) - I(v)\}$$

(75) $$\phi - \theta = C\cos\eta\{I(V) - I(v)\}$$

(76) $$\phi^\circ - \theta^\circ = C\cos\eta\{D(V) - D(v)\}$$

(77) $$\frac{y}{x} = \tan\phi - C\sec\eta\left\{I(V) - \frac{A(V) - A(v)}{S(V) - S(v)}\right\},$$

so that the pseudo-velocity u is replaced throughout by the real velocity v.

For still smaller values of θ and ϕ, that is in the immediate neighbourhood of the vertex of the trajectory, we may put the mean angle $\eta = 0$; and we thus obtain again, in a slightly different manner, the equations employed in problems of direct fire.

The Tenuity Correction at Great Altitudes.

Having determined the value of the coefficient τ at the ground by means of Table XI, from observations of the barometer and thermometer, its value $\tau(y)$ at a height of y feet in the atmosphere must be inferred from the formula for the density of the air.

The formula usually employed is

$$\frac{\tau(y)}{\tau} = \frac{\delta(y)}{\delta} = e^{-\frac{y}{k}},$$

obtained on the theoretical assumption that the temperature is uniform, in which case the density diminishes at compound discount in ascending in the air.

Here k denotes the height in feet of the *homogeneous* atmosphere, that is the height of an atmosphere of uniform density δ which will give the barometric pressure at the ground, and for moderate values of y it is usual to assume that the barometer falls one inch per thousand feet of height, implying a height of the homogeneous atmosphere of 30,000 feet with a barometric height of 30 inches.,

At the freezing temperature $k = 26{,}214$ feet; and, as air expands uniformly by one-492 part for a rise in temperature of one degree Fahrenheit, therefore at a temperature F,

$$k = 26214\frac{460 + F}{492}.$$

A good average value of k is 27,800 feet, corresponding to a temperature 62° F.

The change in the coefficient for tenuity τ becomes considerable in high angle fire at long ranges; and in the calculations it is advisable to divide up the arcs of the trajectory by horizontal lines, say 1000 feet apart, and to take the coefficient of tenuity in an arc as that due to the mean density of the air in the corresponding stratum.

Thus at the beginning and end of the trajectory, where the inclination is considerable and the shot is ascending or descending rapidly, steps must be taken by arcs of small curvature, say of 1° or 2°; but in the middle portion of the trajectory, when the shot is flying more horizontally, the curvature of an arc may be increased to 18°, 20°, or 22°.

When a long trajectory of this nature has been calculated for a certain initial velocity, and it is desired to know the effect of an increased velocity, portions can easily be added at the beginning and end of the trajectory, to allow for this effect; the trajectory is thereby raised on stilts, as it were.

Considering that the resistance of the air is much reduced in the higher strata of the atmosphere, it seems probable that the greatest ranges will be obtained in very long trajectories by firing at an elevation of over 45°, as the shot is thereby carried more rapidly through the lower denser strata.

(*Calculation of the Trajectory of the Jubilee Shot fired from the* 9·2-*inch B.L. Wire Gun.* By Lieut. A. H. Wolley-Dod, R.A. Proc. R.A. Institution, Vol. XVI, 1888.)

The labour of the calculations is very much increased by this necessity of dividing up the trajectory into a number of smaller arcs, each rising or descending about one or two thousand feet; thus Lieut. Wooley-Dod employed 18 separate arcs.

A convenient rule has been given by Captain James M. Ingalls, U.S.A., for approximating to a high angle trajectory in a single arc, which assumes that the mean density of the air may be taken as the density at two-thirds of the height of the vertex; the rule is founded upon the fact that in an unresisted parabolic trajectory the average height of a projectile is two-thirds of the height of the vertex, as illustrated in a jet of water, or in a stream of bullets from a Maxim gun (p. 207).

On this assumption Captain Ingalls was able to make a very accurate calculation of the trajectory of the Jubilee shot by a small fraction of the labour employed by other calculators.

For instance, if it was estimated that the shot would go 3 miles high, take an average density as that at a height of 2 miles; then

$$\frac{\tau_y}{\tau_0} = 0·68 \text{ about.}$$

(*Handbook of Problems in Direct Fire*, by Captain James M. Ingalls, 1890, p. 304.)

The results of Lieutenant Wolley-Dod's computations are embodied in the following table, the data being

$$d = 9·15,\ w = 380,\ V = 2375,\ \phi = 40°.$$

Barometer 29·5 inches.

Thermometer 55° F.

A correction κ was introduced for the shape of head, such that

$$\log \frac{1}{\kappa} = 0·03329, \text{ for velocities above 1330 f/s.;}$$

$$= 0·05555, \text{ for velocities between 1330 and 1120;}$$

$$= 0·10206, \text{ for velocities between 1120 and 790.}$$

TRAJECTORY OF THE JUBILEE SHOT.

Arc.	Mean y.	Log C.	Log sec η.	U.	u.	x.	y.	t.	v.
40—39	1125	0·70923	0·11262	2357·9	2137·2	2728·4	2225·6	1·582	2121·9
39—38	3153	·74065	·10653	2107·4	1957·5	2235·9	1779·8	1·409	1943·7
38—37	4775	·76624	·10046	1390·3	1820·7	1906·6	1463·7	1·284	1809·0
37—35	6649	·79553	·09216	1787·5	1635·1	3162·3	2299·1	2·292	1614·4
35—33	8666	·82708	·08153	1595·5	1493·7	2584·3	1745·8	2·015	1476·2
33—31	10220	·85132	·07166	1460·1	1386·4	2197·0	1374·4	1·818	1371·1
31—28	11710	·89686	·06041	1351·1	1275·0	2802·5	1589·0	2·453	1256·5
28—24	13290	·92154	·04657	1235·0	1169·5	3127·6	1526·3	2·905	1150·0
24—18	14705	·99140	·03022	1126·3	1073·3	3925·9	1516·2	3·833	1052·7
18—0	16320	1·01538	·00746	1018·5	950·3	9425·4	1565·4	9·749	934·1
40—0	—	—	—	—	—	34095·9	17110·6	29·340	—
0—22	16120	1·01226	0·01139	950·9	893·5	10195·5	2012·1	11·315	938·7
22—30	14130	0·98117	·04722	970·3	938·9	3967·5	1933·1	4·625	972·4
30—36	12125	·94985	·07712	1005·3	973·6	3180·9	2069·4	3·867	1007·6
36—40	10210	·91993	·10386	1035·4	1006·0	2258·9	1767·8	2·819	1033·9
40—44	8260	·88947	·12943	1067·0	1027·1	2373·3	2142·8	3·024	1059·9
44—47	6240	·85791	·15471	1088·7	1049·3	1856·5	1891·0	2·480	1077·5
47—50	4215	·82628	·17919	1110·1	1060·3	1911·5	2162·1	2·674	1091·9
50—52° 20′ ..	2200	·79482	·20320	1120·2	1072·8	1518·3	1887·1	2·220	1100·1
52° 20′—53° 40′ ..	620	·77009	·22600	1117·2	1086·3	877·5	1164·7	1·331	1103·2
53° 40′—53° 50′ ..	—	—	—	—	—	60·1	80·7	0·092	—
0—53° 50′ ..	—	—	—	—	—	28200·0	17110·6	34·447	—

Hence, total range 62,295·9 feet, maximum height 17,110·6 feet, time of flight 63·787 seconds, angle of descent 53° 50′.

The results obtained by the same method for lower elevations were a range of 18,345 yards with 30° elevation, and 19,830 yards with 35° elevation.

The calculation of any one of these arcs can serve as a numerical exercise, the method of working being shown in the exercise on High Angle Fire on p. 44, Chapter II, Part I.

Similar computations by Major James M. Ingalls, U.S.A., for an estimated range of 20 miles with the new American 16-inch gun, will be found in the "Engineer," 19th October, 1900, p. 399.

In the following examples, compiled by Mr. A. G. Hadcock, late R.A., it will be sufficient to work to ten times the accuracy observable in practice; so that times of flight are given to hundredth of a second, distances to one-tenth of a foot or yard, and angles to the nearest minute.

Four significant figures and four-figure logarithms are thus in general sufficient; but cases occur occasionally where a larger number of figures must be retained, in consequence of the disappearance of digits in the process of subtraction; for instance in the subtraction of $\Delta A/\Delta S$ from I_V.

EXAMPLES.

1. Firing, on a horizontal plane, with the 15-pr. B.L. gun, at 2,000 yards range, it is required to know at what height a shrapnel shell will be if burst 200, 150, 100, and 50 yards short; also the angle of descent at each point and at the end of the range.

   Here $d = 3$, $w = 14$ lbs., $V = 1574$, $s = 6000$ are given.

2. In Example 1, find the time of flight to each point, and thence find the height of the burst by Sladen's formula.

3. Find the elevation and the heights at the several distances given in Example 1, using Table X, and working with the slide rule.

4. A 12-inch gun was being used at a range of 3,000 yards for attacking a position 1,200 feet above the sea level. Find the quadrant and tangent elevations for the full charge, which gave a velocity of 2367 f/s., and for the half charge, giving a velocity of 1450 f/s.

   Here $d = 12$, $w = 850$ lbs.

5. Supposing the gun in the last example to be placed in position, 1,200 feet above the sea level, and is firing at an enemy's ship at 3,000 yards range, what will now be the quadrant and tangent elevation for the full and half charge? Find also the angle of descent and remaining velocity.

6. Using the data of the two previous examples, show that the trajectory is practically rigid for medium ranges when firing at objects on the horizontal plane or at a higher or lower level.

7. The 6-inch Q.F. gun, firing a cordite charge of $13\frac{1}{4}$ lbs., has a muzzle velocity at normal temperature of 2154 f/s. It was, however, found during cold weather that the actual range obtained with an elevation of 2° 10′ was 2,530 yards, whereas it should, by the range table, have been 2,780 yards. What extra elevation had to be given to the gun in order to obtain the correct range? The jump is nil.

   Here $d = 6$, $w = 100$ lbs., $s = 7590$, $\phi = 2° 10'$.

8. An escarp had to be breached at the Siege of Strasburg by a gun, equivalent to an 8-inch howitzer of 70 cwt., on the same level. From information received from a spy, the ditch was known to be about 50 feet wide; in consequence of which the necessary angle of descent was calculated to be 14°. The howitzer was using common shell and delay-action fuze, and the engineers required that the striking velocity was not to fall short of 600 f/s.

9. Determine the proportions of weight of bullet to calibre in a new rifle, to fulfil the following conditions: at 1,000 yards range the bullet shall have a velocity of 850 f/s, with a maximum height of trajectory of 25 feet above the horizontal plane of the rifle.

Compare these results with those obtainable with the Mauser 7 mm. rifle, the bullet of which weighs about $12\frac{1}{4}$ grams, and has an initial velocity of 700 m/s (mètres per second).

10. A 10-inch B.L. gun is being fired from a battery 80 feet high above the sea level, against the side of an armour-clad 12 feet above the water line, at a range of 2,500 yards. The muzzle velocity of the gun is 2040 f/s., and the weight of the projectile is 500 lbs. What error made in finding the range will admit of the projectile striking the side (*a*) when the line of sight is on the water line of the ship, (*b*) when it is half way up the ship's side?

11. A ship is attacking a fort 1,400 feet high, situated on a cliff which is practically vertical. The distance from the foot of the cliff to the ship is 1,400 yards, and the vessel is using a 6-inch Q.F. gun, which fires a projectile of 100 lbs., with a muzzle velocity of 2154 f/s. Find the necessary tangent elevation, also the quadrant elevation.

12. In an experiment with the Boulengé Chronograph it was found that the height fallen through by the chronograph was marked at 10·517 inches. The disjunctor reading was corrected to 4·345 inches, which corresponds to a time of 0·15 second. The gun was a 6-inch B.L., firing a flat-headed proof cylinder weighing 100 lbs.; and the screens were 150 feet apart, the nearest being 75 feet from the gun.

Find the velocity at 2 feet from the muzzle.

13. A 9-inch gun was fired at an elevation of 10°, and gave a range of 7,876 yards. Determine the muzzle velocity, supposing the projectile weighed (*a*) 300 lbs., (*b*) 400 lbs.

14. An enemy's captive balloon is found to be making observations, and it is thought desirable to fire at it with time shrapnel from a 15-pr. B.L. gun. The R.E. report, from observations with the plane table, that the height of the balloon is 1,312 feet, and its horizontal distance 3,280 yards from the gun, a range of about 3,310 yards. Find the requisite elevation of the gun.

15. Find the length of the dangerous zone on a horizontal plane for the Lee-Metford rifle, fired from the prone position at a range of 700 yards. Take the average height of a man to be 5·5 feet.

Find also the dangerous zone when the marksman fires from a height of 150 feet at the same range.

16. A 12-inch gun, fired with an initial velocity of 2,400 f/s., gave a range of 5,250 yards, with a tangent elevation of 3° 13′ and a jump of − 8′. The barometer was 29·2 inches, the temperature 48° F., and the projectile weighed 860 lbs.

What would be the range for a projectile weighing 850 lbs., with the barometer standing at 30 inches and a temperature of 60° F.? The velocity need not be corrected for temperature.

17. Find the angle of descent at a range of 4,000 yards for a projectile which requires an elevation of 4° 6′ for a range of 3,900 yards, 4° 16′ for a range of 4,000 yards, and 4° 26′ for a range of 4,100 yards. The jump is $3\frac{1}{2}'$. What would you expect the weight and calibre of the projectile to be, supposing the muzzle velocity is 2,150 f/s?

18. A 6-inch B.L. howitzer is to be used for attacking a magazine, protected in such a way that it is advisable to have an angle of descent of 25°, and a remaining velocity of not less than 600 f/s. Find the muzzle velocity and the position of the gun, supposing (*a*) that the gun and magazine are in the same horizontal plane, (*b*) that the magazine is at a level of 200 feet higher than the gun.

19. In the last example suppose the length of the magazine to be 30 feet parallel to the range, and its width 20 feet, covered by a mound of earth which allows of only the top being penetrated. How many rounds should be provided, considering at least three direct hits required to blow up the magazine?

20. A 12-pr. 12 cwt. Q.F. gun is mounted in a position 100 feet above the mean sea level, and it is fitted with an automatic sight 12 inches above the axis of the gun. Find the quadrant elevation of the gun for ranges of 1,000, 2,000, and 3,000 yards from the gun, and the corresponding angles of sight.

21. During the operations round Colesberg, 4,200 feet above sea level, two 15-pr. field guns were hauled to the top of Coleskop, 800 feet above the surrounding plain. Find the extra range due to this height when the guns are fired with the maximum elevation of 16°.

CHAPTER V.—ACCURACY OF FIRE.

THE consideration of Accuracy of Fire, discussed briefly in Chapter III, Part I, is resumed here, and the theoretical basis of the rules employed is explained in detail.

Take as co-ordinate axes the line drawn from the gun to the centre of the target and the horizontal line through the gun at right angles to the former.

Let the latter be the axis of x and the former the axis of y.

The ordinates of the points of impact give the ranges actually obtained, and the arithmetic mean of the ordinates (or the average ordinate) yields the ordinate of the centre of impact.

Similarly the arithmetic mean or average of the abscissæ gives the abscissa of the point of impact.

To be precise, if

$$x_1, x_2, x_3, \ldots . x_n$$

$$y_1, y_2, y_3, \ldots . y_n$$

represent the abscissæ and ordinates of the n points of impact, and X_0 Y_0 the coordinates of the centre of impact,

$$X_0 = \frac{x_1 + x_2 + x_3 + \ldots . + x_n}{n} = \frac{\Sigma x}{n}.$$

$$Y_0 = \frac{y_1 + y_2 + y_3 + \ldots . + y_n}{n} = \frac{\Sigma y}{n}.$$

Hence the position of the centre of impact is determined.

The choice of coordinate axes is quite arbitrary. It may be convenient sometimes to choose an origin of coordinates on the target itself; this is frequently done, and is, of course, necessary when the target is vertical. Occasionally, however, it is useful to put the successive ranges in evidence as has been done above, so that the ordinate of the centre of impact gives the mean range of the gun as fired.

Now transfer the origin to the centre of impact without altering the directions of the axes.

Let
$$a_1, a_2, a_3, \ldots . a_n$$
$$b_1, b_2, b_3, \ldots . b_n$$

denote respectively the abscissas and ordinates of the points of impact referred to the new axes.

Since the centre of impact is now at the origin,

$$0 = \frac{a_1 + a_2 + a_3 + \ldots . + a_n}{n} = \frac{\Sigma a}{n},$$

$$0 = \frac{b_1 + b_2 + b_3 + \ldots . + b_n}{n} = \frac{\Sigma b}{n},$$

and $$\Sigma a = \Sigma b = 0.$$

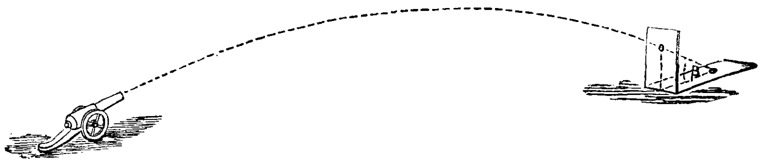
The numbers $a_1, a_2, a_3, \ldots . a_n$ are called the *horizontal or lateral deviations* of the points of impact; and the numbers $b_1, b_2, b_3, \ldots . b_n$ the *longitudinal deviations* of the points of impact.

Observe that *deviati*[illegible]ave always reference to the centre of impact.

The result reached in[illegible]tes that the algebraic sum of the horizontal or longitudinal dev[illegible]ons is zero.

Also we gather at once that the sum of the positive deviations (in either direction) is equal to the sum of the negative deviations, numerical value being alone attended to. When the position of the centre of impact on the horizontal plane is known, fig. 1 shows how the magnitude of the angle of descent determines the position of the centre of impact and of all the points of impact upon a vertical target.

Fig. 1.



Thus, if β be the angle of descent, and if the horizontal target is struck at a distance l from the vertical one, the latter will be struck at a height $l \tan \beta$.

The centre of impact has an important property connected with what is known as the "theory of least squares."

The sum of the squares of the longitudinal (or horizontal) deviations with reference to the centre of impact is a minimum; that is, less than if a point, other than the centre of impact, were taken as origin of the coordinate axes with reference to which the deviations are measured.

This can easily be proved, because

$$a_1 = x_1 - X_0,\ a_2 = x_2 - X_0,\ \ldots\ a_n = x_n - X_0,$$

and therefore

$$a_1^2 + a_2^2 + \ldots + a_n^2 = (x_1 - X_0)^2 + (x_2 - X_0)^2 + \ldots + (x_n - X_0)^2,$$

or $$\Sigma a^2 = \Sigma x^2 - 2X_0 \Sigma x + nX_0^2,$$

and since $$\Sigma x = nX_0,$$

$$\Sigma a^2 = \Sigma x^2 - nX_0^2,$$

Showing that Σa^2 is always less than Σx^2, the defect being nX_0^2, an essentially positive quantity unless $X_0 = 0$, when, obviously, there must be equality.

Hence Σx^2 is a minimum when the origin is the centre of impact.

It follows that the sum of the squares of the absolute deviations has the minimum value

$$\Sigma a^2 + \Sigma b^2,$$

when the deviations are taken with respect to the centre of impact.

Certain definitions are now necessary in order that we may connect the dispersion of the points of impact with the accuracy and precision of the weapon.

The *mean horizontal deviation* is the arithmetical mean of the absolute values of the horizontal deviations. By absolute value is meant numerical value with abstraction of algebraic sign.

This is calculated either by dividing the of the absolute values by the number of shots or by dividing the m of the values of the positive deviations by half the number of ts.

With abstraction of sign, the expressio s

$$\frac{\Sigma a}{n}.$$

The *mean horizontal quadratic deviation*, as found by theory, is

$$\sqrt{\frac{\Sigma a^2}{n-1}},$$

which, when n is not very small so that $n - 1$ may be replaced by n, is practically the square root of the arithmetic mean of the squares of the horizontal deviations.

The *probable horizontal deviation* is that, with respect to which the probabilities of obtaining greater and less deviations are equal; that is to say, in the results of a large number of shots of the same series, half of the horizontal deviations would be less than the probable deviation, and the other half greater; and the probability of obtaining a deviation less than the probable deviation from any particular shot would be one-half.

The same definitions apply, *mutatis mutandis*, to longitudinal, vertical, and absolute deviations.

Similar definitions are employed with regard to "errors" in the "Theory of Errors of Observation."

Write $e(x)$ for mean horizontal deviation,
$e(y)$,, longitudinal (or vertical) deviation,
$E(x)$,, horizontal quadratic deviation,
$E(y)$,, longitudinal (or vertical) deviation,
$r(x)$ for probable horizontal deviation,
$r(y)$,, longitudinal (or vertical) deviation,

and note that when n is large the following results have been established in the "Theory of Probabilities," as given on p. 242.

$$r = 0{\cdot}6745\,E, \qquad E = 1{\cdot}4826\,r.$$
$$r = 0{\cdot}8453\,e, \qquad e = 1{\cdot}1829\,r.$$
$$E = 1{\cdot}2533\,e, \qquad e = 0{\cdot}7978\,E;$$

where all the letters may refer either to x or y.

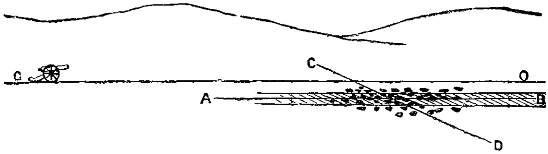
Of the three quantities e, E, and r the probable deviation r is usually chosen as a means of comparison of different guns or different series of shots with the same gun.

From the results of a series of shots both e and E may be calculated by measurements connected with the group of impacts, and from either or both of these quantities r may be deduced by multiplication by a simple decimal number. The calculation of e being more simple than that of E, r is deduced with greater facility from e than from E; but, unless the number of shots is very great, the calculation from E has a greater guarantee of accuracy than that from e.

Suppose that lines are drawn parallel to the line joining the gun with the centre of impact and distant r_x to the right and left of it; we obtain (looking to the definition of r_x) a breadth zone of width $2r_x$ and of indefinite length in which 50 % of the shots (the number being large) will probably fall (fig. 2).

Fig. 2.

Showing 50 °/$_\circ$ breadth zone.



This is termed the 50 °/$_\circ$ *breadth zone.*

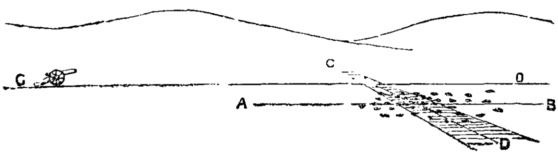
The width of the zone is

$$2r_x = 1\cdot6906\, e_x = 1\cdot349\, E_x.$$

Similarly, by drawing two lines at right angles to the former distant r_y from and on either side of the centre of impact we obtain the 50 °/$_\circ$ *length zone* (fig. 3).

Fig. 3.

Showing 50 % length zone.



The width of the zone is

$$2r_y = 1\cdot6906\, e_y = 1\cdot349\, E_y.$$

So, also, on a vertical target we construct a 50 °/$_\circ$ *height zone.*

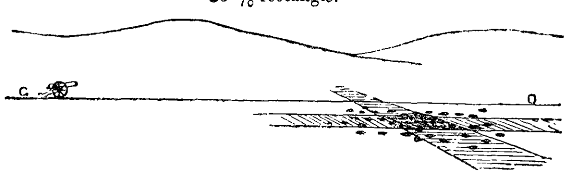
If the 50 °/$_\circ$ breadth and length zones be superposed we obtain a rectangle which must contain 50 °/$_\circ$ of 50 °/$_\circ$ or 25 °/$_\circ$ of the total number of hits. This is called a 25 °/$_\circ$ *rectangle.*

In a similar manner there is a 25 °/$_\circ$ rectangle on a vertical target derived from the 50 °/$_\circ$ breadth and height zones.

The relative accuracy of different guns at different ranges is frequently estimated by the dimensions of this rectangle.

Fig. 4.

Showing 50 °/$_\circ$ length zone and 50 °/$_\circ$ breadth zone intersecting and forming a 25 °/$_\circ$ rectangle.



From data obtained at Sandy Hook with a M.L. rifled mortar at a mean range of 3,357 yards the following values of x and y were obtained, the origin being on the horizontal target, at the shortest range ("Handbook of Problems of Direct Fire," by Captain James M. Ingalls). **Example 1.**

No. of round.	Range.	x.	y.	a.	b.
		yards.	yards.		
178	3264	4	0	−4·67	− 93·11
179	3348	16	84	+7·33	− 9·11
180	3296	9	32	+0·33	− 61·11
181	3427	12	163	+3·33	+ 69·89
182	3473	0	209	−8·67	+115·89
183	3318	6	54	−2·67	− 39·11
184	3320	10	56	+1·33	− 37·11
185	3408	12	144	+3·33	+ 50·89
186	3360	9	96	+0·33	+ 2·89

Here $\Sigma x = 78, \ \Sigma y = 838.$

$$\therefore X_0 = \frac{1}{9}\Sigma x = 8·67\,; \quad Y_0 = \frac{1}{9}\Sigma y = 93·11,$$

giving the position of the centre of impact.

Since $a_1 = x_1 - X_0$, &c., $b_1 = y_1 - Y_0$, &c.,

we calculate the a and b columns which give the coordinates of the points of impact referred to the centre of impact as origin.

The sum of the absolute values of the deviations a is 31·99, and that of the deviations b is 479·11.

Hence $e(x) = \frac{31·99}{9} = 3·55\,; \quad e(y) = \frac{479·11}{9} = 53·23,$

and from the numerical formulas

$$\begin{aligned} r(x) &= 0·845\,e(x) = 2·99 \text{ (yards)},\\ 2r(x) &= 1·69 \ \ e(x) = 5·99 \text{ (yards)},\\ r(y) &= 0·845\,e(y) = 44·98 \text{ (yards)},\\ 2r(y) &= 1·69 \ \ e(y) = 89·96 \text{ (yards)}, \end{aligned}$$

giving the probable horizontal and longitudinal deviations and the width of the 50 °/₀ breadth and length zones as computed from the mean deviations.

Also $\Sigma a^2 = 182\,; \ \Sigma b^2 = 36306·9,$

$$E(x) = \sqrt{\frac{\Sigma a^2}{8}} = 4·77.$$

$$E(y) = \sqrt{\frac{\Sigma b^2}{8}} = 67·37.$$

$$\begin{aligned} \therefore r(x) &= 0·6745\,E(x) = 3·215 \text{ (yards)},\\ 2r(x) &= 1·349\,E(x) = 6·43 \text{ (yards)},\\ r(y) &= 0·6745\,E(y) = 45·44 \text{ (yards)},\\ 2r(y) &= 1·349\,E(y) = 90·88 \text{ (yards)}\,; \end{aligned}$$

the similar results computed from the mean quadratic deviations, and it will be seen that they differ but slightly from those obtained from the mean deviations.

A 25 % rectangle made by the overlapping of the 50 % zones is 90·88 yards by 6·43 yards.

The percentage of hits in other zones, which are symmetrical about the centre of impact in the direction of either axis may be determined; and also the width of zone that may be expected to include a given percentage of hits.

To do this, we require a table of probability factors deduced from theoretical considerations, explained on p. 242.

Table of Probability Factors.

The following gives the proportional width of other zones (containing a different percentage of hits) to one of 50 % as unity.

Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.
1	0·02	21	0·40	41	0·80	61	1·27	81	1·94
2	0·04	22	0·41	42	0·82	62	1·30	82	1·98
3	0·06	23	0·43	43	0·84	63	1·33	83	2·03
4	0·07	24	0·45	44	0·86	64	1·36	84	2·08
5	0·09	25	0·47	45	0·89	65	1·39	85	2·13
6	0·11	26	0·49	46	0·91	66	1·42	86	2·18
7	0·13	27	0·51	47	0·93	67	1·45	87	2·24
8	0·15	28	0·53	48	0·95	68	1·48	88	2·30
9	0·17	29	0·55	49	0·98	69	1·51	89	2·37
10	0·18	30	0·57	50	1·00	70	1·54	90	2·44
11	0·20	31	0·59	51	1·02	71	1·57	91	2·52
12	0·22	32	0·61	52	1·04	72	1·60	92	2·60
13	0·24	33	0·63	53	1·07	73	1·64	93	2·69
14	0·26	34	0·65	54	1·09	74	1·67	94	2·78
15	0·28	35	0·67	55	1·12	75	1·71	95	2·91
16	0·30	36	0·70	56	1·14	76	1·74	96	3·04
17	0·32	37	0·72	57	1·17	77	1·78	97	3·32
18	0·34	38	0·74	58	1·19	78	1·82	98	3·45
19	0·36	39	0·76	59	1·22	79	1·86	99	3·82
20	0·38	40	0·78	60	1·25	80	1·90	100	Infinite.*

* As a factor of 4 contains more than 99 % of the rounds fired, it may be taken for practical purposes to contain the total of 100 %.

In the first column will be found numbers representing the percentages of hits that may be expected in the zones; the corresponding factors represent the multiples that the widths of the zones are of the width of the 50 % zone.

To find the width of the length zone that will contain 75 % of the hits, we enter the table at the number 75 in the column headed "Per cent.," and find the corresponding factor to be 1·71. We deduce, therefore, that the width of the required zone is 1·71 times the width of the 50 % length zone.

Also to find the percentage of hits that will be included in breadth zone 1·25 times the width of the 50 % breadth zone, we enter the table at the number 1·25 in the column headed "Factor," and find the corresponding percentage to be 60. We conclude that 60 % of hits will be found in the given breadth zone.

Intermediate results can be obtained from the table by interpolation.

Rectangles containing a given percentage of hits can be obtained, and conversely we can determine the percentage of hits that will be found in any given rectangle which is symmetrical about the centre of impact.

Suppose a rectangle to be obtained by superposition of a breadth zone of p °/。 and a length zone of q °/。, then the rectangle will contain p °/。 of q °/。, or $\frac{pq}{100}$ °/。 of the hits.

For the design of a rectangle to contain R °/。 of hits we have the relation

$$\frac{pq}{100} = \text{R}.$$

for the determination of p and q. The equation has an infinite number of solutions, so that we can design an infinite number of rectangles containing the given percentage R of hits. We may give q any value we please, and thence determine p from the equation

$$p = \frac{100\text{R}}{q}.$$

We look out q and $\frac{100\text{R}}{q}$ in the column of the table headed "Per cent.," and thence find the widths of the length and breadth zones, which, by superposition, give an R °/。 rectangle. These widths are the longitudinal and horizontal sides of the rectangle.

The 25 °/。 rectangle already met with is thus only one of an infinite number of 25 °/。 rectangles. For its design we excluded 50 °/。 of hits for horizontal deviations and 50 °/。 for longitudinal deviations.

It is frequently desired, as in this case, to exclude the same number of hits for horizontal as for longitudinal deviations, and then the determination of the rectangle rests upon the equation

$$p^2 = 100\,\text{R},$$

or

$$p = 10\sqrt{\text{R}}.$$

An example will make the subject clearer.

Example 2. Find a rectangle containing 50 °/。 of hits such that the same number of hits may be excluded for horizontal as for longitudinal deviations. Here R = 50, and if p be the percentage of hits in the breadth and length zones which, by superposition, give the rectangle

$$p = 10\sqrt{50} = 70{\cdot}7\,;$$

entering the table we find, by interpolation, the factor 1·56, so that the widths of the zones are 1·56 times the widths of the corresponding 50 °/。 zones. Hence the sides of the rectangle are,

$$1{\cdot}56 \times 2r_x = 3{\cdot}12\,r_x,$$

and

$$1{\cdot}56 \times 2r_y = 3{\cdot}12\,r_y.$$

A study of the table shows that a zone four times the width of the 50 °/。 zone practically contains the whole of the hits. This zone is termed the "*enveloping zone*." By superposition of the enveloping breadth and length zones we obtain the *enveloping rectangle*, which may be shown to comprise 98·6 °/。 (practically all) of the hits.

It is obvious that in many cases the horizontal deviations will not be of so much importance as those in the longitudinal direction, and that it will be useful to calculate rectangles which give relatively small importance to the horizontal deviations. In the extreme case of a gun which shoots practically perfectly as to line we need only consider the length zones which are the extreme cases of the rectangles.

Example 3. The numbers of hits excluded for horizontal and longitudinal deviations respectively being in the ratio of 2 to 3, determine the dimensions of the 50 °/₀ rectangle.

p and q having the meanings before assigned, we have the relation

$$100 - p = \frac{2}{3}(100 - q),$$

or

$$3p = 2q + 100;$$

and since

$$pq = 5000,$$

we are led to the quadratic

$$2q^2 + 100q = 15000,$$

from which

$$q = 65{\cdot}14$$
$$p = 76{\cdot}76.$$

From the table the factors are found to be 1·40 and 1·77.

Hence the sides of the rectangle are

$$1{\cdot}77 \times 2r(x) = 3{\cdot}54\,r(x).$$
$$1{\cdot}40 \times 2r(y) = 2{\cdot}80\,r(y).$$

The actual number of hits obtained upon a *given* target depends upon the position of the centre of impact relative to the target.

Examples illustrative of the foregoing principles are now given.

Example 4. What percentage of hits would be obtained on a long wall 12 feet high if fired at by the 8-inch howitzer of 70 cwt. at a range of 1600 yards with a charge of $10\frac{1}{2}$ lbs., supposing the centre of impact half way up the wall?

The range table gives the width of 50 °/₀ height zone as 6·93 feet.

The ratio to this of the height of the wall is $12 \div 6{\cdot}93 = 1{\cdot}73$.

Corresponding to this number in the table we find (by interpolation) the number 75·6. Hence 75·6 °/₀ of the shots may be expected to strike the wall.

Example 5. In the last example, what length of wall, symmetrical about the centre of impact, would be struck by 25 °/₀ of the shots?

The wall itself is a 75·6 °/₀ height zone; we have to superpose a breadth zone so as to form a 25 °/₀ rectangle. Let this zone contain p °/₀ of hits.

Then

$$p \times 75{\cdot}6 = 100 \times 25,$$

or

$$p = \frac{2500}{75{\cdot}6} = 33{\cdot}1.$$

Hence the breadth zone of width equal to the length of the wall receives 33·1 °/₀ of the hits.

Opposite 33·1 in the table is found the number 0·63, indicating that the width of the zone, and therefore the length of the wall, is 0·63 of the width of the 50 °/₀ breadth zone.

By the range table this is 1·86 feet. Hence the length of the wall is

$$0{\cdot}63 \times 1{\cdot}86 = 1{\cdot}17 \text{ (feet)}.$$

Example 6. If a zone of a certain width receives 20 °/₀ of the hits, how wider must another zone be that it may receive 80 °/₀?

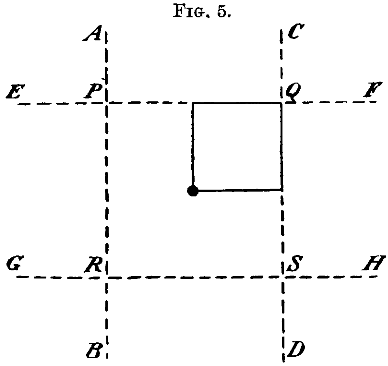
In the table we find the probability factors corresponding to 20 and 80 °/₀ 0·38 and 1·90 respectively.

$$\text{Hence} \qquad \frac{\text{width of } 80\ \%\text{ zone}}{\text{width of } 20\ \%\text{ zone}} = \frac{1{\cdot}90}{0{\cdot}38} = 5$$

that is to say, the 80 % zone is five times as wide as the 20 % zone.

Example 7. If the 50 % breadth and height zones are each 6 feet wide, what percentage of hits may be expected on a vertical target 6 feet square if the centre of impact be at the lower left hand corner?

FIG. 5.



The breadth zone, which includes the whole of the target, is bounded by the lines AB, CD, and the height zone, which has the same property by the lines EF, GH.

From the table we see that each of these zones, being 12 feet wide, includes 82·27 % of the hits.

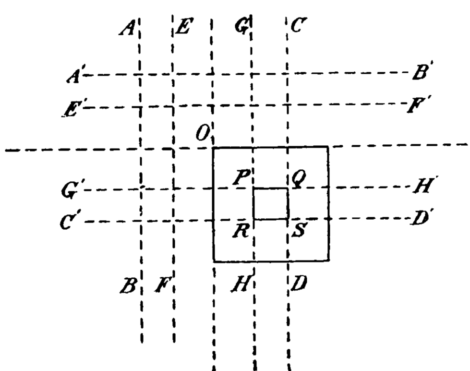
Therefore the rectangle PQRS formed by superposing the zones includes 82·27 % of 82·27 %, or 67·7 % of the hits.

By symmetry only a quarter of these will hit the target.

Hence the required percentage is

$\frac{1}{4}$ of 67·7 or 16·9.

FIG. 6.



Example 8. A vertical target is 8 feet square with a bullseye 2 feet square. If the breadth and height zones are each 6 feet wide and the centre of impact is at the left hand top corner of the target, find the percentage of hits on the bullseye.

O being the centre of impact and PQRS the bullseye, draw symmetrical breadth zones ABCD, EFGH, and height zones A′B′C′D′, E′F′G′H′.

The zone ABCD has factor $\frac{0}{6}$ and includes 73·51 °/ₒ.

EFGH „ $\frac{6}{6}$ „ 50·00 °/ₒ.

Hence the zone GHCD includes

$$\tfrac{1}{2}(73{\cdot}51 - 50) \text{ or } 11{\cdot}75\ \%.$$

Similarly the zone G′H′C′D′ includes 11·75 per cent., and hence by superposition the bullseye PQRS includes

11·75 °/ₒ of 11·75 °/ₒ,

or 1·38 °/ₒ of hits.

If the mean longitudinal deviation (that is the mean error in range) be 15·3 yards, and the mean horizontal deviation (or mean lateral error) be 1·07 yards, find the probability of a single shot striking a horizontal target, 41 yards by 2 yards, the longer side being parallel to the plane of fire and its centre coinciding with the centre of impact. Example 9.

Here

$$e(x) = 1{\cdot}07,\ e(y) = 15{\cdot}3.$$
$$\therefore r(x) = 0{\cdot}845 \times 1{\cdot}07 = 0{\cdot}9 \text{ (yards)}.$$
$$r(y) = 0{\cdot}845 \times 15{\cdot}3 = 12{\cdot}9 \text{ (yards)}.$$

Therefore the widths of the 50 °/ₒ zones are

$$2r_x = 1{\cdot}8 \text{ (yards)}.$$
$$2r_y = 25{\cdot}8 \text{ (yards)}.$$

The breadth zone, which includes the given rectangle, has a factor—

$$\frac{2}{1{\cdot}8} = 1{\cdot}11,$$

and the length zone a factor—

$$\frac{41}{25{\cdot}8} = 1{\cdot}58.$$

By the table these zones include 54·7 °/ₒ and 71·4 °/ₒ of the hits respectively.

Hence, by superposition, the given rectangle 41 yards by 2 yards, includes

54·7 °/ₒ of 71·4 °/ₒ or 39 °/ₒ of the hits.

Therefore the probability of a single shot striking the rectangle is—

$$\frac{39}{100} \text{ or } 0{\cdot}39.$$

The coordinates of the centre of impact have been denoted by X_0, Y_0. If the target is horizontal and the origin at the firing point; Y_0 is the arithmetic mean of the several ranges actually obtained; only when the number of rounds is increased indefinitely does Y_0 represent the exact range appertaining to the gun as laid.

The probable deviation of a single point of impact has been denoted by $r(y)$; this also is deduced from the rounds fired and is only exact when the number of rounds increases without limit. The probable deviation of the centre of impact, deduced from a series of n rounds, from the true centre of impact is found by dividing the probable deviation of a single shot, deduced from the series, by the square root of the number of shots.

Thus if $r(y)$ be the probable deviation in range of a single point of impact

$$\frac{r(y)}{\sqrt{n}}$$

is the probable deviation in range of the centre of impact of a group of n shots.

As an example take the data of example 1. From 9 rounds a mean range of 3,357 yards was obtained, and the probable deviation in range of a single shot was found to be 45·44 yards. The range that might be obtained from a single shot would be denoted by

$$3357 \pm 45{\cdot}44 \text{ yards};$$

but the arithmetic mean of the 9 ranges would be represented by

$$3357 \pm \frac{45{\cdot}44}{\sqrt{9}},$$

or

$$3357 \pm 15{\cdot}15 \text{ yards}.$$

Correction of Fire.

In actual practice the gun should be so laid that the centre of impact is as near as possible to the point on the target that it is desirable to strike. If the range is accurately known, the weapon, ammunition, &c., perfect, the physical conditions ideal, and the marksman expert, the centre of impact will necessarily be very close to the point in question, and the gun may be fired continually without any correction whatever. Some or all of the above mentioned conditions, however, may not be satisfied, and it becomes necessary to evolve the principles which should guide correction of fire.

Consider merely errors in range.

Let the true range be y yards; assume the first round to be laid for a range R yards, and that the point of impact is p_1 yards over. The actual range obtained is $y + p_1$ yards, and the experience of this single round leads to the conclusion that the most probable range appertaining to the gun as laid is

$$y + p_1 \pm r(y) \text{ yards},$$

$r(y)$ being the probable longitudinal deviation of the given gun at the given range as deduced from the range table of the gun.

Observe that p_1 may be positive or negative; it will be negative if the point of impact is short of the desired range.

Although the probable deviation of this first shot is $r(y)$ yards, the theory shows us that the point distant $y + p_1$ yards from the gun, is more likely to be the centre of impact of a number of rounds fired for a range of R yards than any other point along the range that can be assigned.

Accordingly the best chance, in view of bringing the centre of impact near to the desired point, is to fire the second round for a range $R - p_1$ yards. Under these circumstances, guided by the experience of the first round, the centre of impact of a series of rounds fired for a range $R - p_1$ yards is more likely to be y yards from the gun than at any other point along the range.

Suppose this second round fired and the actual range obtained to be $y + p_2$ yards where, as before, p_2 may be negative.

We have obtained a range of $y + p_2$ yards by laying the gun for a range $R - p_1$ yards. We may assume without sensible error that had this round been fired for a range R yards, the range obtained would have been $y + p_1 + p_2$ yards.

Practically we have before us the results of two rounds fired for a range R yards. The range appertaining to the gun, when laid for R yards range, is now most probably the arithmetic mean of the two ranges, or

$$\tfrac{1}{2}(y + p_1 + y + p_1 + p_2) = y + p_1 + \tfrac{1}{2}p_2 \text{ yards.}$$

Hence, for the third shot, the best chance is to lay for a range

$$R - p_1 - \tfrac{1}{2}p_2 \text{ yards.}$$

Observe that the corrections in range for the 2nd and 3rd rounds have been

$$- p_1 \text{ yards and } - \tfrac{1}{2}p_2 \text{ yards}$$

respectively.

If the range obtained by the 3rd round is $y + p_3$ yards, we have virtually obtained a range

$$y + p_1 + \tfrac{1}{2}p_2 + p_3 \text{ yards}$$

by firing for a range R yards.

The arithmetic mean of the ranges obtained on firing for R yards is now

$$\tfrac{1}{3}(y + p_1 + y + p_1 + p_2 + y + p_1 + \tfrac{1}{2}p_2 + p_3) \text{ yards,}$$

or

$$y + p_1 + \tfrac{1}{2}p_2 + \tfrac{1}{3}p_3 \text{ yards.}$$

Hence the 4th round should be fired for a range

$$R - p_1 - \tfrac{1}{2}p_2 - \tfrac{1}{3}p_3 \text{ yards,}$$

that is, the correction for the 4th round is $-\frac{1}{3}p_3$ yards.

Similarly, if this round realises a range

$$y + p_4 \text{ yards,}$$

the correction for the 5th round is $-\frac{1}{4}p_4$ yards.

In general, if the $(n-1)^{\text{th}}$ round realises a range $y + p_{n-1}$ yards, the correction for the nth round is $-\dfrac{1}{n-1}p_{n-1}$ yards.

Probabitity of Fire.

According to the theory of Probability, a certain curve, called the *error curve*, of the shape in fig. 19, p. 74, can be drawn representing graphically by its area the percentage (°/₀) of shots which in the long run can be expected on a target of given dimensions.

Suppose, for instance, that $x'Ox$ in fig. 19 is drawn along the line of mean direction so that O is at the point of mean impact, and that in a very large series of practice all the shot which struck on the line QQ′ are arranged in contact along the ordinate MP; if this is done with all the shot, they will be found arranged in a certain area, bounded by the straight line $x'Ox$ and the error curve $x'Ax$.

This curve of error can be realised experimentally by an instrument (fig. 20, p. 75) invented by Mr. Francis Galton, which he calls the *Quicunx*, from the Latin word describing the arrangement of trees in an orchard.

A charge of small shot is allowed to pour through the funnel at the top; the shot knock against pins arranged like trees, and are scattered thereby in an arbitrary manner; but it is found that the shot always group themselves in the stall at the bottom in a manner which imitates closely the profile of the error curve.

In accordance with abstruse theoretical principles, the error curve can be best represented by an equation of the form

(1) $$y = a\, e^{-\frac{x}{c^2}}, \text{ or } a \exp\left(-\frac{x^2}{c^2}\right);$$

and then the area OMP is given by

(2) $$\text{A}(x) = a\int_0^x \exp\left(-\frac{x^2}{c^2}\right)dx,$$

but as this integral is not the antidifferential of any known function, it must be evaluated by approximate numerical computation.

But denoting the whole area to the right of OA, extending to infinity, by A,

(3) $$\text{A} = a\int_0^\infty \exp\left(-\frac{x^2}{c^2}\right)dx = \tfrac{1}{2}\sqrt{\pi}\, ac,$$

a well known definite integral.

In the long run the mean error e is the abscissa of the C.G. of the area A, so that

(4) $$e\text{A} = a\int_0^\infty x\exp\left(-\frac{x^2}{c^2}\right)dx = \tfrac{1}{2}\, ac^2.$$

(5) $$\frac{e}{c} = \frac{1}{\sqrt{\pi}}.$$

So also

(6) $$\text{E}^2\text{A} = a\int_0^\infty x^2\exp\left(-\frac{x^2}{c^2}\right)dx = \tfrac{1}{4}\sqrt{\pi}\, ac^2;$$

so that E is the radius of gyration of the area A about Oy; and

(7) $$\text{E} = \tfrac{1}{2}c^2,\quad \frac{e}{\text{E}} = \sqrt{\frac{2}{\pi}} = 0{\cdot}7978.$$

The ratio of A (x) to A is the probability that the error will be less than x; denoting this by P, and putting

(8) $$\frac{x}{c} = t,$$

(9) $$\mathrm{A}(x) = ac \int_0^t e^{-t^2}\, dt,$$

so that

(10) $$\frac{\mathrm{A}(x)}{\mathrm{A}} = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2}\, dt,$$

and P has been calculated as a function of t, by approximate numerical computation, and tabulated below :—

t.	P.	t.	P.	t.	P.	t.	P.
0·00	0·00000	0·60	0·60386	1·20	0·91031	1·80	0·98909
0·02	0·02256	0·62	0·61941	1·22	0·91553	1·82	0·98994
0·04	0·04511	0·64	0·63458	1·24	0·92050	1·84	0·99073
0·06	0·06762	0·66	0·64938	1·26	0·92523	1·86	0·99147
0·08	0·09008	0·68	0·66378	1·28	0·92973	1·88	0·99216
0·10	0·11246	0·70	0·67780	1·30	0·93401	1·90	0·99279
0·12	0·13476	0·72	0·69143	1·32	0·93806	1·92	0·99338
0·14	0·15695	0·74	0·70468	1·34	0·94191	1·94	0·99392
0·16	0·17901	0·76	0·71754	1·36	0·94556	1·96	0·99443
0·18	0·20093	0·78	0·73001	1·38	0·94902	1·98	0·99489
0·20	0·22270	0·80	0·74210	1·40	0·95228	2·00	0·99532
0·22	0·24429	0·82	0·75381	1·42	0·95537		
0·24	0·26570	0·84	0·76514	1·44	0·95830		
0·26	0·28690	0·86	0·77610	1·46	0·96105		
0·28	0·30788	0·88	0·78669	1·48	0·96365		
0·30	0·32863	0·90	0·79691	1·50	0·96610	3·00	0·99998
0·32	0·34912	0·92	0·80677	1·52	0·96841		
0·34	0·36936	0·94	0·81627	1·54	0·97058		
0·86	0·38933	0·96	0·[illegible]2542	1·56	0·97263		
0·38	0·40901	0·98	0·83423	1·58	0·97455		
0·40	0·42839	1·00	0·84270	1·60	0·97635		
0·42	0·44747	1·02	0·85084	1·62	0·97804		
0·44	0·46622	1·04	0·85865	1·64	0·97962		
0·46	0·48465	1·06	0·86614	1·66	0·98110		
0·48	0·50275	1·08	0·87333	1·68	0·98249		
0·50	0·52050	1·10	0·88020	1·70	0·98379		
0·52	0·53790	1·12	0·88679	1·72	0·98500		
0·54	0·55494	1·14	0·89308	1·74	0·98613		
0·56	0·57161	1·16	0·89910	1·76	0·98719		
0·58	0·58792	1·18	0·90484	1·78	0·98817	∞	1·00000

The abscissa ρ of the ordinate BC which cuts the area A in half is called the *probable* error, because in the long run half the shots have a greater error and the other half a less error.

Since p/c is the value of t corresponding to $\mathrm{P} = 0{\cdot}5$, and this value of P lies between 0·48465 and 0·50275, corresponding to the values 0·46 and 0·48 of t, it is found by calculation and approximation in this Table of P and t, that

(11) $$\frac{\rho}{c} = 0{\cdot}4769,$$

so that

(12) $$\frac{\rho}{e} = 0{\cdot}4769 \times \sqrt{\pi} = 0{\cdot}8453.$$

The line BM and the parallel symmetrical line AL cut out the middle half of the whole area 2A of the error curve, and thus enclose a zone which will catch 50°/。 of the shot; of the remainder, 25°/。 are beyond BM and 25°/。 beyond AL.

This zone is called the 50 % zone, and its breadth is

$$2 \times 0{\cdot}8453 = 1{\cdot}6906$$

times the mean error obtained by the analysis of all available practice.

To determine the % of hits to be expected in a zone bounded by any ordinate MP and its symmetrical ordinate M′P′, the ratio of the breadth MM′ of this zone to AB, the breadth of the 50 % zone, is calculated, and called the *probability factor*, and a *Table of Probability Factors* is calculated giving the % which the area MPP′M′ of the error curve bears to the whole area and the corresponding probability factor.

The area A in fig. 19 may be shown divided into ten equal areas by the 10, 20, 30, ... % ordinates, and the probability factor, as the abscissa of the ordinates dividing A into 100 equal parts, is given in the numerical table, the abscissa of the 50 % ordinate being taken as unity.

When the ordinates PM and P′M′ which limit the zone occupied by the target are not symmetrical with respect to the line of mean impact AO, the % of hits to be expected on each part AOMP and AOM′P′ must be calculated separately, and these % are added or subtracted according as PM and P′M′ are on opposite sides of AO or on the same side.

Thus if the number of hits on a zone bounded by PM and P′M′ is less than what should be expected, the inference is that the gun is not laid properly so as to bring the line of mean impact AO midway between PM and P′M′.

If the target fired at is limited by two dimensions, say length and breadth, or breadth and height, it is treated as the overlapping of two such unlimited zones, for which the separate % of hits is calculated, and the product of these gives the required percentage.

Modern range tables contain three columns, giving at each range the size of the 50 % zone for errors in range, direction, and vertical deviation; and now the probability factor enables us to calculate the % of hits to be expected on a zone of given depth or length in range, or of breadth in direction, or of given vertical height; thence we infer the number of shots required to make an assigned number of hits, and can decide whether the object is worth the ammunition to be expended; exaggerated stories of wonderful practice can also be discounted.

The theory of Probability is also useful in the design of match targets, and in comparing the results of competitive artillery practice carried out under different conditions.

In designing a vertical target for rifle shooting, the breadth and height may be taken as four times that of the 50 % zones, as more than 99 % of the shots should now be caught by the target, if the rifle is properly aimed.

The overlapping of the two 50 % zones will give a 25 % rectangle, appropriate for the *bulls-eye;* two 70·7 % zones will enclose a 50 % rectangle, which will serve as the boundary of the *centre;* while two 86·6 % zones will enclose a 75 % rectangle, appropriate for the *inner*, the space between this and the enveloping rectangle being the *outer*.

On a circular target the radius of the bulls-eye, centre, inner, and outer would be obtained by the revolution of the error curve round Ox, and determining the radius of the cylinder which cuts out 25 %, 50 %, 75 %, and 99 % of the total volume enclosed by the surface generated.

Then if r is the radius of the circle, with centre at the point of mean impact on the target, which catches 100 P °/₀ of the shots,

$$P = \frac{V(r)}{V(\infty)} = \int_0^r e^{-\frac{r^2}{c^2}} 2\pi r dr \Big/ \int_0^\infty e^{-\frac{r^2}{c^2}} 2\pi r dr$$

$$= 1 - e^{-\frac{r^2}{c^2}};$$

$$\frac{r}{c} = \sqrt{\left(\log_e \frac{1}{1-P}\right)}$$

$$\frac{r}{\rho} = \frac{c}{\rho} \sqrt{\left(\log_e \frac{1}{1-P}\right)}$$

and

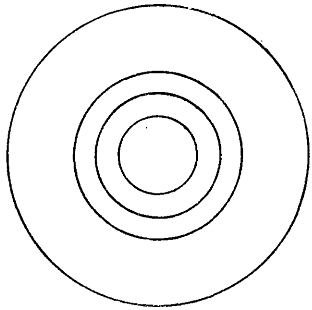
$$\frac{c}{\rho} = \frac{1}{0\cdot4769} = 2\cdot097,$$

so that the radii of the circles for the various °/₀ are easily calculated in accordance with the following scheme :—

P	0·25	0·5	0·75	0·99
$\frac{1}{1-P}$	$\frac{4}{3}$	2	4	100
$\log \frac{1}{1-P}$	0·1249	0·3010	0·6021	2·0000
$\log \log \frac{1}{1-P}$	$\bar{1}$·0965	$\bar{1}$·4786	$\bar{1}$·7797	0·3010
log M	$\bar{1}$·6378	$\bar{1}$·6378	$\bar{1}$·6378	$\bar{1}$·6378
$\log \log_e \frac{1}{1-P}$	$\bar{1}$·4587	$\bar{1}$·8408	0·1619	0·6632
$\log \sqrt{\log_e \frac{1}{1-P}}$	$\bar{1}$·7293	$\bar{1}$·9204	0·0809	0·3316
$\log \frac{c}{\rho}$	0·3216	0·3216	0·3216	0·3216
$\log \frac{r}{\rho}$	0·0509	0·2420	0·4025	0·6542
$\frac{r}{\rho}$	1·124	1·746	2·526	4·5

Thus with a rifle at 500 yards range the probable deviation ρ might be about 8 inches, thus making the radii of the 25, 50, 75, and 99 °/₀ circles about 9, 14, 20, and 36 inches, as shown in fig. 7, drawn to a scale of $\frac{1}{40}$. An expert marksman should bring the centre of impact very close to the centre of the target, and then 99 °/₀ of the shots should be on the target, and 25, 25, 25, and 24 °/₀ in each compartment.

FIG. 7.



If four marks are scored for a bullseye and 3, 2, 1 marks for the other compartments, the probable maximum score for 100 shots would be

$$24 \times 4 + 25 \times 3 + 25 \times 2 + 24 = 249.$$

The same thing will hold when the two errors, lateral and vertical, $\rho(x)$ and $\rho(y)$, are not equal; and now the circular curves must be replaced by similar ellipses.

As another application, determine the height of site above sea level of the 9-inch R.M.L. gun required to put it on even terms with a 6-inch B.L. gun firing at sea level, in a competition of firing over a range of 2,000 yards at a horizontal target on the water, the 50 °/$_0$ zone for errors in range of the 9-inch gun being 23 yards across, and the angle of descent in the range table 3° 45′, while the 50°/$_0$ zone of the 6-inch gun is only 18 yards across.

Let h denote the requisite height in feet and D the angular depression in minutes, so that

$$h = \frac{\text{DR}}{1146}, \text{ with K} = 2000.$$

If β denotes the range table angle of descent, the shot strikes the water at an angle $\beta + \text{D}$, and to catch 50 °/$_0$ of the shots in a length CC′ of 18 yards, which would stretch to a length cc' of 23 yards on the line of sight, we have

$$\frac{\sin(\beta + \text{D})}{\sin \beta} = \frac{cc'}{\text{CC}'} = \frac{23}{18},$$

or, as the angles β and D are small,

$$\frac{\beta + \text{D}}{\beta} = \frac{23}{18}, \ \frac{\text{D}}{\beta} = \frac{5}{18}$$

With $\beta = 225'$, $\text{D} = 62'\cdot 5$, and the requisite height of site is

$$h = 100 \text{ feet}.$$

CHAPTER VI.—THE STRENGTH OF GUNS.

WE resume here the detailed calculation of the stress set up in a gun by the pressure of the gunpowder, or by the shrinkage of the hoops, or by the tension from winding in a wire gun, in continuation of the sketch of the theory in Part I, Chapter V

Fig. 1, p. 125, is again taken to represent a typical state of stress in a cylinder, and now equation (3) on p. 125 can be written in the notation of the integral calculus.

$$(1) \qquad \int_{r_0}^{r_1} t\,dr = p_0 r_0 - p_1 r_1.$$

The upper limit r_1 may be replaced by r, the radius of any interior coaxial cylindrical surface; so that

$$(2) \qquad \int_{r_0}^{r} t\,dr = p_0 r_0 - pr;$$

and differentiating with respect to r,

$$(3) \qquad t = -\frac{d}{dr}(pr)$$

$$= -\frac{dp}{dr}r - p.$$

$$(4) \qquad t + p = -\frac{dp}{dr}r,$$

which, interpreted geometrically, shows that

$$(5) \qquad \mathrm{TP} = \mathrm{NV},$$

if V is the point where the tangent of the curve of radial pressure cuts ON.

This can be seen from elementary geometrical considerations, by supposing the outside radius r, and the inside radius r_0, to close in on the radius r, when the chord P_1P_0 becomes ultimately the tangent of the curve of radial pressure at P, while AB, which is equally inclined with P_0P_1 to ON, ultimately coincides with PL, thus making NL = NV in the limit.

Hence the curve of hoop tension can be drawn when the curve of radial pressure is assigned, and *vice versâ;* thus, for instance, if the hoop tension t is assumed constant, as in the wire gun, the curve of radial pressure P_1P_0 is a hyperbola, with LN and LT as asymptotes.

When the metal of the tube is homogeneous, the most general solution of equation (3) due to arbitrary internal and external pressures, p_0 and p_1 tons/in.^{2}, can be obtained by the combination in various proportions of two separate solutions, obtained by hypotheses due to Barlow and Rankine.

I. On Barlow's hypothesis the metal is squeezed radially as much as it is stretched circumferentially, so that

$$p = t;$$

as in a state of electric stress, in which the tension along the lines of forces and the pressure in all directions perpendicular to the lines of force are equal.

Then from equation (5),

$$\text{NV} = \text{TP} = 2\text{PR}, \tag{6}$$

which is the property of a curve in which

$$p = t = ar^{-2}, \tag{7}$$

where a is an arbitrary constant.

Or otherwise, putting $t = p$ in equation (4),

$$2p = -r\frac{dp}{dr},$$

$$\frac{dp}{p} + 2\frac{dr}{r} = 0; \tag{8}$$

and integrating,

$$\log p + 2 \log r = \log pr^2 \text{ is constant,}$$

or

$$pr^2 = a, \text{ a constant.} \tag{9}$$

Thus, if the radial pressure p and the circumferential tension t are equal, each of them is inversely proportional to the square of the radius, or distance from the axis of the tube.

Take fig. 1, p. 125, to represent this state of stress when C denotes the centre of the cross section of the tube and CM the trace of the diametral section, bisecting all such lines as Pt.

Now, if MT denotes the mean ordinate of the whole curve T_0T_1', when p_0 and p_1 are the applied internal and external pressures, connected by the relation

$$t_0 = p_0 = ar_0^{-2},\ t_1 = p_1 = ar_1^{-2}; \tag{10}$$

$$\text{MT} = \frac{p_0r_0 - p_1r_1}{r_1 - r_0} = a\frac{r_0^{-1} - r_1^{-1}}{r_1 - r_0} = \frac{a}{r_1r_0} = \sqrt{(t_0t_1)}, \tag{11}$$

so that the mean tension is now the G.M. (geometric mean) of the extreme tension t_0 and t_1, and it is the actual tension at a radius r, where

$$r^2 = r_0r_1, \tag{12}$$

or at a radius which is the G.M. of the internal and external radii.

This solution was first given by Mr. Peter Barlow, F.R.S., of the Royal Military Academy, when called upon to calculate the stresses in the metal of a hydraulic press in 1825; and the corresponding curves are called, after him, *Barlow curves.*

To construct the Barlow curves geometrically (as on p. 134) for given applied internal pressure p_0 and equal circumferential tension t_0, say to find the point T_1 on the curve T_0T_1 for circumferential tension, proceed as fig. 2, p. 254, draw T_0s_1 parallel to CM meeting the line through M_1 parallel to M_0P_0 in s_1; join Cs_1, cutting M_0T_0 in h_1, and draw h_1q_1 parallel to CM meeting M_1s_1 in q_1; again join Cq_1, cutting M_0T_0 in k_1, and draw k_1T_1 parallel to CM, cutting M_1s_1 in T_1; then T_1 shall be the required point on the Barlow curve.
For

$$(13)\qquad \frac{M_1T_1}{M_1Q_1} = \frac{CM_0}{CM_1}, \text{ and } \frac{M_1q_1}{M_0T_0} = \frac{CM_0}{CM_1};$$

and therefore

$$(14)\qquad \frac{M_1T_1}{M_0T_0} = \frac{CM_0^2}{CM_1^2} = \frac{CM_1^{-2}}{CM_0^{-2}},$$

the property of the Barlow curve.

Similarly, the point T corresponding to any other radius CM can be determined; and the curve P_0PP_1 for radial pressure, being an equal similar curve, is constructed in the same manner.

It will be noticed that q_1 lies on the hyperbola passing through T_0, and having CM and CV as asymptotes; hence the above method gives incidentally a geometrical method of describing a hyperbola through a given point, and having given asymptotes, as required in the theory of the wire gun.

Also M_1q_1 is the G.M. of M_0T_0 and M_1T_1, and therefore h_1q_1 cuts the Barlow curve T_0T_1 in a point T, corresponding to a radius CM, which is the G.M. of CM_0 and CM_1; M is found geometrically by describing a circle on CM_1 as diameter, cutting M_0T_0 in D, and drawing the circle DM with centre C; and now MT is the mean ordinate of the curve T_0T_1, such that the rectangle $M_0h_1q_1M_1$ is equal to the area $M_0T_0T_1M_1$; and N_0q_1 passes through the point of intersection of P_0M and P_1N_1.

It will be noticed in this Barlow state of stress that the radial pressure, although it diminishes rapidly towards the exterior, never actually vanishes, as is practically the case in a cylinder such as a boiler or a gun, in which the state of stress is due to an internal pressure.

To complete the solution another hypothesis was made by Rankine.

II. On Rankine's hypothesis the metal is squeezed uniformly by the application of equal internal and external pressures, such as would be the case if the tube was placed inside the water of a hydraulic press; a hydrostatic state of stress is now set up in the metal in which the circumferential stress becomes a pressure, equal to the radial pressure; or, algebraically, in which

$$t = -p, p = -t;$$

t now being negative, if estimated positively when it denotes a tension.

Equation (4) now becomes

$$(15)\qquad \frac{dp}{dr} = 0, \text{ or } p = b, \text{ a constant};$$

$$\text{and then } t = -b;$$

and now the average and the actual stress are the same at every point.

The most general state of stress can now be represented by a combination of Barlow's state I and Rankine's state II; if we suppose Rankine's stress is *removed* from Barlow's stress, then

$$p = ar^{-2} - b \tag{16}$$

$$t = ar^{-2} + b \tag{17}$$

equivalent to sliding the Barlow curves horizontally a distance b.

Then

$$t + p = 2ar^{-2}, \tag{18}$$

$$t - p = 2b \tag{19}$$

and the two arbitrary constants a and b are at our disposal to satisfy any two arbitrary conditions.

Suppose, for instance, that the internal and external pressures p_0 and p_1 are arbitrarily assigned; then a and b must be determined from the equations

$$p_0 = ar_0^{-2} - b,$$

$$p_1 = ar_1^{-2} - b; \tag{20}$$

so that

$$a = \frac{p_0 - p_1}{r_0^{-2} - r_1^{-2}} \qquad b = \frac{p_0 r_1^{-2} - p_1 r_0^{-2}}{r_0^{-2} - r_1^{-2}}; \tag{21}$$

and thus generally, in the interior of the metal,

$$p = \frac{p_0(r^{-2} - r_1^{-2}) + p_1(r_0^{-2} - r^{-2})}{r_0^{-2} - r_1^{-2}} \tag{22}$$

$$t = \frac{p_0(r^{-2} + r_1^{-2}) - p_1(r_0^{-2} + r^{-2})}{r_0^{-2} - r_1^{-2}} \tag{23}$$

Thus, if the exterior pressure p_1 is zero,

$$p = p_0 \frac{r^{-2} - r_1^{-2}}{r_0^{-2} - r_1^{-2}},$$

$$t = p_0 \frac{r^{-2} + r_1^{-2}}{r_0^{-2} + r_1^{-2}}; \tag{24}$$

so that if t_0 is fixed by the working tension of the metal

$$\frac{r_0^{-2} + r_1^{-2}}{r_0^{-2} - r_1^{-2}} = \frac{t_0}{p_0}.$$

$$\frac{r_1}{r_0} = \sqrt{\left(\frac{t_0 + p_0}{t_0 - p_0}\right)}, \tag{25}$$

thus determining the requisite thickness of the tube.

We see from this that no thickness is sufficient to stand an internal pressure p_0 greater than t_0, if the exterior of the tube is unsupported; but this drawback is overcome in modern ordnance by exterior reinforcing hoops, shrunk on to an assigned initial tension.

The quantities relating to the different hoops are distinguished by suffixes; thus

$$r_0, r_1, r_2, \dots r_n, \dots,$$

denote the radii in inches of the cylindrical surfaces of the hoops, r_0 denoting the internal radius of the A tube which is exposed to the powder pressure, and r_n denoting the common cylindrical surface, which is the exterior surface of the nth hoop and the interior surface of the $(n+1)$th.

Similarly, $$p_0, p_1, p_2, \dots p_n, \dots,$$

denote the radial pressures, in tons/in.², at these surfaces; but as there is a sudden change in the value of the circumferential tension in passing from one hoop to the next, due to the initial shrinkage, we use t_n to denote the circumferential tension, in tons/in.², in the inner fibres of the $(n+1)$th coil at its inner radius r_n, and t'_n to denote the circumferential tension in the outer fibres of the nth hoop at its outer radius r_n.

Considering the nth hoop,

$$t'_n - p_n = 2b,$$

(26) $$t_{n-1} - p_{n-1} = 2b;$$

so that, eliminating b,

(27) $$t'_n - p_n = t_{n-1} - p_{n-1}.$$

Also $$p_{n-1} - p_n = \frac{a}{r_{n-1}^2} - \frac{a}{r_n^2},$$

(28) $$t_{n-1} + p_n = \frac{a}{r_{n-1}^2} + \frac{a}{r_n^2};$$

and eliminating a,

$$\frac{p_{n-1} - p_n}{t_{n-1} + p_n} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2}$$

(29) $$p_{n-1} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2}(t_{n-1} + p_n) + p_n,$$

the gunmaker's formula employed in the design of built up ordnance, given already in equation (17), p. 127.

For if r_n denotes the external radius of the gun, and if

$$t_{n-1}, t_{n-2}, \dots$$

denote the given maximum allowable tensions in the material of the hoops, then starting from the exterior, where $p_n = 0$,

(30) $$p_{n-1} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_n^{-2}} t_{n-1}$$

(31) $$p_{n-2} = \frac{r_{n-1}^2 - r_{n-2}^2}{r_{n-1}^2 + r_{n-2}^2}(t_{n-2} + p_{n-1}) + p_{n-1}$$

....

and finally

(32) $$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1;$$

whence $p_{n-1}, p_{n-2}, \dots$ can be calculated, and finally p_0, the maximum pressure allowable in the bore.

Or, conversely, supposing p_0 is given, then working the equations backwards we determine $p_1, p_2, \dots, p_{n-1}$, when $t_0, t_1, t_2, \dots, t_{n-1}$, and

$r_0, r_1, r_2, \ldots r_{n-1}$ are given; and then equation (32) determines r_n, the external radius of the outside jacket.

In gun construction, t_{n-1}, t_{n-2}, ... t_2, t_1 are generally taken at 18 tons/in.^{2}, but t_0 is taken at 15 tons/in.^{2}, to allow for erosion.

As the numerical calculations are laborious, the following geometrical construction may be substituted.

When p_0 and t_0 are given for a tube or hoop of internal and external radius r_0 and r_1, represented by the ordinates R_0P_0 and R_0T_0 at the radius Or_0 in fig. 1, p. 125, we need only bisect P_0T_0 in M_0 and draw CM_0 parallel to OR_0 to obtain the axis of the Barlow curves; and now the determination of P_1 and T_1 is effected as on p. 135.

If it is required, as on p. 254, to determine the external radius r' of this tube where the radial pressure p' is zero, we have to determine the point R′ when the curve of radial pressure P_3P_1 cuts the line Ox_0.

Take M_0B the G.M. of M_0P_0 and M_0R_0, and produce CB to meet N_0P_0 produced in A; the AR_1 drawn parallel to ON_0 will cut OR_0 produced in the required point R_1; we thus obtain a geometrical construction for the requisite thickness of a tube of calibre $2r_0$, composed of metal with given tenacity t_0, required to carry a given pressure p_0.

This is the problem required in the determination of the thickness $r_1 - r_0$ of steel, necessary in the chase portion of a light field or quick-firing gun in order to stand a pressure P_0 without straining the metal at any part of the surface of the bore beyond a certain working tension $T_0 = 15$ tons/in^2.

The figures 1 and 2 on p. 254 annexed show the application to a 3-inch field gun.

Let $r_0 = 1{\cdot}5$ inches, and let the gaseous pressures at O, O′ and O‴ to be expected from the propellant used, be 4·4, 4 and 3·6 tons/in.^{2} respectively as show by the upper curve; then considering first the point 0, measure off in fig. 2, $r_0T_0 = 15$, and for safety take P_0 equal to double the pressure expected, making $r_0P_0 = 8{\cdot}8$; draw Cm_0 passing through m_0, the middle point of P_0T_0, and take CB_0 the geometric mean of CO and Cn_0, or m_0B, the geometric mean of m_0r_0 and m_0P_0, so that B_0B is parallel to Or; then CB produced will meet n_0P_0 produced in a point A such that Ar, drawn parallel to OC, will cut off the required outside radius $Or_1 (= 2{\cdot}94$ ins.); this follows from the preceding theory.

Making successively $r_0P_0' = 8$ and $r_0P_0'' = 7{\cdot}2$, while T_0 always remains the same, viz., 15, will give new centres C′ and C″, and proceeding by similar construction, lines drawn horizontally through the new points A′, A″ obtained on n_0P_0 produced, will cut off the required radii

$$Or_2 = 2{\cdot}7, \text{ and } Or_2'' = 2{\cdot}5 \text{ ins.}$$

Thus the outside diameters d_1, d'_1, d_1'' in the figure should be 5·9, 5·4 and 5 inches respectively, at the points O, O′, and O″.

But if p_1 and t_0 are given, and we have to determine p_0 from equation (29) by means of a geometrical construction, take a third proportional x to r_1 and r_0, represented by Od in fig. A, p. 264; then from equation (29),

$$(33) \qquad \frac{p_0 - p_1}{t_0 + p_1} = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2} = \frac{r_1^2 - xr_1}{r_1^2 + xr_1} = \frac{r_1 - x}{r_1 + x},$$

or, as represented in Fig. A,

$$(34) \qquad \frac{A_1P_1}{N_1L_0} = \frac{r_1d}{dR_1}.$$

Hence the point P_0 is determined by drawing P_1N_1 parallel to Or to meet dD, perpendicular to Or, in D, and producing S′D to meet r_1P_1 produced in A_1; then A_1N_0, parallel to Or, will cut off the length r_0P_0.

If the line A_1S_1 cuts ON_1 in C_0, then C_0 will be the centre, and C_0m_0 the axis of the Barlow curves P_0P_1 and T_0T_1'; for now

$$(35) \qquad \frac{m_0P_0}{fD} = \frac{Or'}{Od} = \frac{Or_1^2}{Or_0^2}.$$

So also if the internal and external pressures, p_0 and p_1, are given, represented in fig. 1, p. 125, by the ordinates R_0P_0 and $R_1 P_1$; draw the diagonal AB of the rectangle AP_0 BP_1 to meet ON in L; then OL or RT represents the average tension of the circumferential fibres.

If TL meets r_0I, parallel to OL, in I, then AI will cut OL in C, the centre of the Barlow curves P_0P_1 and T_0T_1; and now these curves can be constructed geometrically.

For
$$\frac{CL}{CN_0} = \frac{r_0}{r_1} = \frac{r_0^2}{r_0r_1},$$

and, from equation (13),

$$CL = \frac{a}{r_0r_1}, \text{ so that } CN_0 = \frac{a}{r_0^2},\ CN_1 = \frac{a}{r_1^2}.$$

A successive application of these geometrical processes, as shown in fig. A, p. 264, will determine the axes of the Barlow curves, and thence all the stresses in the successive hoops of the gun, and determine for given working tenacities t_0, t_1, t_2, ... t_n, ... either the maximum allowable interior pressure p_0 for given radii r_1, r_2, ... r_n, ..., of the hoops; or the outside radius of the external jacket when the interior pressure p_0 is assigned.

The stress thus determined is called the *firing stress* of the gun; and to ensure the proper distribution of the firing stress, the hoops are shrunk on in the process of manufacture so as to set up an appropriate state of stress, called *initial stress* or *stress of repose*, such that the addition of the stress due to the application of the internal powder pressure p_0, called the *powder stress*, produces the *firing stress.*

It is assumed that the powder stress is that which would be produced in a homogeneous tube of the same bore and external diameter as the gun, by an internal pressure p_0; and this powder stress is, therefore, easily calculated or constructed geometrically by the preceding methods, as exhibited in fig. B, p. 264.

Deducting the *powder stress* from the *firing stress*, we are left with the *initial stress* of the gun in repose, which is the stress to be imparted in manufacture by the shrinkage of the hoops.

Figs. (3A), (3B), (3C), Chap. V, Part I, shows the *firing stress*, the *powder stress*, and the *initial stress* in a section across the powder chamber of a 6-inch gun, due to a pressure of 24·7 tons/in.², the working tenacities of the steel being limited to 18 tons/in.² in the hoops and jackets, and to 15 tons-in². in the tube; taking, in inches,

$$r_0 = 4,\ r_1 = 5{\cdot}6,\ r_2 = 8{\cdot}7,\ r_3 = 11{\cdot}8.$$

The initial stress and strain set up in the manufacture of the gun by shrinking on the coils (p. 136).

If r_n denotes the exterior radius of the nth hoop and the interior radius of the $(n + 1)$th hoop in the completed gun, then in the manu-

FIG. 1.

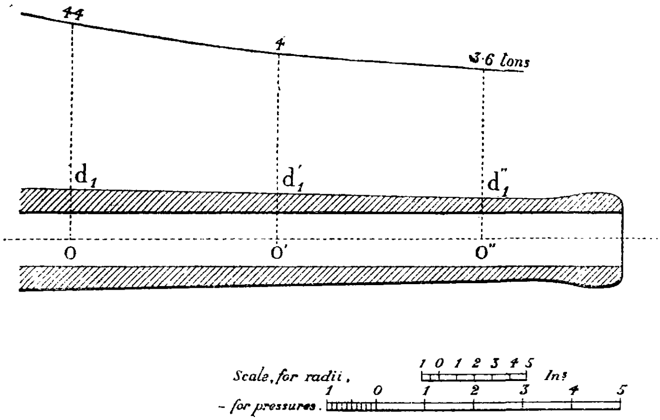
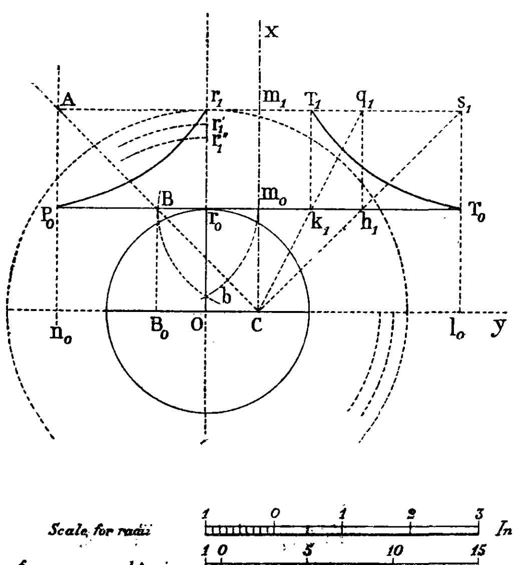

4·4
4
3·6 tons
d_1
d_1'
d_1''
O
O'
O''
Scale, for radii.
1 0 1 2 3 4 5
Ins
– for pressures.
1 0 1 2 3 4 5

Fig. 2


x
A
r_1
m_1
T_1
q_1
s_1
r_1'
r_1''
B
m_0
P_0
r_0
k_1
h_1
T_0
b
n_0
B_0
O
C
l_0
y
Scale, for radii
1 0 1 2 3
Ins
– for pressures and tensions.
1 0 5 10 15

facture these cylindrical surfaces are turned to different radii; we denote these radii by

$$r_n + u'_n \text{ and } r_n - u_n,$$

so that, before assemblage of the parts, there would be an overlap of thickness $u_n + u'_n$.

But the outer hoop can be expanded by heat so that its internal radius exceeds $r_n + u'_n$; and now it can be slipped over the inner hoop; and on cooling a pressure is set up between the surfaces in contact, producing an initial state of stress.

The difference $2(u_n + u'_n)$ of the diameters of the surfaces before assemblage is called the *shrinkage*, and denoted by ${}_nS_{n+1}$; and to determine the appropriate shrinkage to set up at a given state of initial stress, it is necessary to make a digression on the relation between the *stresses* and accompanying *strains* in the interior of an elastic body; in particular for a homogeneous cylindrical tube, due to given applied internal and external pressures.

The reader is referred to Thomson and Tait's "Natural Philosophy," §§ 682, 683 ...., for a complete treatment; the parts bearing on the question of gun construction may be presented as follows ·

When a piece of metal is pulled, as for instance a test piece of steel in a testing machine, it is found that the *extension*, measured by the ratio of the *elongation* to the original length, is proportional to the *tension*, which we shall measure in tons per square inch of cross section.

Thus doubling the tension doubles the extension; and so on in proportion, provided the elastic limit is not exceeded.

This experimental law is called "Hooke's Law," and it is the axiomatic foundation of the Mathematical Theory of Elasticity. Expressed in an algebraical form, if a pull of P tons in a bar, K in.² in cross section, stretches the length from L to L + l, then the tension $\frac{P}{K}$ tons/in.², and the extension $\frac{l}{L}$, are, by Hooke's law, connected by the relation

$$(36) \qquad \frac{\frac{P}{K}}{\frac{l}{L}} = M, \text{ a constant,}$$

where M denotes a number of tons/in.², called Young's modulus of elasticity of the material; thus for steel we may put (p. 6)

$$(37) \qquad M = 12{,}500 \text{ tons/in.}^2$$

In this case the metal is subject to a single tension, and a certain amount of lateral contraction takes place; but now consider the strains which take place in a small brick shaped portion of metal, of which the length, breadth, and height, are denoted by x, y, z, due to tensions P, Q, R tons/in.², acting parallel to the edges, across the faces.

The metal will be strained into a slightly enlarged brick shape, of which the lengths of the edges are

(38) $$x(1 + e),\ y(1 + f),\ z(1 + g),$$

suppose; so that the *extension* of the edges is represented by the numbers

$$e, f, g.$$

In consequence of Hooke's law and the homogeneity of the material, we shall have

(39) $$P = Ae + Bf + Bg,$$

(40) $$Q = Be + Af + Bg,$$

(41) $$R = Be + Bf + Ag,$$

where A and B are two constants depending on the elasticity of the material.

By solution of these equations,

$$P - \frac{B}{A + B}(Q + R) = \left(A - \frac{2B^2}{A + B}\right) e,$$

or

(42) $$P - \sigma(Q + R) = Me,$$

(43) $$Q - \sigma(R + P) = Mf,$$

(44) $$R - \sigma(P + Q) = Mg.$$

where

(45) $$M = A - \frac{2B^2}{A + B},$$

(46) $$\sigma = \frac{B}{A + B}.$$

For a simple tension, $Q = 0$, $R = 0$, and then

(47) $$P = Me,$$

so that M is, as before, Young's modulus of elasticity of the substance as determined in the testing machine; and then

(48) $$f = g = -\sigma e,$$

so that σ, called *Poisson's ratio*, is the ratio of the lateral contraction to the linear extension of the test piece, under simple tension.

But if lateral contraction is prevented by appropriate lateral tension, so that $f = 0$, $g = 0$, and the strain is a *pure extension* e, then

$$P = Ae,$$

(49) $$Q = R = Be,$$

and the modulus of elasticity P/e now appears as A; and

(50) $$\frac{M}{A} = 1 - \frac{2B^2}{A^2 + AB}.$$

For steel we find that $\sigma = \frac{1}{4}$, so that A = 3B, and

(51) $$\frac{M}{A} = \frac{5}{6}.$$

and now

(52) $$P = A(e + \tfrac{1}{3}f + \tfrac{1}{3}g),$$
$$Q = A(\tfrac{1}{3}e + f + \tfrac{1}{3}g),$$
$$R = A(\tfrac{1}{3}e + \tfrac{1}{3}f + g),$$

(53) $$Me = P - \tfrac{1}{4}(Q + R),$$
$$Mf = Q - \tfrac{1}{4}(R + P),$$
$$Mg = R - \tfrac{1}{4}(P + Q).$$

Consider now the stresses and strains in a small brick-shaped piece of metal cut out from the material of a hoop—

(i) by two adjacent concentric cylinders of radii r and $r + dr$;
(ii) by two consecutive radial planes;
(iii) by two consecutive transverse plane cross sections.

Then taking x, y, z in the circumferential, radial, and longitudinal directions, we put

(54) $$P = \quad t = \quad ar^{-2} + b,$$
$$Q = -p = -ar^{-2} + b,$$

leaving R undetermined.

But it is usually assumed that R is constant, the constant value of R being taken as equal to the total longitudinal thrust $p_0\, \pi r_0^2$ tons of the interior pressure p_0 tons/in.^{2}, divided by the area in inches of the cross section of the material of the gun; or, in considering the *initial stresses* in a state of repose, we may put R equal to zero.

For the determinations of the corresponding strains, denote by u the increase of the radius r of the circumferential fibre; then the fibre is stretched from a length $2\pi r$ to a length of $2\pi(r + u)$, so that the circumferential extension

(55) $$e = \frac{2\pi u}{2\pi r} = \frac{u}{r}.$$

The radial extension

(56) $$f = \frac{du}{dr};$$

while the longitudinal extension g is left undetermined at present.

But since $P + Q = 2b$, a constant, the third equation of (53) shows that $Mg - R$ is constant, so that g may also be taken as constant.

Then

(57) $$Me = M\frac{u}{r} = P - \sigma(Q + R),$$

$$Mu = (P - \sigma Q)\, r - \sigma R r$$

In the outer circumferential fibre of the nth hoop the radius has been diminished from $r_n + u_n'$ to r_n, so that, as it is immaterial whether we take r to refer to the unstrained or the strained radius of the fibre, we may put

$$r = r_n,\ u = -u'_n,$$

in equation (57), and at the same time put

$$P = t_n',\ Q = -p_n;$$

so that

(58) $$-Mu_n' = (t_n' + \sigma p_n)r_n - \sigma R r_n.$$

Again, in the adjacent inner circumferential fibre of the $(n + 1)$th hoop, the radius has been increased from

$$r_n - u_n \text{ to } r_n,$$

we put $r = r_n, u = u_n$, $P = t_n$, $Q = -p_n$ in equation (57); and thus

(59) $$Mu_n = (t_n + \sigma p_n)r_n - \sigma R r_n;$$

so that, subtracting,

(60) $$M(u_n + u_n') = (t_n - t_n')r_n.$$

Therefore, denoting the shrinkage between the nth and $(n + 1)$th coils by ${}_nS_{n+1}$, we have

$${}_nS_{n+1} = 2(u_n + u_n')$$

(61) $$= (t_n - t_n')\frac{2r_n}{M};$$

so that the shrinkage is the elongation produced in a bar of the metal $2r_0$ inches long, due to a tension of $t_n - t_n'$ tons-in.^{2}

Also from equation (30)

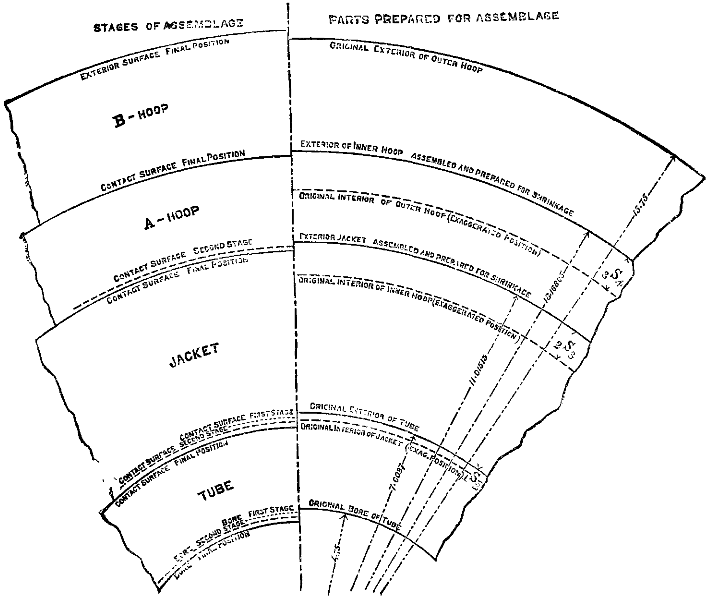
$$t_n - t_n' = (t_n - t_{n-1}) - (p_n - p_{n-1})$$

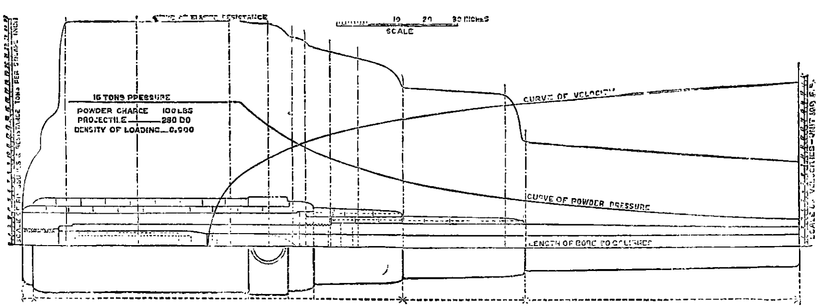
(62) $$= t_n - t_{n-1} + \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2}(t_{n-1} + p_n).$$

Considering that the curve of circumferential tension is continuous for the powder stresses, the addition or subtraction of the powder stresses does not alter the difference $t_n - t_n'$; so that the shrinkage ${}_nS_{n+1}$ can be calculated from the diagram and values either of the firing stresses or of the initial stresses; and it is independent of the shrinkage imparted at other surfaces of contact of the coils, provided it is calculated as the shrinkage of the parts before assemblage.

If, however, the shrinkage is estimated for the difference between the internal diameter of a coil and the external diameter of the finished portion of the gun, then the initial stresses already set up in the gun must be taken into accouut and deducted.

This is illustrated in diagrams in the American "Notes on the Construction of Ordnance," Nos. 31, 33, 35 by Lieutenant Rogers Birnie, showing the shrinkage (enlarged fifty times) of the different finished parts, and the intermediate states during assemblage, and the final state, when a jacket and two hoops are shrunk over the A tube of an 8-inch gun, shown in longitudinal section in the annexed figure.





With these dimensions we find with $t_0 = 15$, $t_1 = t_2 = t_3 = 18$,

$$r_0 = 5,\ p_0 = 28{\cdot}7,$$
$$r_1 = 7,\ t_1 - t_1' = 12{\cdot}7,$$
$$r_2 = 11,\ t_2 - t_2' = 10{\cdot}7,$$
$$r_3 = 13,\ t_3 - t_3' = 3{\cdot}6\,;$$

so that, with M = 12,500 tons/in^2.,

$$_1S_2 = 12{\cdot}7 \times 14 \div 12{,}500 = 0{\cdot}014,$$
$$_2S_3 = 10{\cdot}7 \times 22 \div 12{,}500 = 0{\cdot}019,$$
$$_3S_4 = 3{\cdot}6 \times 26 \div 12{,}500 = 0{\cdot}007\,;$$

or 14, 19, and 7 thousandths of an inch.

In a state of repose the tension of the outer fibres of the outside hoop is 8·1 tons/in.², and the circumferential pressure in the interior of the bore is 19·9 tons-in.²; so that, with $r_0 = 5$, $r_5 = 16$,

$${}_0S_1 = 19{\cdot}9 \times 10 \div 12{,}500 = 0{\cdot}016,$$

or the contraction of the calibre is 16 thousandths of an inch, in consequence of the shrinkage; while

$${}_4S_5 = 8{\cdot}1 \times 32 \div 12{,}500 = 0{\cdot}021,$$

or the elongation of the external diameter due to the shrinkage is 21 thousandths of an inch.

To lay off the shrinkages geometrically in fig. 2, p. 254, mark off a length T_2M on T_2P_2 to represent to scale a tension of 12·5 tons/in.², one-thousandth of the modulus of elasticity M; join MS_2, and draw $T_2'S_2'$ parallel to MS_2 to meet S_2T_2 in S_2'; then T_2S_2' will represent in inches the thousandths of inches in the shrinkage ${}_2S_3$; and similarly for the other shrinkages. Thus, in fig. A, T_1S_1' represents the shrinkage between the A tube and the jacket, enlarged 1,000 times.

The coefficient of expansion of steel per 1° F. is about 1 ÷ 150,000; so that if ${}_nS_{n+1}$ denotes the shrinkage during manufacture, the temperature must be raised

$$150{,}000 \frac{{}_nS_{n+1}}{2r_n}$$

degrees Fahrenheit for the $(n + 1)$th coil to be expanded sufficiently so as to slip over the nth coil; and this rise of temperature can also be represented geometrically in a similar manner.

Wire Gun Construction.

An inspection of the firing stresses in fig. 3A, p. 128, and of the serrated edge of the curve of circumferential tension, shows that the inner fibre only of each coil is doing its full share of resistance when the gun is fired, the lost resistance of the breech-piece being represented by the area $T_1T_2T'_2$.

Great economy of material can be effected if we can make all the circumferential fibres take up a full uniform working tension (say of 18 tons/in.²) when the gun is fired; but to secure this condition only approximately, the number of coils must be largely increased, and the cost, complication and time of manufacture of a gun would be enormous.

But by adapting Mr. J A Longridge's plan of strengthening the inner tube A by steel wire, wound on with appropriately varying tension, we are able theoretically to make the circumferential firing tension t uniform, or the curve T_1T_2 a straight line; and now all parts of the wire coil are equally strained, and take an equal share in the resistance.

The subject has been investigated theoretically by Mr. Longridge, assisted by Mr. C. H. Brooks, beginning in 1855; and his theories are set forth in the "Proceedings of the Institution of Civil Engineers," 1860, 1879, 1884; and in a "Treatise on the Application of Wire to the Construction of Ordnance," 1884; and in "Further Investigations regarding Wire Gun Construction," 1887.

Besides Mr. Longridge's treatises, the most important is a long article in the "Revue d'Artillerie" on "Steel Wire Guns," by Lieutenant G. Moch, since published as a separate book, "Les Canons à fil d'acier," and also translated in the American "Notes on the Construction of Ordnance," No. 48, 1888.

Let fig. C, p. 265, represent the firing stresses in a wire gun, composed of an inner tube A, the wire coil B, and an outer jacket C.

The jacket C is merely required for the protection of the wire, so that it may be supposed fitted over the wire without any appreciable shrinkage; when the gun is at rest, the jacket will then be in a state of repose, free from stress; but when the gun is fired, we may suppose the stress in C to be the powder stress, on the usual assumption that the gun when fired behaves as if homogeneous.

In the wire coil B the firing stress is represented by a given uniform circumferential tension t, and by the radial pressure p, which will be represented by the ordinates of a hyperbola.

For if the straight line $T_2'TT_1$ parallel to Or, representing the uniform circumferential tension t of the wire, meets NO in O_1, then the condition of equilibrium of the section r_2r of the wire coil is expressed by equation (3) as

(63) the rectangle OP $-$ rectangle OP_2

$=$ rectangle $rTT_2'r_2$,

or the rectangle O_1P

$=$ rectangle O_1P_2

which proves that P lies on the hyperbola, having O_1O, O_1T as asymptotes, and starting from the point P_2, where the curve of radial pressure of the powder stresses cuts r_2P_2.

To find the point P_1 where this hyperbola of radial pressure cuts T_1r_1, draw O_1A_2 through the point of intersection of T_1r_1 and P_2N_2 to meet r_2P_2 produced in A_2; then the line through A_2 parallel to OR will cut T_1r_1 in P_1.

The point P_0 is known from the given powder pressure p_0, and P_1P_0 must be joined by a Barlow curve to represent the radial pressure at any point of the tube; the centre, C_1 of this Barlow curve will be determined by drawing A_1B_0, the other diagonal of the rectangle P_1P_0, to meet ON in I, drawing IL at right angles to ON of length Or_0, and joining A_1L; this will cut ON in C_1; and now the circumferential firing stresses of the tube can be laid off on the diagram.

It will be noticed in the diagram that these circumferential stresses are pressures, showing that the tube is slightly compressed even when the gun is fired; this property is utilised in the Brown segmental wire gun, in which the inner tube is constructed in segments.

The curves of the powder stresses are the Barlow curves $P_0P_2r_3$ and $T_0T_2T_3$, of which the centre C can be found in the manner already explained, and the curves thence drawn by geometrical construction; and, stripping off these powder stresses, the outside jacket is left unstrained, and the wire coil and the tube have the state of initial stress which it is requisite to give the gun during the process of manufacture, as shown in fig. D, p. 265.

The curve of initial circumferential tension in the wire is obtained by subtracting the ordinates of the Barlow curve T_2T_1'' from the uniform ordinates of the straight line $T_2'T_1$; thence we obtain the symmetrical Barlow curve $\phi_2\phi_1$ of initial circumferential tension, the reflexion of the curve T_2T_1'' in the straight line bisecting MT at right angles.

The curve of initial radial pressure is obtained by subtracting the ordinates of a Barlow curve from the ordinates of a hyperbola; this curve is thus easily plotted, although of a more complicated analytical nature.

Finally we come to the state of initial stress of repose in the tube obtained by deducting the powder stresses from the firing stresses; these will be represented by the Barlow curves $\varpi_1 r_0$ for radial pressure, and $\tau_1'\tau_0$ for circumferential pressure; and these Barlow curves for given ϖ_1 are constructed in the manner already explained.

It will be noticed that τ_0 is considerable, and, with imperfect design, may become dangerously near the crushing pressure of the steel of the tube; practically, however, the great initial pressure τ_0 at the interior of the tube is considered advantageous, as tending to improve the resisting power of the material against the erosion of the bore.

In the Severn tunnel, as another exemplification of these principles, the head of the water of the adjacent land springs, if not kept down by pumping, is sufficient to crush the bricks on the interior of the tunnel.

We have still to determine the varying tension at which the wire must be wound on, so that in the finished gun the curve of initial circumferential tension may assume the requisite form $\phi_2\phi_1$.

Calling this the *winding* tension of the wire, and denoting it by θ, we assume that this winding tension θ is equal to the finished initial tension ϕ, increased by the circumferential stress due to the initial radial pressure ϖ at the radius r, acting on the partly finished tube and coil between the radii r_9 and r; and thus, from equation (34),

$$(64) \qquad \theta = \tau + \phi \frac{r^2 + r_0^2}{r^2 - r_0^2}$$

In other words, it is assumed that the tension of repose, τ, is less than the winding tension, θ, by the amount due to the radial pressure ϕ at a radius r, and zero pressure at the radius r_0 in a homogeneous tube.

Now, from equations (24), (25),

$$(65) \qquad \phi = t - p_0 \frac{r^{-2} + r_3^{-2}}{r_0^{-2} - r_3^{-2}}$$

$$(66) \qquad \phi = p - p_0 \frac{r^{-2} - r_3^{-2}}{r_0^{-2} - r_3^{-2}},$$

and

$$(67) \qquad t + p = (t + p_2)\frac{r_2}{r},$$

so that

$$\theta = t - p_0 \frac{r_0^2}{r^2}\frac{r_3^2 + r^2}{r_3^2 - r_0^2}$$

$$(68) \qquad + \left(t\frac{r_2 - r}{r} + p_2\frac{r_2}{r} - p_0\frac{r_0^2}{r^2}\frac{r_3^2 - r^2}{r_3^2 - r_0^2}\right)\frac{r^2 + r_0^2}{r^2 - r_0^2}$$

and, after algebraical reduction, this can be expressed in the form

$$(69) \qquad \theta = \frac{L}{r} + \frac{M}{r - r_9} + \frac{N}{r + r_0},$$

where

$$(70) \qquad L = -(t + p_2)r_2,$$

(71) $$M = t(r_2 - r_0) - p_0 r_0 + p_2 r_2$$
$$= (t + p_2) r_2 - (t + p_0) r_0,$$

(72) $$N = t(r_2 + r_0) + p_0 r_0 + p_2 r_2$$
$$= (t + p_2) r_2 + (t + p_0) r_0.$$

The curve representing θ can thus be plotted out by ordinates equal to the sum of the three ordinates

(73) $$\frac{L}{r}, \quad \frac{M}{r - r_0}, \quad \frac{N}{r + r_0},$$

of corresponding hyperbolas; the first of these three being the hyperbola of firing radial pressure in the wire.

Putting $r = r_2$ gives $\theta_2 = \phi_2$, as is obviously the case, since the winding tension of the last layer of wire is the same as the tension in repose.

Having plotted out the curve $\theta_1\theta_2$ for the winding tension of the wire, it is found near enough for practical purposes to replace this curve by the straight line $\theta_1\theta_2$; and now, in winding the coil, the difference of the weights which give the winding tension for two consecutive layers may be taken as constant.

The theory of the longitudinal strength of the wire gun has not been touched upon, because it is still a point in dispute as to whether the tube alone should be strong enough to provide the whole longitudinal strength, or whether the outside jacket should be fitted so as to take part of the longitudinal tension.

For an experimental verification of the above theory of the wire gun, the reader may consult "Notes on the Construction of Ordnance, No. 38," on Winding and Dismantling an Experimental Wire-wound Gun Cylinder, by Lieutenant N. Crozier, 1886.

To conclude with a general exercise on the preceding principles, we take figs. A and B to represent sections across about the centre of the cartridge chamber of a 4·7-inch Q.F. steel gun (Mark III); this gun being selected for illustration of the geometrical method of finding the stresses in order to make a comparison with those set up by the same chamber pressure in a wire gun of the same calibre and weights, shown in section in figs. C and D.

The diameter of the chamber at the point chosen is 5 inches, the thickness of metal in the A-tube being 1·6 and in the jacket 3·4 inches give $r_0 = 2·5$, $r_2 = 4·1$, and $r_1 = 7·5$ inches. Similarly to the case of fig. 2, Part I, we suppose that it is required to calculate the maximum pressure in the chamber that will exactly strain the interior surfaces of the A-tube and jacket to 15 and 18 tons/in.^{2} respectively.

Having set off the radii Or_0, Or_1 and Or_2, also $\mathrm{r}_0\mathrm{T}_0$ and $\mathrm{r}_1\mathrm{T}_1 = 15$ and 18 respectively on the scale for stresses, from T_1 draw $\mathrm{T}_1\mathrm{S}_2$ parallel to Or, then making Od_1 a third proportional to Or_2 and Or_1, in the manner explained above, $\mathrm{S}_2\mathrm{d}_1$ produced will meet $\mathrm{T}_2\mathrm{r}_2$ produced in A_2, such that $\mathrm{A}_2\mathrm{P}_1$ parallel to Or will cut off $\mathrm{r}_1\mathrm{P}_1$ representing P_1 (= 9·72 tons/in.^{2}) the missing lines in fig. A must be supplied. Halving $\mathrm{P}_1\mathrm{T}_1$ gives the point m_1, a vertical drawn through which gives C_1, on the horizontal line through O and m_2 on a horizontal through r_2; doubling $\mathrm{r}_2\mathrm{m}_2$ gives T'_2 (= 8·28 tons/in.^{2}). C_1 is the centre required for completing the Barlow curves $\mathrm{T}_1\mathrm{T}_2$ for hoop tension, and $\mathrm{P}_1\mathrm{r}_2$ for radial pressure by the method explained above; thus at any radius $r = 5·5$ inches, the construction shown by dotted lines gives $\mathrm{P} = 3·5$, and $\mathrm{T} = 11·8$ tons/in.^{2}.

FIG. A.

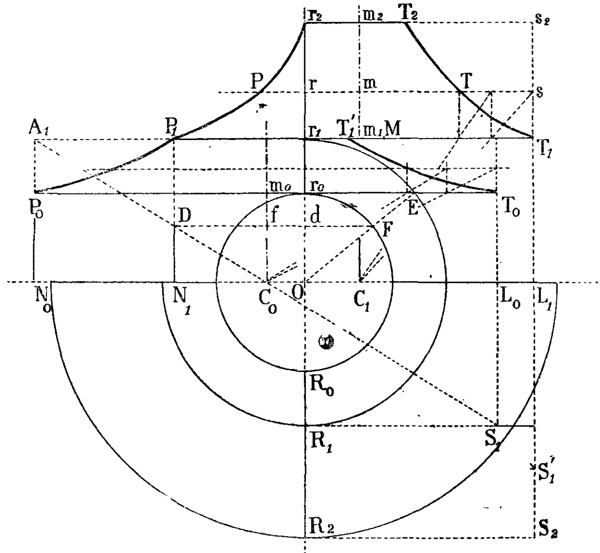

r2
m2
T2
s2
P
r
m
T
s
A1
P1
r1
T1'
m1M
T1
m0
r0
P0
T0
D
f
d
E
F
N0
N1
C0
O
C1
L0
L1
R0
R1
S1
S1'
R2
S2

FIG. B.

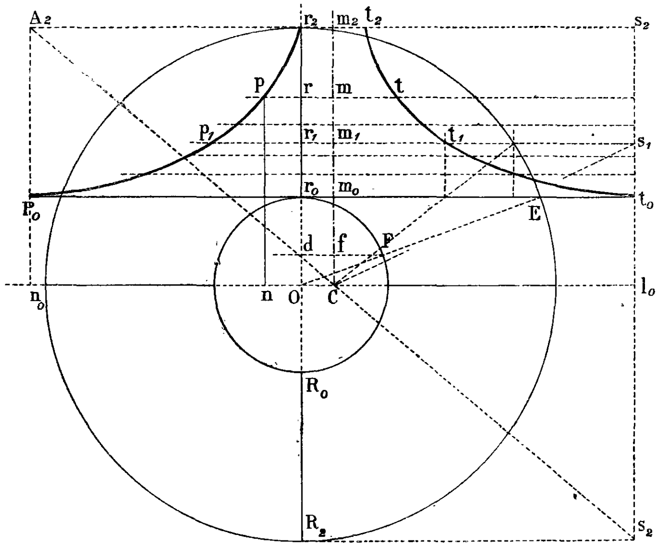
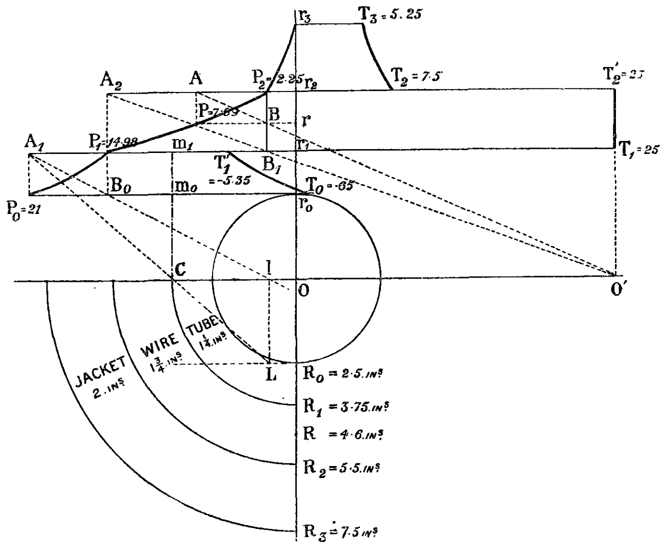

A2
r2
m2
t2
s2
p
r
m
t
p1
r1
m1
t1
s1
r0
m0
P0
t0
E
d
f
F
n0
n
O
C
l0
R0
R2
s2

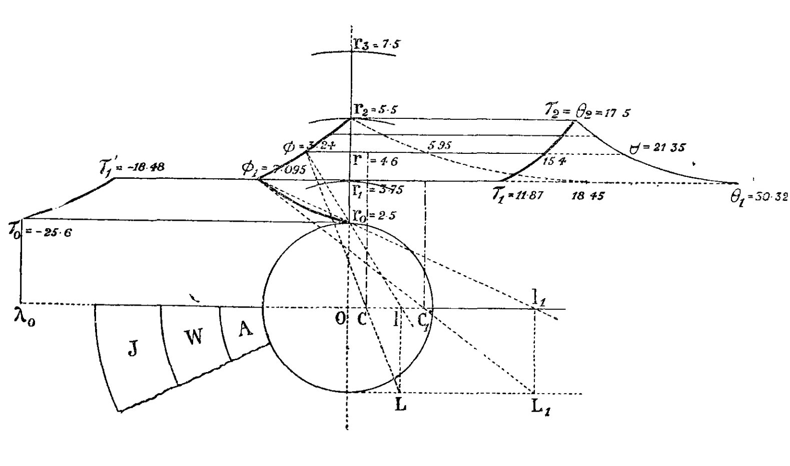
FIG. C.



4·7-inch Wire Gun. Section across Powder Chamber.

Suppose the data to be that—Under a chamber pressure of 21 tons/in.² the wire-coil is to be strained uniformly to 25 tons/in.²

FIG. D.



Next, in the A-tube, join EO cutting the chamber circumference in F; from T_0 draw T_0S_1 parallel to Or, and a horizontal through F, cutting Or, in d; from P_1 draw a vertical line cutting Od produced, in D; join S_1D and produce it to meet r_1 P_1 produced in A_1, a vertical from A_1 will now meet T_0r_0 produced, in P_0, which is the maximum pressure required to be found; measurement of r_0P_0 on the stress scale gives $P_0 = 21$ tons/in.².

Halving P_0T_0 gives m_0 and a vertical through this point gives C_0, the centre for constructing the stress curves in the tube; it is observed that C_0 is also given by the line S_1D which intersects the horizontal through O in O_0.

We thus have,

at $r_1 = 4{\cdot}1$ ins	$P_1 = 9{\cdot}72$	and	$T_1' = 3{\cdot}72$	tons/in.².
$r = 3{\cdot}2$,,	$P = 14$	,,	$T = 8$	,,
$r_0 = 2{\cdot}5$,,	$P = 21$	,,	$T_0 = 15$	,,

In fig. B, showing the powder stresses, we suppose the gun to be homogeneous as in figs. 2B, 3B, 5B, Part I. The data are $r_0 = 2{\cdot}5, r_2 = 7{\cdot}5$, and $p_0 = 21$, to find t_0, t_1, &c., and to complete the curves p_0r_2 and t_0t_2 of radial pressures and hoop tensions. The point A_2 is obtained by drawing vertical and horizontal lines through p_0 and r_2 respectively, and determining Od, the third proportional to Or_2 and Or_0, by the same construction as that described above.

Then A_2d produced will cut a horizontal line through R_2 in S_2, and S_2s_2 drawn parallel to R_2r_2 will cut p_0r_0 produced in t_0, measurement of r_0t_0 on the stress scale gives $t_0 = 26{\cdot}245$ tons/in.², halving p_0t_0 gives m_0, a vertical through which gives C; or C is also obtained by dS_2, which cuts the horizontal through O in C.

Joining Cs_1, the dotted lines show the construction to find t_2 and $m_1t_1 = m_1p_1$ to give p_1; horizontal lines are drawn through $r = 3{\cdot}75$ and 4·6 inches in order that fig. B may also serve for calculating the powder stress of the 4·7-inch wire gun of fig. C, which has the same internal and external diameters as the point considered.

By the construction explained above, the details of which may now for the sake of clearness be omitted from the figure, and measurement on the stress scale, we find :—

at $r_0 = 2{\cdot}5$ ins.	$p_0 = 21$	and	$t_0 = 26{\cdot}25$	tons/in.
$r = 3{\cdot}2$,,	$p = 11{\cdot}8$	,,	$t = 17{\cdot}04$	,,
$r = 3{\cdot}75$,,	$p = 7{\cdot}89$	,,	$t = 13{\cdot}13$	,,
$r = 4{\cdot}1$,,	$p_1 = 6{\cdot}16$	,,	$t_1 = 11{\cdot}42$	,,
$r = 4{\cdot}6$,,	$p_1 = 4{\cdot}35$	,,	$t = 9{\cdot}6$	,,
$r = 5{\cdot}5$,,	$p_1 = 2{\cdot}25$	,,	$t = 7{\cdot}51$	,,
$r = 7{\cdot}5$,,	$p_2 = 0$	,,	$t_2 = 5{\cdot}25$	,,

Deducting the powder stresses p and t from the firing stresses P and T, gives the initial stresses ϖ and τ, such that—

(i) In the jacket, $T_2 = 3{\cdot}03$, τ (at $r = 5{\cdot}5) = 4{\cdot}29$, $\tau_1 = 6{\cdot}58$;
(ii) ,, tube, $T'_1 = -7{\cdot}7$, T (at $r = 3{\cdot}2) = -9{\cdot}04$, $T_0 = -11{\cdot}25$.

The firing stress of the corresponding 4·7-inch wire gun of the same calibre is shown in section across the powder chamber in fig. C.

In the jacket the stress of repose is nil; the firing stress is, therefore, obtained directly from fig. B (where the gun is supposed to be homogeneous).

We have therefore

$$P_2 = 2{\cdot}25 \text{ tons/in}^2.$$
$$T_3 = 5{\cdot}25 \quad ,,$$
$$T_2 = 7{\cdot}5 \quad ,,$$

In the wire coil we have

$$P_2 = 2{\cdot}25 \text{ tons/in.}^2, \text{ and } T'_2 = T_1 = 25 \text{ tons/in.}^2.$$

Draw the lines T'_2T_1O' and P_2BB_1 vertically, r (= 4·6 inch in the diagram) denoting *any* radius in the wire coil. Join O′B, and produce it to meet r_2P_2 produced in A; from this point draw a vertical, this will cut rP produced in P, such that rP = 7·59 tons/in.^{2}, a point on the curve of radial pressure, in the wire coil.

Next join O′B, and produce it to meet r_2P_2 produced in A_2; from this point draw a vertical downwards; this will cut r_1B_1 produced in P_0, such that r_1P_1 = 14·98 tons/in.^{2}.

Any intermediate point on the wire radial pressure curve can be similarly obtained.

In the tube we have found that P_1 = 14·98, and we know by the data that P_0 = 21 tons/in.^{2}, and the problem is to find by construction the hoop stresses, viz., T_0 and T'_1 at the inner and outer surfaces of the tube.

Produce the line A_2P_1 to cut r_0P_0 in B_0; join A_1B_0, and produce it to cut the horizontal line through O in l, draw lL vertically, join A_1L, cutting Ol produced in C, draw Cm_0m_1 vertically upwards, now measuring off $m_0T_0 = m_0P_0$, and $m_1T'_1 = m_1P'$ gives

$$T_0 = 0{\cdot}65 \text{ tons/in.}^2, \text{ and } T'_1 = -5{\cdot}35 \text{ tons/in.}^2$$

With C as a centre for the two Barlow curves P_0P_1 and $T_0T'_1$, by the method of construction already explained with fig. B, any point on these curves can be obtained corresponding to any given radius between r_0 and r_1.

Fig. D shows the curves of winding tension to be employed in the construction of the wire gun.

The theory of Rifling may be resumed at this stage, as part of theory of Gun Construction, so far as the thrust on the grooves is concerned.

Taking coordinate axes Ox in the radial and Oy in the tangential direction of the cross section of the bore, at the point P where the rifled surface bears on the driving band of the shot, and Oz in direction of the axis of the bore, the thrust producing rotation was calculated by Sir Andrew Noble from the formula (*Phil. Mag.*, 1863 and 1873)—

$$(1) \qquad R = \frac{G \tan\theta + \dfrac{wv^2}{2240g}\dfrac{d^2y}{dz^2}}{\dfrac{\dfrac{r^2}{\rho^2} + \tan^2\theta}{\sqrt{(1+\sin^2\delta\tan^2\theta)}}\sin\delta - \left(\dfrac{r^2}{\rho^2} - 1\right)\mu\sin\theta}$$

giving R the total rotating thrust in tons on the bearing surfaces; and G denotes the powder gas thrust on the base of the shot in tons,

θ the angle of the rifling,
w the weight of the shot expressed in lb.,
v the velocity of advance of the shot up the bore in f/s;
$r = d \div 24$ the radius of the bore in feet,
ρ the radius of gyration of the shot in feet about the axis of figure;
μ the coefficient of friction;

while δ denotes the angle between the normal of the cross-section of the groove and the radius of the cross-section to the bearing point of the groove.

The spherical triangles in the figure show the direction in space of the various lines drawn from the origin O; OT is tangential to the spiral of the rifling, ON is normal to the bearing surfaces. while $90° - \delta$ is the angle YZN between the planes ZY and ZN.

The equations of motion of the shot, resolving parallel to Oz, and taking moments round the axis Oz are

$$(2) \qquad \frac{w}{2240g}\frac{d^2z}{dt} = \mathrm{G} + \mathrm{R}\cos \mathrm{NZ} - \mu\,\mathrm{R}\cos \mathrm{TZ}.$$

$$(3) \qquad \frac{w}{2240g}\rho^2\frac{d^2\omega}{dt} = r\mathrm{R}\cos \mathrm{NY} - r\,\mu\,\mathrm{R}\cos \mathrm{TY},$$

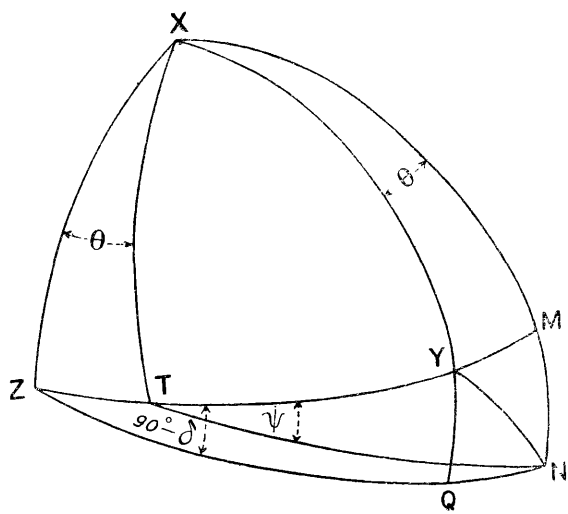
ω denoting the radians which the shot turns through while it advances z feet.

But for any curve of rifling bar

$$(4) \qquad r\frac{d\omega}{dt} = \frac{dy}{dt} = \frac{dy}{dz}\frac{dz}{dt} = \frac{dy}{dz}v.$$

$$(5) \qquad r\frac{d^2\omega}{dt^2} = \frac{d^2y}{dt^2} = \frac{d^2y}{dz^2}\left(\frac{dz}{dt}\right)^3 + \frac{dy}{dz}\frac{d^2z}{dt^2}, = \frac{d^2y}{dz^2}v^2 + \tan\theta\frac{d^2z}{dt^2};$$

so that eliminating $\frac{d^2z}{dt}$ and $\frac{d^2\omega}{dt^2}$ in (2) and (3).



(6) $$\frac{w}{2240g}\frac{\rho^2}{r^2}\frac{d^2y}{dz^2}v^2 + \frac{\rho^2}{r^2}\tan\theta\,(G + R\cos NZ - \mu R\cos TZ) = R\cos NY - \mu R\cos TY.$$

(7) $$R = \frac{\frac{\rho^2}{r^2}\left(G\tan\theta + \frac{wv^2}{2240g}\frac{d^2y}{dz^2}\right)}{\cos NY - \mu\cos TY - \frac{\rho^2}{r^2}\tan\theta\,(\cos NZ - \mu\cos TZ)}.$$

In the spherical triangles of the figure, ψ denoting the angle MTN,

(8) $$\cos TY = \sin\theta,\ \cos TZ = \cos\theta.$$

(9) $$\cos NY = \cos\theta\cos\psi,\ \cos NZ = -\sin\theta\cos\psi$$

(10) $$\tan\psi = \cos\theta\cot\delta = \frac{\cos\theta\cos\delta}{\sin\delta}.$$

so also

(11) $$\cos\psi = \frac{\sin\delta}{\sqrt{(\sin^2\delta + \cos^2\theta\cos^2\delta)}}$$

and substituting these values, the expression for R in (1) is arrived at.

In the uniform rifling, the rifling bar is straight,[a]

(12) $$\frac{d^2y}{dz^2} = 0,\ \frac{dy}{dz} = \tan\theta = \frac{\pi}{n}.$$

In the parabolic rifling, the rifling bar is curved to a parabola,

(13) $$y = \frac{z^2}{p},\ \frac{dy}{dz} = \tan\theta = 2\frac{z}{p},\ \frac{d^2y}{dz^2} = \frac{2}{p}.$$

In the numerical example of which the results are given in Table II, p. 156,

$$r = 4\cdot 7 \div 24 = 0\cdot 2 \text{ foot},$$

$$\frac{\rho^2}{r^2} = \tfrac{1}{2}$$

for a solid cylindrical shot: also $w = 45$ lb.

In the uniform twist the pitch of the rifling was 35 calibres; while in the parabolic rifling the pitch diminished from 100 calibres at the breech to 35 calibres at the muzzle.

CHAPTER VII.—INTERIOR BALLISTICS.

WE resume here the consideration of the problem of Interior Ballistics, of which the elementary details have been given in Chapter IV, Part I.

The experiments of Noble and Abel, described in the *Phil. Trans.* 1875–80–92–94, are the foundation of the modern theory of the action of fired gunpowder.

In these experiments a charge of P lb. of powder is fired in an explosion chamber (figs. 8, 9, 10, pp. 95, 96), of which the capacity, C in^3, is accurately known, and the pressure, p tons/in.^{2}, was recorded by a crusher gauge (figs. 5, 6, 7, pp. 90, 93) for the corresponding density of the powder gas P/C lb./in.^{3}, at the temperature of explosion.

The results were plotted in figs. 14, 15, p. 108 in curves, fig. 15. showing the relation between the pressure p and the gravimetric density, G.D., where

(1) $$\text{G.D.} = 27{\cdot}73\,\frac{P}{C},$$

the G.D. being the specific gravity of the P lb. of powder when filling the volume C in.^{3} in a state of gas, referred to water, which bulks 277·3 in.^{3} to the gallon, or 27·73 in.^{3}/lb.

The diagram, fig. 14, shows also the relation between p and v, the reciprocal of the G.D., which may be called the gravimetric volume (G.V.), being the ratio of the volume of the gas to the volume of an equal weight of water.

The results are also embodied in the table given in Part I, Chap. IV, p. 104.

At the standard temperature of 62° F. the volume of the gallon of 10 lb. of water is 277·3 in.^{3}; or otherwise 1 ft.^{3}, or 1728 in.^{3} of water at this temperature weighs 62·35 lb. and, therefore, 1 lb. of water bulks

$$1728 \div 62{\cdot}35 = 27{\cdot}73 \text{ in}^3.$$

Thus, if a charge of P lb. is placed in a chamber of capacity C in.^{3}, the

(2) $$\text{G.D.} = 27{\cdot}73\,\frac{P}{C},$$

and the

$$\text{G.V.} = \frac{C}{27{\cdot}73\,P}.$$

Sometimes Noble employs the factor 27·68, corresponding to a density of water of about 62·4 lb/ft.^{3}, and a temperature of 54 or 55° F.

With metric units, measuring P in kg. and C in litres or dm.^{3}, or P in g and C in cm.^{3}, the

$$\text{(3)} \qquad \text{G.D.} = \frac{P}{C}, \qquad \text{G.V.} = \frac{C}{P},$$

no factor being required.

After the explosion of a charge of P lb. of gunpowder, it was found in these experiments that a fraction α P lb. remained in a liquid state at unit S.G., and therefore of volume 27·73 α P in.^{3}; the remaining $(1 - \alpha)$ P lb. of the charge was converted into the gas which filled the remaining

$$\text{(4)} \qquad C - 27{\cdot}73\,\alpha\,P = C\,(1 - \alpha\,D) \text{ in.}^3,$$

of the chamber, D denoting the G.D. of the charge; so that the S.G. of the gas was

$$\text{(5)} \qquad \frac{27{\cdot}73\,(1-\alpha)\,P}{C\,(1-\alpha\,D)} = \frac{(1-\alpha)\,D}{1-\alpha\,D} = \frac{1-\alpha}{v-\alpha},$$

where $v = 1 \div D$, the G.V. of the gas.

On the assumption that the gas obeyed Boyle's law, and that the temperature of the explosion was constant,

$$\text{(6)} \qquad \frac{p}{p_0} = \frac{(1-\alpha)\,D}{1-\alpha\,D} = \frac{1-\alpha}{v-\alpha},$$

where p_0 denotes the pressure, when $D = 1$, $v = 1$; and α may now be called the *covolume*.

In Noble and Abel's experiments it was found on the average that $p_0 = 43$ tons/in.^{2}, while the liquid residue was 57 °/$_\circ$ by weight of the charge, so that $\alpha = 0{\cdot}57$, $1 - \alpha = 0{\cdot}43$; this makes

$$\text{(7)} \qquad p = \frac{43 \times 0{\cdot}43}{v - 0{\cdot}57},$$

and the dotted line in fig. 15, p. 108 shows the theoretical curve of relation between p and v calculated by this formula; the actual realised curve is seen to lie slightly below.

From the Table of Pressure on p. 104, or by a quadrature of the curve in fig. 14, p. 108, the work E in foot-tons realised by the expansion of 1 lb. of the powder gas from one gravimetric volume or density to another can be inferred, on the assumption that the pressure in the closed vessel is the same as when the gas is expanded in the bore of the gun.

For if the average pressure is p tons/in.^{2} at an average G.V. v, then while the G.V. changes by Δv from $v - \frac{1}{2}\Delta v$ to $v + \frac{1}{4}\Delta v$, a change of volume of 27·73 Δv in.^{3}, the work done is 27·73 $p\Delta v$ inch-tons, or in foot-tons,

(8) $$\Delta E = 2{\cdot}31\, p\Delta v\,;$$

and the difference ΔE being calculated from the observed experimental values of p, a summation, as in the Ballistic Tables, gives E, as tabulated in Table XIV.

Conversely, from a table of E in terms of v, as in Table XIV, we can infer the value of p from the formula

(9) $$p = \frac{1}{2{\cdot}31}\frac{\Delta E}{\Delta v}\,.$$

For instance, as v changes from 4·9 to 5·1, so that $\Delta v = 0{\cdot}2$, then, from Table XIV,

(10) $$\begin{aligned}\Delta E &= 92{\cdot}186\\ &\quad - 90{\cdot}565\\ &= 1{\cdot}621,\end{aligned}$$

making

(11) $$p = \frac{1{\cdot}621}{2{\cdot}31 \times 0{\cdot}2} = 3{\cdot}5 \text{ tons/in.}^2,$$

agreeing closely with the experimental value.

On drawing off a little of the gas from the explosion vessel, it was found that a gramme of powder gas (or cordite), at 0° C and standard atmospheric pressure of 14·7 lb./in.^{2}, occupied 280 cm.^{3} (cordite 703 cm.^{3}), while the same gramme of powder gas, compressed into 0·43 cm.^{3} at the temperature of explosion, had a pressure of 43 tons/in.^{2}, or $43 \times 2240 \div 14{\cdot}7 = 6552$ atmospheres.

The absolute centigrade temperature T of explosion is thence inferred from the gas equation

(12) $$R = \frac{pv}{T} = \frac{p_0 v_0}{273}\,,$$

which, with $p = 6552$, $v = 0{\cdot}43$, $p_0 = 1$, $v_0 = 280$, makes

(13) $$T = 273\,\frac{6552 \times 0{\cdot}43}{280} = 2748,$$

a temperature of 2475° C or 4487° F.

These calculations are made for the case of a charge of powder fired in a closed explosion chamber; but if the powder gas expands in the bore of a gun according to the ordinary adiabatic law equation (6) must be changed to

(14) $$\frac{p}{p_0} = \left(\frac{1-\alpha}{v-\alpha}\right)^{\gamma},$$

where the index γ is the ratio C_p/C_v of C_p the specific heat (S.H.) at constant pressure to C_v the S.H. at constant volume; and $\gamma = 1{\cdot}4$ on the average.

But, contrary to the anticipation based on this adiabatic law, Noble and Abel found, at an early stage of their researches, that the pressure observed in a closed vessel, as given isothermally by equation (7), did not differ greatly from the pressure in the bore of the gun itself as deduced from experiments with crusher gauges inserted in plugs up the bore; so that the pressure falls off much more slowly than according to the ordinary adiabatic law, and more in accordance with the isothermal expansion law; and they came to the conclusion that this departure from expectation was due to the heat stored up in the liquid and solid residue, which forms the smoke particles.

Denoting by β the ratio by weight of the non-gaseous to the gaseous products of 1 lb. of the charge, and by λ the S.H. of the non-gaseous portion supposed to be distributed in a finely divided state throughout the gas, the heat dH, in B.T.U. (British thermal units), given out by β lb. of the non-gaseous part during a rise of temperature dT is such that

$$(15) \qquad dH = -\beta\lambda dT.$$

The gaseous part, $1 - \beta$ lb., obeys the gas equation

$$(16) \qquad R = \frac{p(v - \alpha)}{T} = \frac{p_0(1 - \alpha)}{T_0}$$

so that, taking logarithmic differentials,

$$(17) \qquad \frac{dp}{p} + \frac{dv}{v - \alpha} - \frac{dT}{T} = 0,$$

or,

$$(18) \qquad \frac{dT}{T} = \frac{dp}{p} + \frac{dv}{v - \alpha},$$

and then

$$(19) \qquad \frac{dH}{T} = -\beta\lambda\left(\frac{dp}{p} + \frac{dv}{v - \alpha}\right).$$

Supposing p and v to vary one at a time,

$$(20) \qquad dH = \frac{\delta H}{\delta p} dp \ (v \text{ constant}) + \frac{\delta H}{\delta v} dv \ (p \text{ constant})$$

and p varying while v is constant,

$$(21) \qquad \frac{\delta H}{\delta p} = \frac{\delta H}{\delta T}\frac{\delta T}{\delta p} = C_v \frac{v-a}{R} = C_v \frac{T}{p}$$

while if p is constant and v varies,

$$(22) \qquad \frac{\delta H}{\delta v} = \frac{\delta H}{\delta T}\frac{\delta T}{\delta p} = C_p \frac{h}{R} = C_p \frac{T}{v-a};$$

so that, in (20),

$$(23) \qquad \frac{dH}{T} = C_v \frac{dh}{p} + C_p \frac{dv}{v-a}.$$

Equating the values of dH/T in (15) and (23),

$$(24) \qquad (C_v + \beta\lambda)\frac{dp}{p} + (C_p + \beta\lambda)\frac{dv}{v-a} = 0$$

a differential relation, leading on integration to

$$(25) \qquad (C_v + \beta\lambda)\log p + (C_p + \beta\lambda)\log(v-a) = \text{constant},$$

or

$$(26) \qquad \frac{p}{p_0} = \left(\frac{1-a}{v-a}\right)^m,$$

where

$$(27) \qquad m = \frac{C_p + \beta\lambda}{C_v + \beta\lambda},$$

reducing to the ordinary adiabatic law, when $\beta = 0$, and there is no liquid or solid residue, as with smokeless powder.

According to the experiments of Noble and Abel,

$$(28) \qquad a = 0{\cdot}57, \qquad 1 - a = 0{\cdot}43, \qquad \beta = \frac{a}{1-a} = 1{\cdot}3256,$$

$$\lambda = 0{\cdot}45,\ C_p = 0{\cdot}2324,\ C_v = 0{\cdot}1762,$$

making

$$(29) \qquad m = 1{\cdot}074.$$

In metric units the work done in g-cm. per gramme of powder, in the expansion from unit G.V. is

$$(30)\quad \int p dv = p_0 \int_1^v \left(\frac{1-\alpha}{v-a}\right)^m dv = \frac{p_0 (1-a)}{m-1} \left\{ 1 - \left(\frac{1-a}{v-a}\right)^{m-1} \right\}$$

$$= \frac{p_0 (1-a) - p (v-a)}{m-1} = \frac{R}{m-1} (T_0 - T)$$

as the temperature falls from T_0 to T.

In British units this must be multiplied by 2·31 to obtain E, the work done in ft.-tons per lb. of powder; and with $p_0 = 43$ tons/in.^{2}.

$$(31)\qquad p = 43 \left(\frac{0{\cdot}43}{v - 0{\cdot}57}\right)^{1{\cdot}074}.$$

$$(32)\qquad \frac{T}{T_0} = \left(\frac{0{\cdot}4}{v - 0{\cdot}57}\right)^{0{\cdot}074},$$

$$(33)\quad E = \frac{2{\cdot}31 \times 43 \times 0{\cdot}43}{0{\cdot}074} \left[1 - \left(\frac{0{\cdot}43}{v - 0{\cdot}57}\right)^{0{\cdot}074}\right] = 577{\cdot}3 \left(1 - \frac{T}{T_0}\right),$$

so that 577·3 ft. tons is the total amount of work realisable from the infinite expansion from unit G.V. of one lb. of gunpowder.

Table XIV is calculated from these formulas (31), (32), (33), and the results are only slightly different to those obtained in the previous manner from the observed pressure in closed vessels.

The Table is carried up to $v = 40$, but can easily be extended in accordance with the scheme of computation given here.

v	2	10	40	50
$v-\alpha$	1·43	9·43	39·43	49·43
$1-\alpha$	0·43			
$\log (v-\alpha)$	0·1553	0·9745	1·5958	1·6940
$\log (1-\alpha)$	$\bar{1}$·6335			
$\log\left(\frac{v-\alpha}{1-\alpha}\right)$	0·5218	1·3410	1·9623	2·0605
$m-1$	0·074			
$\log\left(\frac{v-\alpha}{1-\alpha}\right)^{m-1}$	0·0386	0 0992	0·1452	0·1525
log 577·3	2·7614			
log (577 3—E)	2·7228	2·6622	2·6162	2·6089
577·3—E	528·2	459·4	413·2	406·3
E	49·1	117·9	164·1	171·0

The agreement of these numbers with those printed in the table is close enough for practical purposes; and a computation by a slide rule would serve equally well.

With cordite the products of combustion are almost all gases, and there is little or no solid and liquid residue, hence the absence of smoke. We can thus put $\alpha = 0$, $\beta = 0$ in formulas (27), (30), (31), (32), (33); and a good average value of m is found by experiment to be

$$(34) \qquad m = 1{\cdot}3.$$

It is also found by experiment in closed vessels (p. 104), that

$$p = 30 \text{ tons/in.}^2 \text{ for } v = 3;$$

thence a table of E can be calculated for cordite, giving E the energy in ft. tons realisable per lb. of cordite.

In the employment of these tables to calculate the muzzle energy and velocity to be expected from a given charge of P lb. of powder or cordite in expanding from the volume, C in.^{3} of the chamber to the total volume, B in.^{3} of the bore, including the chamber, the initial and final gravimetric volumes (G.V.) denoted by v_0 and v are calculated from

$$(35) \qquad v_0 = \frac{C}{27{\cdot}73\,P}, \quad v = \frac{B}{27{\cdot}73\,P},$$

and then the difference

$$(36) \qquad E(v) - E(v_0)$$

of the corresponding values of E, multiplied by P, the charge in lb., gives the maximum realisable work in ft-tons.

In practice a factor f, called the *factor of effect*, varying from 0·9 to 0·7, equivalent to a discount of 10 to 30 °/$_\circ$, is employed to obtain the actual net realised work stored up in the shot on leaving the muzzle.

Mr. Longridge (*Interior Ballistics*) points out the reason for some such reduction, from the time occupied by the charge in combustion, during which the pressure rises to its maximum; the direct employment of the Table assuming that the charge was completely consumed before the shot began to move.

The dotted line in fig. 4, p. 85, shows the upper theoretical line of pressure, the area of which is tabulated in Table XIV; and the area between this curve and the actual pressure curve while the combustion of the charge is in progress will represent the work to be deducted on Mr. Longridge's theory in consequence of the pressure rising gradually to a maximum P along the portion of the pressure curve P_0P_1.

A knowledge of the maximum pressure p_1 to be expected in the bore will enable us to settle v_1, the G.V. of the powder gas at the point of maximum pressure; and now

$$(37) \qquad P[E(v) - E(v_1)]$$

will give the work realised in foot-tons during the further stage of expansion up to the muzzle.

In the absence of an exact knowledge of the curve P_0P_1 along which the pressure rises during combustion, we may assume the pressure equal to p_1; and now the work realised during combustion will be given by

$$(38) \qquad 2{\cdot}31\, Pp_1(v_1 - v_0) \text{ ft-tons};$$

in reality somewhat less.

Thus, with Mr. Longridge's modification, the total work realised will be

$$(39) \qquad 2{\cdot}31\, Pp_1(v_1 - v_0) + P[E(v) - E(v_0)].$$

Thus, for instance, in the 15-pr. B.L. guns, in which

$$C = 117, \quad B = 647 \text{ in.}^3,$$

a charge of 4 lb. of gunpowder expands between the G.D.'s,

$$D_0 = 0{\cdot}9482, \quad D_1 = 0{\cdot}1715$$

or between the G.V.'s,

$$v_0 = 1{\cdot}054, \quad v_1 = 5{\cdot}83;$$

so that

$$(40) \qquad E(v) - E(v_0) = 97{\cdot}5113 - 5{\cdot}0232 = 92{\cdot}4881;$$

and with a factor of effect 0·7, the net muzzle energy

$$(41) \qquad \frac{wV^2}{2g \times 2240} = 0{\cdot}7 \times 4 \times 92{\cdot}4881 = 259 \text{ ft-tons.}$$

so that, if $w = 14\frac{1}{16}$ lb., this makes $V = 1629$ f/s.

In Longridge's method, assuming a maximum pressure

$$p_1 = 15 \text{ tons/in.}^2, \text{ corresponding to a G.V. of about } v_1 = 1{\cdot}7,$$

$$(42) \qquad E(v_1) - E(v) = 97{\cdot}511 - 39{\cdot}778 = 57{\cdot}733,$$

$$(43) \qquad P[E(v_1) - E(v)] = 4 \times 57{\cdot}733 = 230{\cdot}932,$$

and

$$(44) \qquad 2{\cdot}31\, Pp_1(v_1 - v_0) = 2{\cdot}31 \times 4 \times 15 \times 0{\cdot}646 = 38{\cdot}8;$$

this makes the total realised work about 269 ft-tons, a much closer agreement with practice.

The charge actually employed is 15 oz. of cordite, expanding between the G.V.'s of

$$(45) \qquad v_0 = 4{\cdot}5,\ v = 24{\cdot}9;$$

and it imparts a muzzle velocity V = 1576 f/s, so that 1 lb. of cordite will give about the same muzzle velocity as 4 lbs. of powder.

Pending further experiments with cordite, and the construction of a Table of Work, some factor of effect as 4 must be employed with cordite charges in calculations based on Table

In these provisional calculations much reliance is placed on empirical formulas resembling those employed for the perforation of Armour Plates.

Sarrau's Monomial Formula is useful, giving the muzzle velocity

$$(46) \qquad V = HP^x w^y d^p D_0^q \left(\frac{B}{C}\right)^r,$$

where the symbols have their previous meaning, and H is a factor depending on the quality and structure of the powder; the indices x, y, p, q, r being settled by experiment; they are determined very readily by plotting a few experimental results on a logarithmic chart.

Interpreted in popular language as before, the formula asserts that for moderate changes, 1 % increase or decrease in P, w, d, D_0, B/C causes x, y, p, q, r % increase or decrease in V.

For the old rule of Robins and Hutton, that—the muzzle energy is simply proportional to the charge—ignoring the efforts of calibre, density of loading, and number of expansions in the bore, we put

$$(47) \qquad x = \tfrac{1}{2},\ y = -\tfrac{1}{2},\ p,\ q,\ r = 0.$$

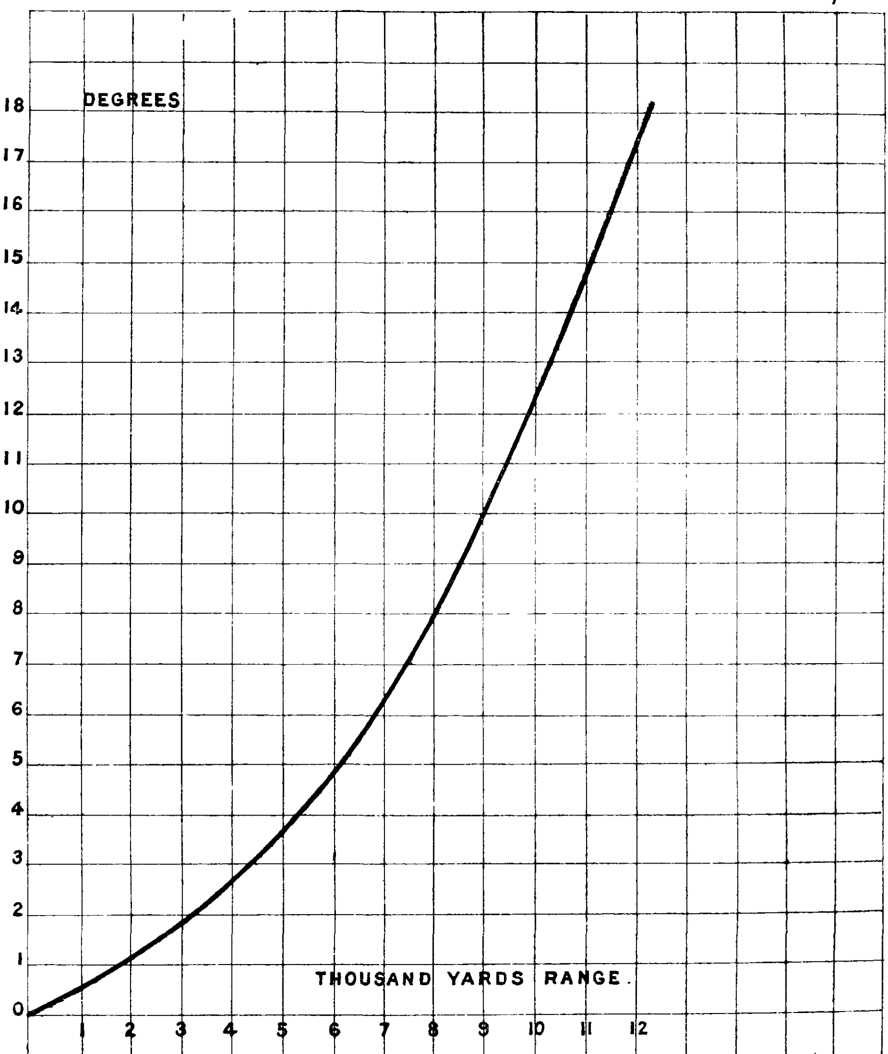
For quick powder, entirely consumed in the bore, Sarrau takes

$$(48) \qquad x = \tfrac{3}{8},\ y = -\tfrac{1}{2},\ p = \tfrac{1}{4},\ q = \tfrac{1}{4},\ r = \tfrac{1}{8}.$$

For slow powder, some of which is blown out unconsumed from the muzzle, he takes

$$(49) \qquad x = \tfrac{3}{8},\ y = -\tfrac{7}{16},\ p = \tfrac{1}{8},\ q = \tfrac{1}{4},\ r = \tfrac{3}{16}.$$

To face p 281.



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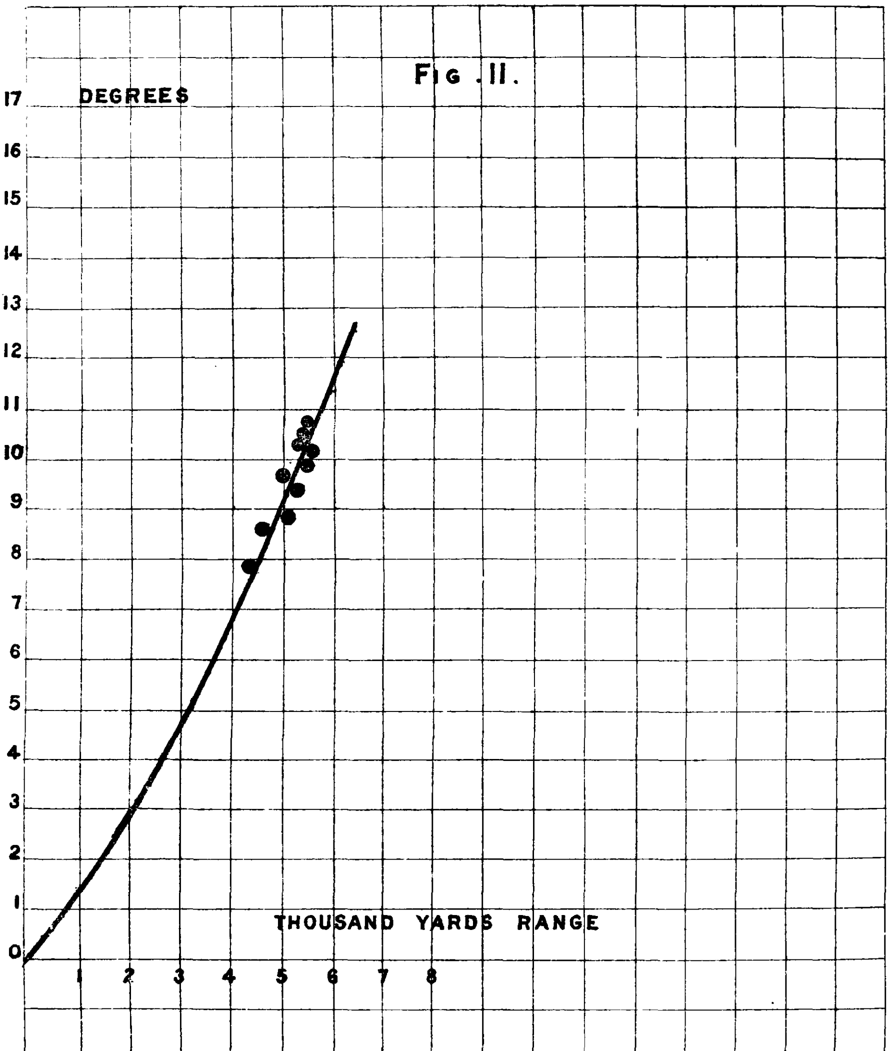
To face p 281.



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APPENDIX I.

CORRECTION OF A RANGE TABLE FOR A LOSS OF MUZZLE VELOCITY.

Fig. I shows range and angle of projection plotted from the official range table of the 6-inch B.L. Mark VII, land service, full charge, based on practice of 7.11.99.

Fig. II shows the results of some practice in which ten rounds were fired at a moving target. These ten rounds are shown by the ten ●'s which are plotted from the ranges to the splashes and the angles of projection as obtained from the practice report.

Now if a tracing is made of fig. II, and if this tracing is placed over fig. I in such a way that the yards' scales coincide, it will at once be seen that the gun has been shooting short, and that a new curve of angles of projection is required.

It is obtained as follows:—

(*a.*) Keeping the yards' scale of the tracing parallel to the yards' scale of fig. I, and at the same time keeping the zero of the tracing on the curve of angles of projection of fig. I, slide the tracing upwards and to the right till about half the shot plotted on the tracing fall above the curve on fig. I.

(*b.*) From fig. I, trace the curves.

The tracing paper will now give the required angles of projection for any given range on the assumption that the shape of the trajectory does not alter for a moderate change of the angle of sight. A guess must be made at the new jump.

Owing to the small size of the plates, a very exaggerated case has purposely been plotted on fig. II. The appearance of the tracing after the curve has been traced from fig. I is illustrated by fig. II.

APPENDIX II.

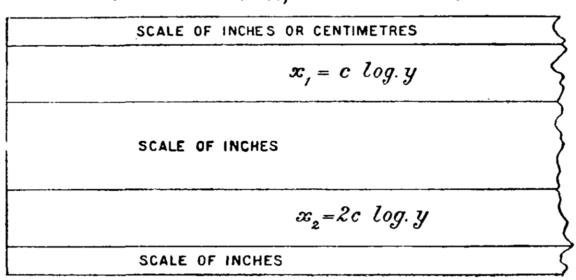
PRACTICAL NOTES ON THE SLIDE-RULE.

In these notes a knowledge of the theory is presumed. They merely deal with a few of the common problems in practical gunnery. The reader is expected to be in possession of a slide-rule, and of the instructions for using it supplied by the makers, John Davis and Co., Victoria Street, W.

The service slide-rule consists of three parts, called the ruler, the slide, and the cursor.

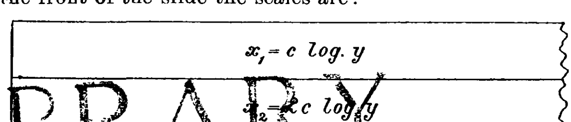
The Ruler.

On the ruler there are five scales, as shown below:—



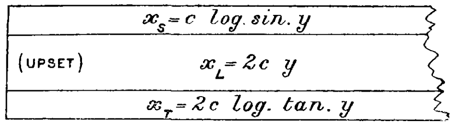
The Slide.

On the front of the slide the scales are:—



While on the back of the slide there are three scales, as below.

(When we turn the slide 180° so as to bring it back up, we speak of it as being "upset.")



The x_s scale is marked with an " S " at sin 90°.
The x_T scale with a " T " at tan 45°.
The scales, except the x_L scale, all read from left to right.

The Cursor.

This consists mainly of a piece of glass on a sliding saddle, the glass having cut on it a line, by means of which it can be set.

The Decimal Point and Plus and Minus Signs.

These must be attended to independently of the slide-rule by ordinary arithmetic, trigonometry, or common sense.

The Use of the Slide-Rule.

The slide-rule will give approximately to three figures the answer to anything that can be done with a book of common logarithms. When using it one thinks of × as +, and of ÷ as −; + being to right, − to the left.

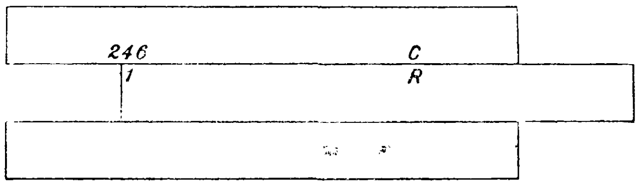
EXAMPLES.

THE x_1 SCALES.

Given a battery h yards high, required the correction in minutes, for R yards of range, to be subtracted from the tangent elevation in order to give quadrant elevation.

First as regards curvature C—

$$C = \frac{\cdot 246R}{1000}$$



R	C
2000	·492
4000	·984
6000	1·476
8000	1·968
10000	2·460

In the above note that for one setting of the slide all the values of C are found.

The equation is usually written—

$$x = \frac{ay}{b} \qquad \text{(i)}$$

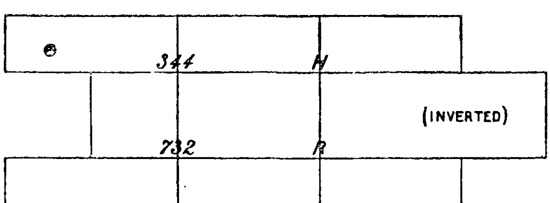
Next as regards the correction H for height of battery—

$$H = \frac{3440h}{R} = \frac{3440 \times 73{\cdot}2}{R} \text{ suppose.}$$

This equation is generally written—

$$x = \frac{ab}{y} \qquad \text{(ii)}$$

Note now that the variable y is in the denominator. Whenever an equation takes this form, it is handy to "invert" the slide, *i.e.*, to turn it 180° round with the clock, or end for end.



Note that the vertical lines through the 344 and 732, and through the H and R represent the cursor, which in this case has to be made use of first for setting, then for reading.

R	H	Brought Forward. C	H + C
2000	126·0	0·5	126·
4000	63·0	1·0	64·
6000	42·0	1·5	43·
8000	31·4	2·0	33·
10000	25·1	2·5	28·

H + C gives the whole correction required to the nearest minute—

Note that in finding the values of H, the slide is only set once, the various readings of H being obtained by sliding the cursor over successive values of R.

Note also equation (ii) gives a hyperbola, and is therefore useful in wire gun construction.

These two equations (i) and (ii) embrace an enormous number of cases in which the slide-rule is useful.

Next let it be required to find

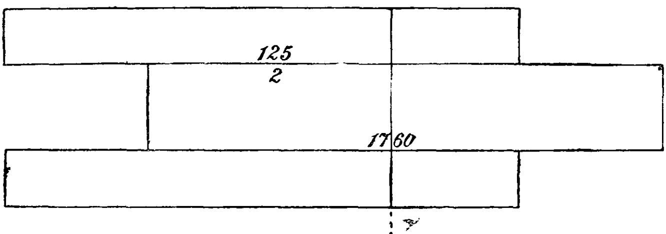
$$\frac{wv^2}{2 \times g \times 2240}.$$

This is usually written

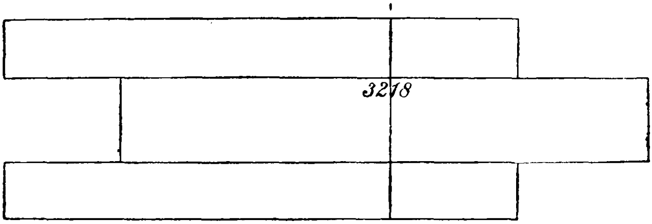
$$x = \frac{ace}{bd} \qquad \text{(iii.)}$$

$$= \frac{12 \cdot 5 \times 1760^2 \times 1 \times 1}{2 \times 32 \cdot 18 \times 2240} \text{ suppose}$$

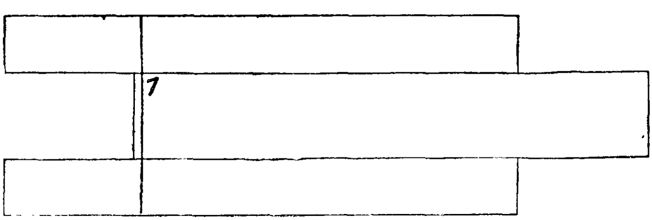
First, put 2 under 125, cursor to 1760, on x_2 scale of the slide—



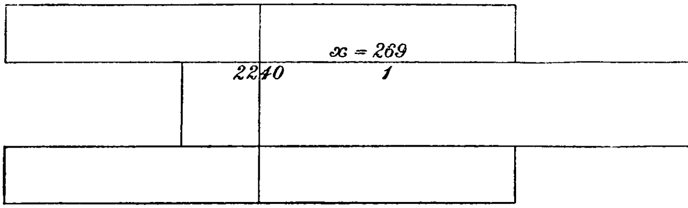
Next, put slide to 3218 without moving cursor—



Then, move the cursor to 1 without moving slide.



Then move the slide to 2240 without moving cursor.



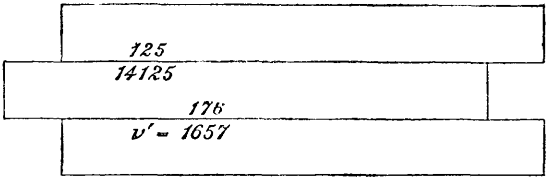
When, looking above the "1" on the slide, we find the answer, $x = 269$.

Now as regards the decimal point, the 2240 cancels the square of the 1760. This leaves $\frac{17000}{60}$, approximately $=300$. Therefore there are three figures in the answer.

Note this example might have been worked out entirely with the x_1 scales, 1760 being taken twice in succession by means of the cursor and the x_1 scale on the slide.

Suppose, as another example,

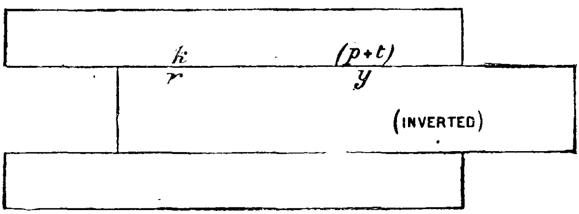
$$12\tfrac{1}{2} \times 1760^2 = 14{\cdot}125\ v'^2$$



Note.—As we discard the decimal points, every number considered merely as a group of figures has two square roots. In this case the other root is 525; but 525 and 5250 are both clearly impossible. Hence, 1657 is the answer.

BARLOW CURVES IN GUN CONSTRUCTION.

$$(p + t) = \frac{kr^2}{y^2}.$$

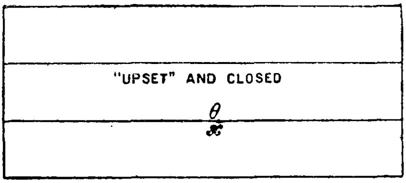


Suppose, $k = 5{\cdot}02$, $r = 9$.

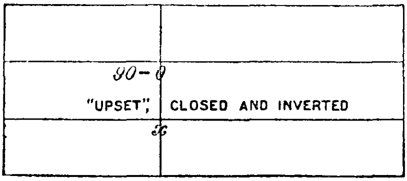
y	$p+t$
3	45·2
5	16·3

Note.—With one setting of the slide any number of values of $(p + t)$ can be read off.

THE X_T SCALE.

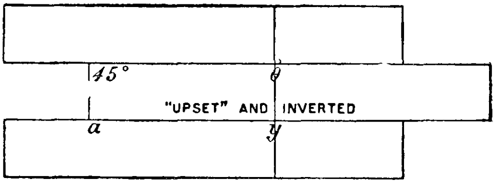


The above gives $x = \tan\theta$ up to 45°.

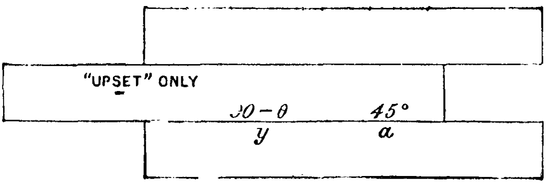


The above gives $x = \tan\theta$ when θ is greater than 45°.

The next gives $\tan^{-1}\frac{a}{y} = \theta$ when $a < y$.



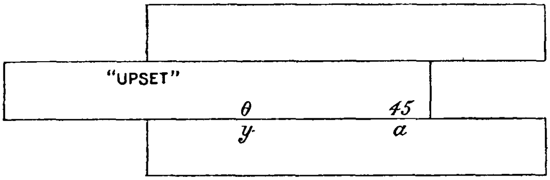
which is useful for finding θ when it is over 6°, for instance—when a = height of battery, y the range



The above gives $\tan^{-1}\frac{a}{y} = \theta$ when $a > y$.

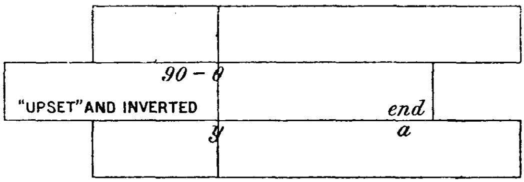
The following gives—

$$\tan^{-1}\frac{y}{a} = \theta \text{ when } y < a.$$



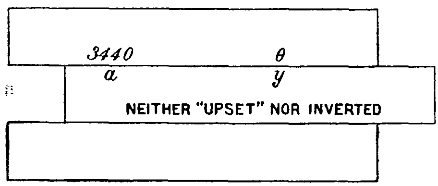
and the following—

$$\tan^{-1}\frac{y}{a} = \theta \text{ when } y > a.$$



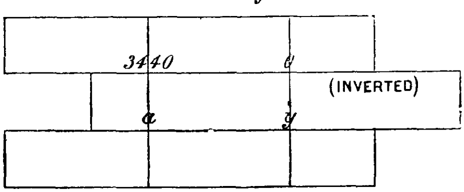
When θ is under 6°, we proceed as in the first example, that is—

$$\tan^{-1}\frac{y}{a} \text{ is given by}$$



θ being now in minutes

$$\text{while } \theta = \tan^{-1}\frac{a}{y} \text{ is give by}$$

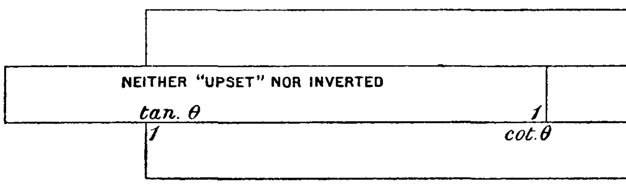


θ being in minutes.

In practice the double inversions (viz., $\frac{y}{a}$, $\frac{a}{y}$ and tan θ, cot θ) are best dealt with by trial: Remember that the tan $26\frac{1}{2}° = ·5$, and that the cot $26\frac{1}{2}° = 2$. Then, if the slide is in wrong after the first trial, take it out and invert it.

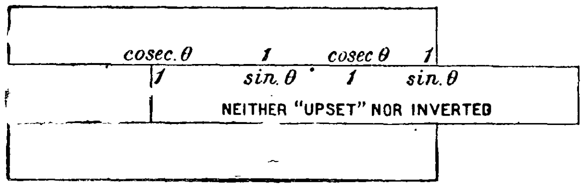
Again—

Pulling the slide out to the left till θ is read on the x_r scale by the index on the back of the ruler, we have for angles up to 45°—



THE x_s SCALE.

Similarly pulling the slide out to the right, till θ is read on the x scale by the index on the back of the ruler, we have—



In order to avoid mistakes with the decimal points, a few sines and tangents should be learnt by heart.

GROUP DIFFERENCE TABLE.

$$a = \phi \pm \sin^{-1} \frac{12\frac{1}{2}n}{d};$$

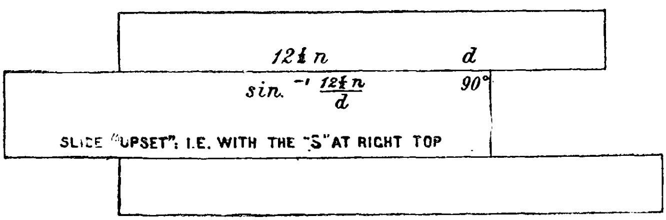
where

a is the limit of training for a group difference in multiples of 25 yards.

ϕ is the angle of training when the gun and DRF both point at right angles to the line joining them to each other.

n is any odd integer.

d is the distance in yards between the gun and DRF.



Suppose $d = 63$ yards.

$12\frac{1}{2}n$	$\sin^{-1}\frac{12\frac{1}{2}n}{d}$
$12\frac{1}{2}$	11°
$37\frac{1}{2}$	37°
$62\frac{1}{2}$	82°
$87\frac{1}{2}$	over 90°

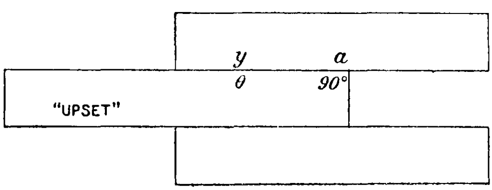
The rest is mere addition and subtraction, for which, of course, the slide rule is useless.

Suppose $\phi = 330°$.

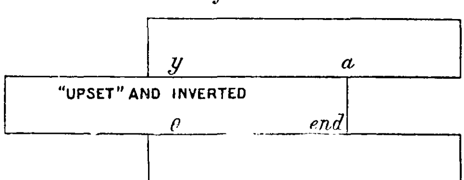
Brought Forward. $\sin^{-1}\frac{12\frac{1}{2}n}{d}$.	a.	Group Differences.	180° + a or a − 180°.
0	330°		150°
		0	
11°	341°		161°
		25	
37°	7°		187°
		50	
82°	52°		232°
		75	
90°	60°		240°
		75	
− 82	248°		68°
		50	
−− 37	293°		113°
		25	
− 11	319°		139°
		0	
− 0	330°		150°

Which group differences should be marked " + " and which " −," the slide-rule will not disclose. But no trigonometry is wanted, for one can always *see* in practice.

$\theta = \sin^{-1}\frac{y}{a}$ is given by



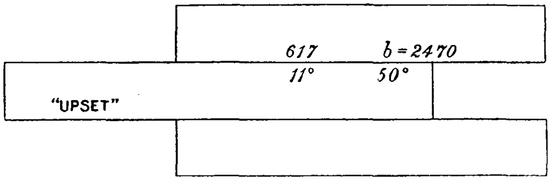
$\theta = \sin^{-1}\frac{a}{y}$ is given by



In a triangle given a, B and C, to find b—

$$b = \frac{a \sin B}{\sin (B + C)}$$

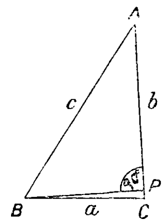
$$\text{suppose } b = \frac{617 \sin 50^\circ}{\sin 11^\circ}$$



The x_s and x_T Scales.

In the triangle ABC, given a, b, C to find c.

Given		Suppose
a	=	420
b	=	601
C	=	70°

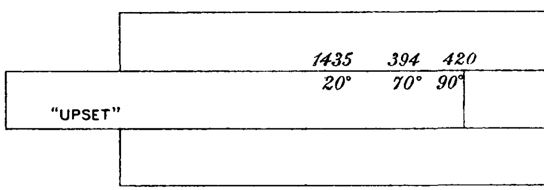


From B draw BP at right angles to AC.

Then

$$BP = 420 \sin 70° = 394$$

$$PC = 420 \sin (90 - 70°) = 143{\cdot}5, \text{ as below.}$$

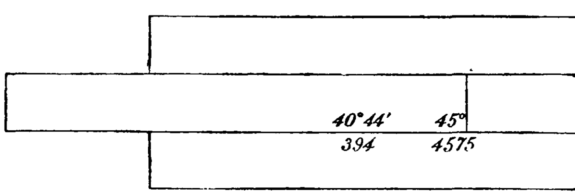


Hence

$$AP = 601 - 143{\cdot}5 = 457{\cdot}5.$$

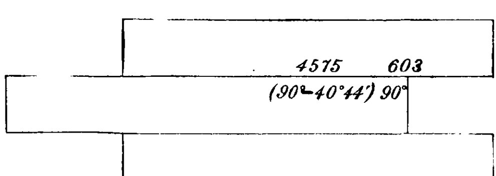
Hence

$$A = \tan^{-1} \frac{394}{457 \cdot 5} = 40° \, 44', \text{ as below.}$$



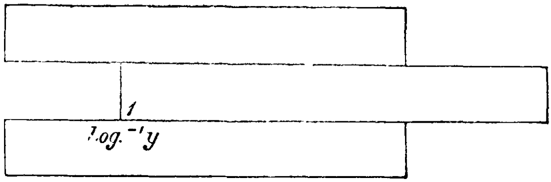
Next,

$$c = \frac{457{\cdot}5}{\cos 40°44'} = \frac{457{\cdot}5}{\sin (90 - 40° \, 44)'} = 603, \text{ as below.}$$



The x_L scale.

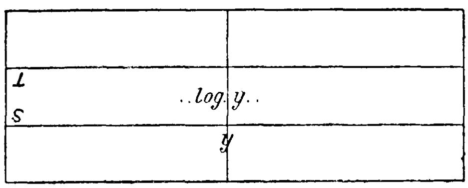
If the slide is pulled out to the right till the index on the back of the right of the ruler reads y on the centre scale of the slide, we have



Thus, failing a logarithm book, the slide-rule can be used to raise numbers to any power.

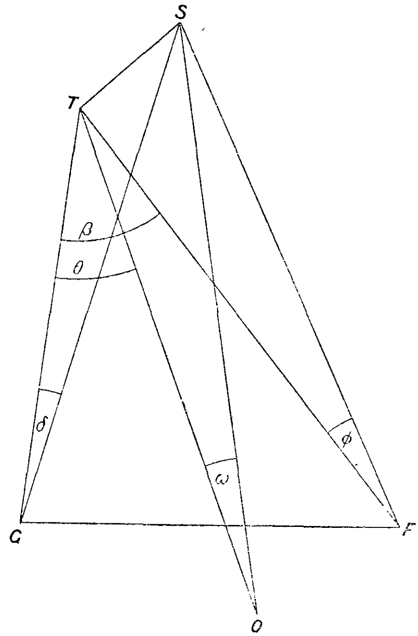
An example of the proper use of x_L scale will be found further on under the heading of "The Chance of Hitting."

If the slide is upset, reversed and run home, we could use the slide-rule and cursor just as if they constituted a table of logarithms, but the latter would be far more convenient—



Correction of Fire from Two Outlying Observing Stations.

The x_s Scale, and subsequently the x_1 Scales.



T = target.

G = gun. S = where the shot fell.

δ = error in direction in minutes, + if to the right of T, – if to the left.

F = the far observing station.

O = the other observing station.

β = GTF, + if to the right of TG, – if to the left.

ϕ = TFS in minutes, + if to the right of FT, – if to the left.

ω = TOS in minutes, + if to the right of OT, – if to the left.

θ = OTG, + if to the right of TG, – if to the left.

e = error of the shot in range in minutes of elevation.

y = number of yards range corresponding to 5 minutes.

(i.) $\delta = \frac{\text{TO}}{\text{TG}} \cdot \frac{\sin \beta}{\sin (\beta - \theta)} \cdot \omega - \frac{\text{TF}}{\text{TG}} \frac{\sin \theta}{\sin (\beta - \theta)} \phi.$

(ii.) $e = \frac{5}{y} \cdot \frac{\text{TF} \sin (90 - \theta)}{3440 \sin (\beta - \theta)} \cdot \phi - \frac{5}{y} \cdot \frac{\text{TO} \sin (90^\circ - \beta)}{3440 \sin (\beta - \theta)} \omega.$

These equations are in a convenient form for using slide-rules, for (i) may be written

$$\delta = \text{constant} \times w - \text{constant} \times \phi\,;$$

while (ii) may be written

$$e = \text{constant} \times \phi - \text{constant} \times w.$$

Thus having, for any given target, worked out the constants, four slide-rules can be kept "set," and corrections can be read off and ordered in a few seconds.

Except in a few cases, no rule has been laid down for pulling the slide *to the right or the left. If the right fails, the left will succeed*, is the best rule.

It must not be understood that in the examples given above there is only one way of working. There are many ways. The best way is that which comes easiest; but it is well to avoid an unnecessary number of settings in ordinary work.

The Chance of Hitting.

$$P = \sqrt{p_1^2 + p_2^2 + p_3^2 + \ldots}$$

where P is the whole probable error due to all causes,
p_1 the probable error due to one cause,
p_2 " " a second cause,
p_3 " " a third cause,
and provided all the errors follow the normal law of error.

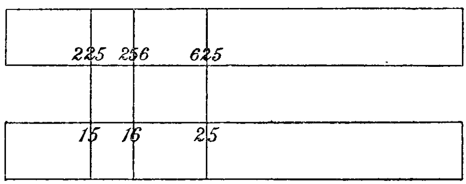
The causes might be

The probable error in ordering the elevation = 25 suppose,
The probable error in laying = 15 "
The gun's probable error = 16 "

Then

$$P = \sqrt{25^2 + 15^2 + 16^2}.$$

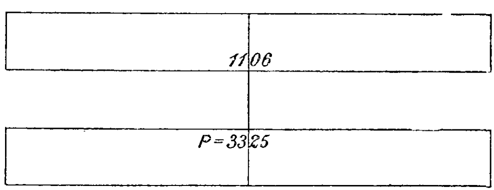
By means of the cursor and rule we have first



Secondly, adding the squares, we have

$$225 + 256 + 625 = 1106.$$

Thirdly, sliding the cursor to 1106, we have

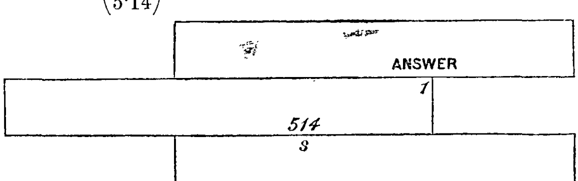


And clearly the answer must be $P = 33{\cdot}25$; for the other square root reads 10·5 or 105, both of which are evidently impossible.

If one of the curves of error is due to the minimum possible correction s, that can be ordered for the sights or fuze.

$$P = \sqrt{p_1^2 + p_2^2 + p_3^2 + \ldots + \left(\frac{s}{5{\cdot}14}\right)^2}.$$

The value of $\left(\frac{s}{5{\cdot}14}\right)^2$ is obtained as below.



The last equation holds good only when

$$s < \sqrt{p_1^2 + p_2^2 + p_3^2 + \ldots}$$

As regards the chances of hitting

$$q^2 = 1 - \log^{-1}\left(-\frac{0{\cdot}03123\ z^2}{P^2}\right).$$

Where q = the chance,

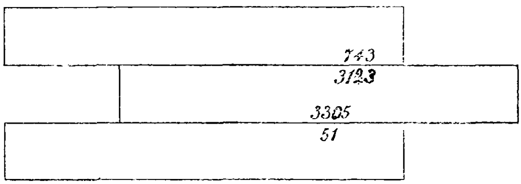
z = the zone,

P = the whole probable error,

putting $z = 51$, $P = 33{\cdot}05$, we have

$$q^2 = 1 - \log^{-1}\left(-\frac{0{\cdot}03123 \times 51^2}{33{\cdot}05^2}\right).$$

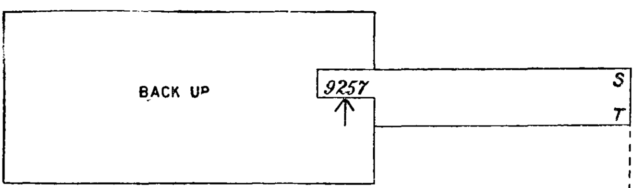
Taking the portion inside the bracket, we have by slide-rule



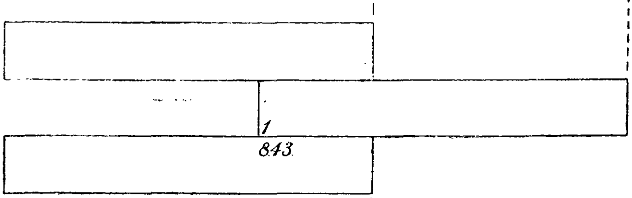
The answer is clearly ·0743. We therefore have

$$q^2 = 1 - \log^{-1}(-{\cdot}0743)$$
$$= 1 - \log^{-1}(\bar{1}{\cdot}9257).$$

Again, taking the portion inside the bracket, we set the slide-rule as below, using the central scale on the back of the slide,



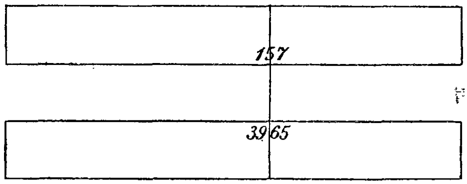
Then, turning the slide-rule right side up, we have



The answer is 0·843. We thus have

$$q^2 = 1 - \cdot 843 = 0 \cdot 157;$$

whence, by the rule and cursor,



Therefore $q = \cdot 3965$.

When Z is less than P, we may simplify the work by writing

$$q = \frac{0 \cdot 269\, Z}{P}.$$

Transposing, we have, if q is under 0·27,

$$Z = \frac{qP}{0 \cdot 269};$$

But if q is over ·27,

$$Z^2 = P^2 \left(\frac{\log (1 - q^2)}{0 \cdot 03123} \right).$$

If it is known for certain that the mean point of impact is at a distance D from the centre of the target T,

$$2q = \sqrt{1 - \log^{-1} \left\{ \frac{0 \cdot 03123\ (2D + T)^2}{P^2} \right\}} - \sqrt{1 - \log^{-1} \left\{ \frac{0 \cdot 03123\ (2D - T)^2}{P^2} \right\}}$$

T, D and P all being measured in the same direction.

But if $4D < P$, we may write the above

$$q^2 = 1 - \log^{-1} \left\{ \frac{0 \cdot 03123 T^2}{P^2 + \left(\frac{4D}{5 \cdot 14} \right)^2} \right\},$$

which is worked out more rapidly.

APPENDIX III.

GUNNERY TABLES.

CONTENTS.

TABLE I.

Values of K for Ogival-headed Projectiles of $1\frac{1}{2}$ diameters for the cubic law of resistance of the air.

(From Supplement *Bashforth's Motion of Projectiles*, 1881.)

v.	K.	v.	K.	v.	K.	v.	K.	v.	K.
f/s.		f/s.		f/s.		f/s.		f/s.	
100	578·1	640	93·5	1180	109·6	1720	81·8	2260	66·5
110	525·5	650	91·1	1190	109·6	1730	81·2	2270	66·4
120	481·7	660	90·5	1200	109·6	1740	80·6	2280	66·2
130	444·7	670	89·1	1210	109·6	1750	80·0	2290	65·9
140	412·9	680	87·7	1220	109·6	1760	79·5	2300	65·5
150	385·4	690	86·3	1230	109·5	1770	78·9	2310	65·0
160	361·3	700	84·9	1240	109·5	1780	78·4	2320	64·4
170	340·1	710	83·7	1250	109·4	1790	77·8	2230	63·8
180	321·2	720	82·6	1260	109·3	1800	77·3	2340	63·2
190	304·3	730	81·6	1270	109·2	1810	76·8	2350	62·6
200	289·0	740	80·6	1280	109·0	1820	76·2	2360	62·0
210	275·3	750	79·6	1290	108·8	1830	75·7	2370	61·4
220	262·8	760	78·7	1300	108·6	1840	75·2	2380	60·8
230	251·3	770	78·0	1310	108·4	1850	74·7	2390	60·2
240	240·9	780	77·4	1320	108·1	1860	74·2	2400	59·6
250	231·2	790	76·8	1330	107·8	1870	73·6	2410	59·0
260	222·4	800	76·2	1340	107·5	1880	73·1	2420	58·4
270	214·1	810	75·6	1350	107·1	1890	72·6	2430	57·8
280	206·5	820	75·2	1360	107·7	1900	72·1	2440	57·2
290	199·3	330	75·1	1370	107·3	1910	71·6	2450	56·7
300	192·7	840	75·0	1380	107·8	1920	71·2	2460	56·2
310	186·5	850	75·0	1390	105·3	1930	70·8	2470	55·7
320	180·8	860	75·0	1400	104·7	1940	70·4	2480	55·2
330	175·5	870	75·0	1410	104·1	1950	70·0	2490	54·8
340	170·6	880	75·0	1420	103·5	1960	69·7	2500	54·4
350	166·0	890	75·0	1430	102·9	1970	69·4	2510	54·0
360	161·9	900	75·0	1440	102·3	1980	69·2	2520	53·7
370	158·0	910	75·0	1450	101·6	1990	69·0	2530	53·4
380	154·4	920	75·0	1460	100·9	2000	68·8	2540	53·1
390	151·1	930	75·0	1470	100·1	2010	68·6	2550	52·9
400	148·0	940	75·0	1480	99·4	2020	68·4	2560	52·7
410	145·2	950	75·0	1490	98·6	2030	68·3	2570	52·6
420	142·5	960	75·0	1500	97·9	2040	68·2	2580	59·5
430	139·8	970	75·0	1510	97·1	2050	68·1	2590	52·5
440	137·2	980	75·0	1520	96·2	2060	68·0	2600	52·4
450	134·6	990	75·0	1530	95·3	2070	67·9	2610	52·4
460	132·0	1000	75·0	1540	94·4	2080	67·9	2620	52·4
470	129·4	1010	75·1	1550	93·6	2090	67·8	2630	52·3
480	126·9	1020	75·3	1560	92·8	2100	67·8	2640	52·3
490	124·4	1030	76·7	1570	92·0	2110	67·7	2650	52·3
500	121·9	1040	80·8	1580	91·2	2120	67·6	2660	52·2
510	119·6	1050	87·3	1590	90·4	2130	67·6	2670	52·2
520	117·3	1060	94·0	1600	89·7	2140	67·5	2680	52·2
530	115·0	1070	98·7	1610	89·0	2150	67·4	2690	52·1
540	112·8	1080	102·2	1620	88·3	2160	67·3	2700	52·1
550	110·7	1090	104·9	1630	87·6	2170	67·2	2710	52·1
560	108·7	1100	106·9	1640	86·9	2180	67·2	2720	52·0
570	106·7	1110	108·4	1650	86·2	2190	67·1	2730	52·0
580	104·6	1120	109·2	1660	85·5	2200	67·0	2740	52·0
590	102·5	1130	109·6	1670	84·8	2210	66·9	2750	52·0
600	100·5	1140	109·6	1680	84·2	2220	66·8	2760	52·0
610	98·6	1150	109·6	1690	83·6	2230	66·8	2770	52·0
620	96·8	1160	109·6	1700	83·0	2240	66·7	2780	52·0
630	95·1	1170	109·6	1710	82·4	2250	66·6	2800	52·0

Table II.

Showing the Resistance of the Air in pounds (p) to a 1-inch Projectile with an Ogival head of $1\frac{1}{2}$ diameters radius under standard conditions of shape, steadiness, and density of air, for velocities from 100 to 2800 f/s.

Calculated by Mr. A. G. Hadcock, late R.A., from Mr. Bashforth's values of K, by the use of the formula

$$p = \frac{K}{g}\left(\frac{v}{1000}\right)^3 = \frac{k}{g}\left(\frac{v}{1000}\right)^2.$$

v	p	v	p	v	p	v	p	v	p
f/s.	lbs.	f/s.	lbs.	f/s.	lbs.	f/s.	lbs.	f/s.	lbs.
100	0·0180	640	0·7615	1180	5·594	1720	12·900	2260	23·848
110	0·0217	650	0·7840	1190	5·738	1730	13·059	2270	24·132
120	0·0259	660	0·8081	1200	5·884	1740	13·191	2280	24·368
130	0·0303	670	0·8325	1210	6·032	1750	13·318	2290	24·583
140	0·0352	680	0·8565	1220	6·183	1760	13·466	2300	24·760
150	0·0404	690	0·8807	1230	6·331	1770	13·591	2310	24·887
160	0·0459	700	0·9048	1240	6·486	1780	13·733	2320	24·987
170	0·0519	710	0·9306	1250	5·637	1790	13·862	2330	25·071
180	0·0582	720	0·9577	1260	6·791	1800	14·002	2340	25·152
190	0·0648	730	0·9861	1270	6·948	1810	14·149	2350	25·242
200	0·0718	740	1·0146	1280	7·101	1820	14·269	2360	25·316
210	0·0792	750	1·0433	1290	7·256	1830	14·414	2370	25·386
220	0·0869	760	1·0733	1300	7·413	1840	14·552	2380	25·467
230	0·0950	770	1·1062	1310	7·569	1850	14·696	2390	25·529
240	0·1035	780	1·1408	1320	7·723	1860	14·832	2400	25·588
250	0·1122	790	1·1764	1330	7·879	1870	14·949	2410	25·658
260	0·1214	800	1·2119	1340	8·034	1880	15·090	2420	25·710
270	0·1310	810	1·248	1350	8·185	1890	15·224	2430	25·772
280	0·1409	820	1·288	1360	8·339	1900	15·354	2440	25·814
290	0·1511	830	1·334	1370	8·490	1910	15·496	2450	25·898
300	0·1616	840	1·381	1380	8·639	1920	15·656	2460	26·008
310	0·1727	850	1·431	1390	8·784	1930	15·809	2470	26·071
320	0·1841	860	1·482	1400	8·924	1940	15·968	2480	26·158
330	0·1959	870	1·534	1410	9·066	1950	16·127	2490	26·276
340	0·2083	880	1·588	1420	9·206	1960	16·302	2500	26·406
350	0·2211	890	1·643	1430	9·349	1970	16·484	2510	26·534
360	0·2346	900	1·699	1440	9·489	1980	16·639	2520	26·709
370	0·2485	910	1·756	1450	9·622	1990	16·838	2530	26·866
380	0·2631	920	1·814	1460	9·753	2000	17·096	2540	27·039
390	0·2784	930	1·874	1470	9·879	2010	17·305	2550	27·243
400	0·2943	940	1·935	1480	10·013	2020	17·515	2560	27·464
410	0·3110	950	1·998	1490	10·133	2030	17·752	2570	27·736
420	0·3280	960	2·061	1500	10·263	2040	17·990	2580	28·010
430	0·3453	970	2·127	1510	10·384	2050	18·229	2590	28·337
440	0·3630	980	2·193	1520	10·493	2060	18·463	2600	28·613
450	0·3810	990	2·261	1530	10·601	2070	18·706	2610	28·945
460	0·3992	1000	2·330	1540	10·712	2080	18·978	2620	29·279
470	0·4174	1010	2·404	1550	10·829	2090	19·227	2630	29·562
480	0·4360	1020	2·482	1560	10·945	2100	19·504	2640	29·899
490	0·4547	1030	2·604	1570	11·060	2110	19·755	2650	30·241
500	0·4734	1040	2·823	1580	11·175	2120	20·010	2660	30·527
510	0·4928	1050	3·139	1590	11·288	2130	20·294	2670	30·873
520	0·5124	1060	3·478	1600	11·416	2140	20·551	2680	31·221
530	0·5318	1070	3·756	1610	11·540	2150	20·811	2690	31·494
540	0·5517	1080	3·999	1620	11·662	2160	21·072	2700	31·846
550	0·5721	1090	4·221	1630	11·784	2170	21·336	2710	32·203
560	0·5931	1100	4·420	1640	11·909	2180	21·633	2720	32·500
570	0·6139	1110	4·605	1650	12·030	2190	21·889	2730	32·859
580	0·6359	1120	4·766	1660	12·150	2200	22·158	2740	33·222
590	0·6539	1130	4·913	1670	12·268	2210	22·429	2750	33·586
600	0·6743	1140	5·041	1680	12·404	2220	22·702	2760	33·955
610	0·6952	1150	5·179	1690	12·536	2230	23·010	2770	34·325
620	0·7166	1160	5·315	1700	12·666	2240	23·288	2780	34·697
630	0·7386	1170	5·454	1710	12·801	2250	23·566	2800	35·453

Table III.

Time t in seconds, between velocities V and v f/s,

$$t = C\,(T_V - T_v).$$

(From Supplement *Bashforth's Motion of Projectiles*, 1881.)

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
10	75·399	77·111	78·790	80·437	82·052	83·636	85·190	86·715	88·212	89·682	1·584
11	91·125	92·542	93·934	95·301	96 644	97·964	99·261	00·536*	01·789*	03·021*	1·320
12	1 04·232	05·423	06·595	07·748	08·883	09·999	11·097	12·178	13·243	14·291	1·116
13	1 15 323	16·339	17·340	18·326	19·297	20·254	21·196	22·124	23·039	23·941	·957
14	24·830	25·706	26·570	27·422	28·262	29·091	29·908	30·714	31·509	32·294	·829
15	33·068	33·832	34·586	35·331	36·066	36·792	37·508	38·215	38·913	39·602	·726
16	1 40·283	40·955	41·618	42·273	42·920	43·559	44·190	44·813	45·429	46·038	·639
17	46·640	47·235	47·823	48·404	48·978	49·546	50·107	50·662	51·211	51·754	·568
18	52·291	52·822	53·347	53·867	54·381	54·890	55·393	55·890	56·382	56·869	·509
19	1 57·351	57·828	58·300	58·767	59·229	59·686	60·138	60·586	61·029	61·468	·457
20	61·902	62·332	62·758	63·180	63·598	64·012	64·422	64·828	65·230	65 628	·414
21	66·022	66·412	66·798	67·181	67·560	67·936	68·308	68·676	69·041	69·403	·376
22	1 69·762	70·118	70·470	70·819	71·165	71·508	71·848	72·185	72·519	72·850	·343
23	73·179	73·505	73·828	74·148	74·465	74·780	75·092	75·401	75·708	76·012	·315
24	76·314	76·613	76·909	77·203	77·494	77·783	78·070	78·354	78·636	78·916	·289
25	1 79·194	79·470	79·743	80·014	80·283	80·550	80·815	81·078	81·339	81·598	·267
26	81·855	82·110	82·363	82·614	82·863	83·110	83·355	83·598	83·839	84·079	·247
27	84·317	84·553	84·787	85·020	85·251	85·481	85·709	85·935	86·160	86·382	·230
28	1 86·604	86·824	87·042	87·259	87·474	87·688	87·900	88·111	88·320	88·528	·214
29	88·734	88 939	89·143	89·345	89·546	89·745	89·943	90·140	90·335	90·529	·199
30	90·721	90·912	91·102	91·291	91·478	91·664	91·849	92·033	92·216	92·397	·186
31	1 92·577	92·756	92·934	93·111	93·287	93·461	93·634	93·806	93·971	94·147	·174
32	94·316	94·484	94·651	94·817	94·982	95·146	95·309	95·471	95·632	95·792	·164
33	95·951	96·109	96·266	96·422	96·577	96·731	96·884	97·036	97·187	97·338	·154
34	1 97·488	97·637	97·785	97·932	98·078	98·223	98·367	98·510	98·652	98·794	·145
35	98·935	99·075	99·214	99·352	99·490	99·627	99·763	99·898	00·032*	00·166*	·137
36	2 00·299	00·431	00·562	00·692	00·822	00·951	01·079	01·206	01·333	01·459	·129
37	2 01·585	01·710	01·834	01·957	02·080	02·202	02·323	02·443	02·563	02·682	·122
38	02 801	02·919	03·336	03·152	03·268	03·383	03·497	03·610	03·723	03·835	·115
39	03 947	04·058	04·168	04·278	04·387	04·496	04·604	04·711	04·818	04·924	·109
40	20 5·0299	5·1349	5·2393	5·3432	5·4466	5·5494	5·6517	5·7534	5·8546	5·9553	·1028
41	6·0554	6·1550	6·2540	6·3525	6 4505	6·5480	6·6450	6·7414	6·8373	6·9327	·0975
42	7·0276	7·1220	7·2159	7·3093	7·4022	7·4947	7·5867	7·6782	7·7693	7·8599	·0925
43	20 7·9501	8·0398	8·1291	8·2179	8·3063	8·3942	8·4817	8·5687	8·6553	8·7415	·0879
44	8·8272	8·9125	8·9974	9·0819	9·1660	9·2497	9·3330	9·4159	9·4984	9·5805	·0837
45	9·6622	9·7435	9·8244	9·9050	9·9852	0·0651*	0·1446*	*0·2237	0·3025*	0·3809*	·0799
46	21 0·4590	0·5367	0·6140	0·6910	0·7677	0·8440	0·9200	0·9956	1·0709	1·1459	·0763
47	1·2205	1·2948	1·3687	1·4423	1·5156	1·5886	1·6613	1·7336	1·8056	1·8773	·0730
48	1·9487	2·0198	2·0906	2·1611	2·2313	2·3012	2·3708	2·4401	2·5091	2·5779	·0699
49	21 2·6464	2·7146	2·7825	2·8501	2·9174	2·9845	3·0513	3·1178	3·1841	3·2501	·0671
50	3·3159	3·3814	3·4466	3·5116	3·5763	3·6608	3·7050	3 7689	3·8326	3·8960	·0645
51	3·9592	4·0221	4·0848	4·1472	4·2094	4·2713	4·3330	4·3944	4·4556	4·5165	·0619
52	21 4·5772	4·6377	4·6979	4·7579	4·8177	4·8773	4·9367	4·9958	5·0547	5·1134	·0596
53	5·1719	5·2302	5·2882	5·3460	5·4036	5·4610	5·5182	5·5752	5·6320	5·6886	·0574
54	5·7450	5·8012	5·8572	5·9130	5·9686	6·0240	6·0792	6·1342	6·1890	6·2436	·0554
55	21 6·2980	6·3522	6·4062	6·4600	6·5136	6·5670	6·6202	6·6732	6 7260	6·7786	·0534
56	6·8311	6·8834	6·9355	6·9874	7·0391	7·0907	7·1421	7·1933	7·2444	7·2953	·0516
57	7·3460	7·3965	7·4469	7·4971	7·5471	7·5970	7·6467	7·6962	7·7456	7·7948	·0499
58	21 7·8433	7·8928	7·9417	7·99[illegible]4	8·0389	8·0873	8·1356	8·1837	8·2316	8·2793	·0483
59	8·3271	8·3746	8·4220	8·4692	8·5163	8·5632	8·6100	8 6566	8·7031	8·7494	·0468
60	8·7957	8·8417	8·8877	8·9334	8·9791	9·0246	9·0700	9·1152	9·1603	9·2052	·0454
61	21 9·2501	9·2947	9·3393	9·3837	9·4280	9·4721	9·5161	9·5600	9·6037	9·6473	·0441
62	9·6908	9·7341	9·7773	9·8204	9·8633	9·9062	9·9489	9·9914	*0·0338	0·0761*	·0428
63	22 0·1183	0·1604	0·2023	0·2441	0·2858	0·3273	0·3687	0·4100	0·4512	0·4922	·0415

Table III—*continued*.

$$t = C\,(T_V - T_v).$$

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
64	22 0·5332	0·5740	0·6147	0·6552	0·6957	0 7360	0·7762	0·8163	0·8563	0·8962	·040
65	0·9359	0·9755	1·0151	1·0544	1·0937	1·1328	1·1718	1·2107	1·2495	1·2881	·039
66	1·3267	1·3651	1·4034	1 4416	1·4797	1·5177	1·5555	1·5933	1·6309	1·6684	·037
67	22 1·7059	1·7432	1·7804	1·8175	1·8545	1·8914	1·9281	1·9648	2·0014	2·0378	·036
68	2·0742	2·1105	2·1466	2·1827	2·2186	2·2545	2·2902	2·3259	2·3614	2 3969	·035
69	2·4322	2·4675	2·5027	2·5377	2·5727	2·6076	2·6424	2·6771	2·7117	2·7462	·034
70	22 2·7806	2·8150	2·8492	2·8833	2·9174	2·9513	2·9852	3·0189	3·0526	3·0862	·033
71	3·1196	3·1530	3·1863	3·2195	3·2526	3·2856	3·3185	3·3513	3·3840	3·4167	·033
72	3·4492	3·4816	3·5140	3·5462	3·5784	3·6105	3·6424	3·6743	3·7061	3·7378	·032
73	22 3·7694	3·8009	3·8323	3·8636	3 8949	3·9260	3·9571	3·9881	4·0189	4·0497	·031
74	4·0804	4·1110	4·1416	4·1720	4·2024	4·2326	4·2628	4·2929	4·3230	4·3529	·030
75	4·3828	4·4125	4·4422	4·4719	4·5014	4·5308	4·5602	4·5895	4·6187	4·6478	·029
76	22 4·6769	4·7058	4·7347	4 7635	4·7922	4·8208	4·8493	4·8777	4·9060	4·9343	·028
77	4·9624	4 9905	5·0185	5·0464	5·0742	5·1020	5·1296	5·1572	5·1847	5·2121	·027
78	5·2394	5 2666	5·2937	5·3208	5·3478	5·3747	5·4015	5·4282	5·4549	5·4814	·026
79	22 5·5079	5·5343	5·5606	5·5869	5·6130	5·6391	5·6652	5·6911	5·7170	5·7428	·026
80	5·7685	5·7941	5·8197	5·8452	5·8706	5·8959	5·9212	5·9463	5·9714	5·9965	·025
81	6·0214	6·0463	6·0711	6·0959	6·1205	6·1451	6·1696	6·1941	6·2184	6·2427	·024
82	22 6·2669	6·2910	6·3151	6·3390	6·3629	6·3867	6·4104	6·4340	6·4576	6·4810	·0237
83	6·5044	6·5277	6·5509	6·5740	6·5971	6·6201	6·6430	6·6658	6·6885	6·7111	·022
84	6·7337	6·7562	6·7786	6·8009	6·8232	6·8454	6·8675	6·8895	6·9114	6·9333	·0221
85	22 6·9551	6·9768	6·9984	7·0200	7·0415	7·0629	7·0842	7·1055	7·1267	7·1478	·0214
86	7·1688	7·1898	7·2107	7·2315	7·2522	7·2729	7·2935	7·3140	7·3345	7·3549	·0206
87	7·3752	7·3954	7·4156	7·4357	7·4558	7·4757	7·4956	7·5155	7·5353	7·5550	·0199
88	22 7·5746	7·5942	7·6137	7 6332	7·6526	7·6719	7·6912	7·7104	7·7295	7·7486	·0193
89	7·7677	7·7866	7·8055	7·8244	7·8431	7·8618	7·8805	7·8991	7·9176	7·9360	·0187
90	7·9544	7·9727	7·9909	8·0091	8·0272	8·0452	8·0632	8·0812	8·0990	8·1168	·0180
91	22 8 1346	8·1523	8·1699	8·1875	8·2050	8·2225	8·2399	8·2573	8·2746	8·2918	·0174
92	8·3090	8·3261	8·3432	8·3602	8·3772	8·3941	8·41 9	8·4277	8·4445	8·4611	·0169
93	8·4778	8·4943	8 5109	8·5273	8·5437	8·5601	8·5764	8·5927	8·6089	8·6250	·0163
94	22 8·6411	8·6572	8·6732	8·6892	8·7051	8·7209	8·7367	8·7525	8·7682	8·7838	·0158
95	8·7994	8·8150	8·8305	8·8459	8·8613	8·8767	8·8920	8·9073	8·9225	8·9376	·0153
96	8·9528	8·9678	8·9828	8·9978	9·0128	9·0276	9·0425	9·0573	9 0720	9·0867	·0149
97	22 9·1014	9·1160	9·1306	9·1451	9·1595	9·1740	9·1884	9·2027	9·2170	9·2312	·0144
98	9·2454	9·2596	9·2737	9·2878	9·3018	9·3158	9·3298	9·3437	9·3575	9·3713	·0140
99	9·3851	9·3989	9·4126	9·4262	9·4398	9·4534	9·4670	9·4805	9·4939	9·5073	·0136
100	22 9·5207	9·5340	9·5473	9·5606	9·5738	9·5869	9·6001	9·6132	9·6262	9·6392	·0132
101	9·6522	9·6651	9·6780	9·6908	9·7036	9·7164	9·7291	9·7418	9·7544	9·7670	·0127
102	9·7796	9·7921	9·8046	9·8170	9·8294	9·8417	9·8540	9·8662	9·8783	9·8904	·0123
103	22 9·9024	9·9144	9·9262	9·9380	9·9496	9·9612	9·9727	9·9841	9·9954	0·0066*	·0115
104	23 0·0177	0·0287	0·0396	0·0504	0·0610	0·0716	0·0820	0·0923	0·1025	0·1126	·0105
105	0·1226	0·1325	0·1423	0·1520	0·1615	0·1710	0·1804	0·1897	0·1988	0·2079	·0094
106	23 0·2170	0·2259	0·2347	0·2435	0·2522	0·2609	0·2694	0·2780	0·2864	0·2948	·0086
107	0·3031	0·3114	0·3196	0·3278	0·3359	0·3439	0·3520	0·3599	0·3678	0·3757	·0080
108	0·3835	0·3913	0·3990	0·4067	0·4143	0·4219	0·4295	0·4370	0·4445	0·4519	·0076
109	23 0·4593	0·4667	0·4740	0·4813	0·4885	0·4958	0·5030	0·5101	0·5172	0·5243	·0072
110	0·5314	0·5384	0·5454	0·5524	0·5593	0·5662	0·5731	0·5800	0·5868	0·5936	·0069
111	0·6004	0·6071	0·6139	0·6206	0·6272	0·6339	0·6405	0·6471	0·6537	0·6603	·0066
112	23 0·6668	0·6733	0·6798	0·6863	0·6928	0·6992	0·7056	0·7120	0·7184	0·7248	·0064
113	0·7311	0·7374	0·7437	0·7500	0·7563	0·7625	0·7688	0·7750	0·7812	0·7874	·0062
114	0·7936	0·7997	0·8059	0·8120	0·8181	0·8242	0·8303	0·8364	0·8424	0·8484	·0061
115	23 0·8545	0·8605	0·8665	0·8726	0·8787	0·8847	0·8906	0·8965	0·9024	0·9083	·0059
116	0·9142	0·9200	0·9259	0·9317	0 9375	0·9433	0·9490	0 9548	0·9605	0·9663	·0058
117	0·9720	0·9777	0·9833	0·9890	0·9947	1·0003	1·0059	1·0115	1·0171	1·0227	·0056
118	23 1·0283	1·0338	1·0394	1·0449	1·0504	1·0559	1·0614	1·0669	1·0723	1·0778	·0055
119	1·0832	1·0886	1·0940	1·0994	1·1048	1·1101	1·1154	1·1208	1·1261	1·1314	·0054
120	1·1367	1·1420	1·1473	1·1525	1·1578	1·1630	1·1682	1·1734	1·1786	1·1838	·0052
121	23 1·1889	1·1941	1·1992	1·2043	1·2095	1·2146	1·2196	1 2247	1·2298	1·2348	·0051
122	1·2399	1·2449	1·2499	1·2549	1·2599	1·2649	1·2698	1·2748	1·2797	1·2847	·0050
123	1·2896	1·2945	1·2994	1·3043	1·3091	1·3140	1·3188	1 3237	1·3285	1·3333	·0049

TABLE III—*continued.*

$$t = C\,(T_{P} - T_{v}).$$

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f 's.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
124	23 1·3381	1·3429	1·3477	1·3524	1·3572	1·3619	1·3667	1·3714	1·3761	1·3808	·0047
125	1·3855	1·3902	1·3948	1·3995	1·4041	1·4088	1·4134	1·4180	1·4226	1·4272	·0046
126	1·4318	1·4364	1·4410	1·4455	1·4501	1·4546	1·4591	1·4636	1·4681	1·4726	·0045
127	23 1·4771	1·4816	1·4860	1·4905	1·4949	1·4993	1·5038	1·5082	1·5126	1·5170	·0044
128	1·5214	1·5257	1·5301	1·5345	1·5388	1·5431	1·5475	1·5518	1·5561	1·5604	·0043
129	1·5647	1·5690	1·5732	1·5775	1·5818	1·5860	1·5902	1·5945	1·5987	1·6029	·0042
130	23 1·6071	1·6113	1·6155	1·6196	1·6238	1·6280	1·6321	1·6362	1·6404	1·6445	·0042
131	1·6486	1·6527	1·6568	1·6609	1·6650	1·6690	1·6731	1·6772	1·6812	1·6852	·0041
132	1·6893	1·6933	1·6973	1·7013	1·7053	1·7093	1·7133	1·7173	1·7212	1·7252	·0040
133	23 1·7291	1·7331	1·7370	1·7410	1·7449	1·7488	1·7527	1·7566	1·7605	1·7644	·0039
134	1·7682	1·7721	1·7760	1·7798	1·7837	1·7875	1·7913	1·7952	1·7990	1·8028	·0038
135	1·8066	1·8104	1·8142	1·8179	1·8217	1·8255	1·8292	1·8330	1·8367	1·8405	·0038
136	23 1·8442	1·8479	1·8517	1·8554	1·8591	1·8628	1·8665	1·8702	1·8738	1·8775	·0037
137	1·8812	1·8848	1·8885	1·8921	1·8958	1·8994	1·9030	1·9067	1·9103	1·9139	·0036
138	1·9175	1·9211	1·9247	1·9282	1·9318	1·9354	1·9390	1·9425	1·9461	1·9496	·0036
139	23 1·9532	1·9567	1·9602	1·9638	1·9673	1·9708	1·9743	1·9778	1·9813	1·9848	·0035
140	1·9883	1·9918	1·9952	1·9987	2·0022	2·0056	2·0091	2·0125	2·0160	2·0194	·0035
141	2·0228	2·0263	2·0297	2·0331	2·0365	2·0399	2·0433	2·0467	2·0501	2·0535	·0034
142	23 2·0569	2·0602	2·0636	2·0670	2·0703	2·0737	2·0770	2·0804	2·0837	2·0870	·0034
143	2·0904	2·0937	2·0970	2·1003	2·1036	2·1069	2·1102	2·1135	2·1168	2·1201	·0033
144	2·1234	2·1267	2·1299	2·1332	2·1364	2·1397	2·1430	2·1462	2·1494	2·1527	·0033
145	23 2·1559	2·1591	2·1624	2·1656	2·1688	2·1720	2·1752	2·1784	2·1816	2·1848	·0032
146	2·1880	2·1912	2·1944	2·1975	2·2007	2·2039	2·2071	2·2102	2·2134	2·2165	·0032
147	2·2197	2·2228	2·2260	2·2291	2·2322	2·2354	2·2385	2·2416	2·2447	2·2478	·0031
148	23 2·2509	2·2540	2·2571	2·2602	2·2633	2·2664	2·2695	2·2726	2·2757	2·2787	·0031
149	2·2818	2·2849	2·2879	2·2910	2·2940	2·2971	2·3001	2·3032	2·3062	2·3093	·0030
150	2·3123	2·3153	2·3183	2·3214	2·3244	2·3274	2·3304	2·3334	2·3364	2·3394	·0030
151	23 2·3424	2·3454	2·3484	2·3514	2·3543	2·3573	2·3603	2·3633	2·3662	2·3692	·0030
152	2·3722	2·3751	2·3781	2·3810	2·3840	2·3869	2·3899	2·3928	2·3958	2·3987	·0029
153	2·4016	2·4046	2·4075	2·4104	2·4133	2·4162	2·4192	2·4221	2·4250	2·4279	·0029
154	23 2·4308	2·4337	2·4366	2·4395	2·4424	2·4453	2·4481	2·4510	2·4539	2·4568	·0029
155	2·4597	2·4625	2·4654	2·4683	2·4711	2·4740	2·4768	2·4797	2·4825	2·4854	·0029
156	2·4882	2·4911	2·4939	2·4967	2·4996	2·5024	2·5052	2·5080	2·5108	2·5137	·0028
157	23 2·5165	2·5193	2·5221	2·5249	2·5277	2·5305	2·5333	2·5361	2·5389	2·5416	·0028
158	2·5444	2·5472	2·5500	2·5528	2·5555	2·5583	2·5611	2·5638	2·5666	2·5693	·0028
159	2·5721	2·5748	2·5776	2·5803	2·5831	2·5858	2·5885	2·5913	2·5940	2·5967	·0027
160	23 2·5994	2·6022	2·6049	2·6076	2·6103	2·6130	2·6157	2·6184	2·6211	2·6238	·0027
161	2·6265	2·6292	2·6319	2·6346	2·6373	2·6400	2·6426	2·6453	2·6480	2·6506	·0027
162	2·6533	2·6560	2·6586	2·6613	2·6640	2·6666	2·6693	2·6719	2·6745	2·6772	·0026
163	23 2·6798	2·6825	2·6851	2·6877	2·6903	2·6930	2·6956	2·6982	2·7008	2·7034	·0026
164	2·7061	2·7087	2·7113	2·7139	2·7165	2·7191	2·7217	2·7243	2·7268	2·7294	·0026
165	2·7320	2·7346	2·7372	2·7398	2·7423	2·7449	2·7475	2·7500	2·7526	2·7552	·0026
166	23 2·7577	2·7603	2·7628	2·7654	2·7679	2·7705	2·7730	2·7756	2·7781	2·7806	·0025
167	2·7832	2·7857	2·7882	2·7908	2·7933	2·7958	2·7983	2·8008	2·8034	2·8059	·0025
168	2·8084	2·8109	2·8134	2·8159	2·8184	2·8209	2·8234	2·8258	2·8283	2·8308	·0025
169	23 2·8333	2·8358	2·8383	2·8407	2·8432	2·8457	2·8481	2·8506	2·8531	2·8555	·0025
170	2·8580	2·8604	2·8629	2·8653	2·8678	2·8702	2·8726	2·8751	2·8775	2·8799	·0024
171	2·8824	2·8848	2·8872	2·8896	2·8921	2·8945	2·8969	2·8993	2·9017	2·9041	·0024
172	23 2·9065	2·9089	2·9113	2·9137	2·9161	2·9185	2·9209	2·9233	2·9257	2·9281	·0024
173	2·9304	2·9328	2·9352	2·9376	2·9399	2·9423	2·9447	2·9470	2·9494	2·9518	·0024
174	2·9541	2·9565	2·9588	2·9612	2·9635	2·9659	2·9682	2·9705	2·9729	2·9752	·0023
175	23 2·9776	2·9799	2·9822	2·9845	2·9869	2·9892	2·9915	2·9938	2·9961	2·9985	·0023
176	3·0008	3·0031	3·0054	3·0077	3·0100	3.0123	3·0146	3·0169	3·0192	3·0215	·0023
177	3·0237	3·0260	3·0283	3·0306	3·0329	3·0351	3·0374	3·0397	3·0420	3·0442	·0023
178	23 3·0465	3·0488	3·0510	3·0533	3·0555	3·0578	3·0600	3·0623	3·0645	3·0668	·0023
179	3·0690	3·0713	3·0735	3·0757	3·0780	3·0802	3·0824	3·0847	3·0869	3·0891	·0022
180	3·0913	3·0935	3·0958	3·0980	3·1002	3·1024	3·1045	3·1068	3·1090	3·1112	·0022
181	23 3·1134	3·1156	3·1178	3·1200	3·1222	3·1244	3·1266	3·1287	3·1309	3·1331	·0022
182	3·1353	3·1375	3·1396	3·1418	3·1440	3·1461	3·1483	3·1505	3·1526	3·1548	·0022
183	3·1569	3·1591	3·1613	3·1634	3·1656	3·1677	3·1698	3·1720	3·1741	3·1763	·0021

TABLE III—*continued.*

$t = C\,(T_V - T_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
184	23 3·1784	3·1805	3·1827	3·1848	3·1869	3·1891	3·1912	3·1933	3·1954	3·1975	·0021
185	3·1997	3·2018	3·2039	3·2060	3·2081	3·2102	3·2123	3·2144	3·2165	3·2186	·0021
186	3·2207	3·2228	3·2249	3·2270	3·2291	3·2312	3·2333	3·2353	3·2374	3·2395	·0021
187	23 3·2416	3·2437	3·2457	3·2478	3·2499	3·2520	3·2540	3·2561	3·2582	3·2602	·0021
188	3·2623	3·2643	3·2664	3·2685	3·2705	3·2726	3·2746	3·2767	3·2787	3·2808	·0021
189	3·2828	3·2848	3·2869	3·2889	3·2909	3·2930	3·2950	3·2970	3·2991	3·3011	·0020
190	23 3·3031	3·3051	3·3072	3·3092	3·3112	3·3132	3·3152	3·3172	3·3192	3·3212	·0020
191	3·3233	3·3253	3 3273	3·3293	3·3313	3·3333	3·3353	3·3372	3·3392	3·3412	·0020
192	3·3432	3·3452	3·3472	3·3492	3·3511	3·3531	3·3551	3·3571	3·3590	3·3610	·0020
193	23 3·3630	3·3649	3·3669	3·3689	3·3708	3·3728	3·3747	3·3767	3·3786	3·3806	·0020
194	3·3825	3·3845	3·3864	3·3884	3·3903	3·3922	3·3942	3·3961	3·3980	3·4000	·0019
195	3·4019	3·4038	3·4057	3·4077	3·4096	3·4115	3·4134	3·4153	3·4172	3·4192	·0019
196	23 3·4211	3·4230	3·4249	3·4268	3·4287	3·4306	3·4325	3·4344	3·4362	3·4381	·0019
197	3·4400	3·4419	3·4438	3·4457	3·4476	3·4494	3·4513	3·4532	3·4550	3·4569	·0019
198	3·4588	3·4606	3·4625	3·4644	3·4662	3·4681	3·4699	3·4718	3·4736	3·4755	·0019
199	23 3·4773	3·4791	3·4810	3·4828	3·4846	3·4865	3·4883	3·4901	3·4920	3·4938	·0018
200	3·4956	3·4974	3·4992	3·5010	3·5028	3·5047	3·5065	3·5083	3·5101	3·5119	·0018
201	3·5137	3·5155	3·5172	3·5190	3·5208	3·5226	3·5244	3·5262	3·5280	3·5297	·0018
202	23 3·5315	3·5333	3·5351	3·5368	3·5386	3·5404	3·5421	3·5439	3·5456	3·5474	·0018
203	3·5492	3·5509	3·5527	3·5544	3·5561	3·5579	3·5596	3·5614	3·5631	3·5648	·0017
204	3·5666	3·5683	3·5700	3·5717	3·5735	3·5752	3·5769	3·5786	3·5803	3·5820	·0017
205	23 3·5837	3·5854	3·5871	3·5888	3·5905	3·5922	3·5939	3·5956	3·5973	3·5990	·0017
206	3·6007	3·6024	3·6040	3·6057	3·6074	3·6091	3·6107	3·6124	3·6141	3·6157	·0017
207	3·6174	3·6191	3·6207	3·6224	3·6240	3·6257	3·6273	3·6290	3·6306	3·6323	·0016
208	23 3·6339	3·6355	3·6372	3·6388	3·6404	3·6420	3·6437	3·6453	3·6469	3·6485	·0016
209	3·6502	3·6518	3·6534	3·6550	3 6566	3·6582	3·6598	3·6614	3·6630	3·6646	·0016
210	3·6662	3·6678	3·6694	3·6710	3·6726	3·6741	3·6757	3·6773	3·6789	3·6805	·0016
211	23 3·6820	3·6836	3·6852	3·6867	3·6883	3·6899	3·6914	3·6930	3·6946	3·6961	·0016
212	3·6977	3·6992	3·7008	3·7023	3·7039	3·7054	3·7070	3·7085	3·7100	3·7116	·0015
213	3·7131	3·7146	3·7162	3·7177	3·7192	3·7207	3·7223	3·7238	3·7253	3·7268	·0015
214	23 3·7283	3·7298	3·7313	3·7329	3·7344	3·7359	3·7374	3·7389	3·7404	3·7419	·0015
215	3·7434	3·7448	3·7463	3·7478	3·7493	3·7508	3·7523	3·7538	3·7552	3·7567	·0015
216	3·7582	3·7597	3·7612	3·7626	3·7641	3·7656	3·7670	3·7685	3·7700	3·7714	·0015
217	23 3·7729	3·7743	3·7758	3·7772	3·7787	3·7801	3·7816	3·7830	3·7845	3·7859	·0014
218	3·7874	3·7888	3·7902	3·7917	3·7931	3·7945	3·7960	3·7974	3·7988	3·8002	·0014
219	3·8016	3·8031	3 8045	3·8059	3·8073	3·8087	3·8101	3·8115	3·8129	3·8144	·0014
220	23 3·8158	3·8172	3·8186	3·8200	3·8214	3·8227	3·8241	3·8255	3·8269	3·8283	0014
221	3·8297	3·8311	3·8325	3·8338	3·8352	3·8366	3·8380	3·8394	3·8407	3·8421	·0014
222	3·8435	3·8448	3·8462	3·8476	3·8489	3·8502	3·8517	3·8530	3·8544	3·8557	·0014
223	23 3·8571	3·8584	3·8598	3·8611	3·8625	3 8638	3·8651	3·8665	3·8678	3·8692	·0013
224	3·8705	3·8718	3·8732	3·8745	3·8758	3·8772	3·8785	3·8798	3·8811	3·8824	·0013
225	3·8838	3·8851	3·8864	3·8877	3·8890	3·8903	3·8916	3·8930	3·8943	3·8956	·0013
226	23 3·8969	3·8982	3·8995	3·9008	3·9021	3·9034	3·9047	3·9059	3·9072	3·9085	·0013
227	3·9098	3·9111	3·9124	3·9137	3·9150	3·9162	3·9175	3·9188	3·9201	3·9214	·0013
228	3·9226	3·9239	3·9252	3·9264	3·9277	3·9290	3·9303	3·9315	3·9328	3·9341	·0013
229	23 3·9353	3·9366	3·9378	3·9391	3·9404	3·9416	3·9429	3·9441	3·9454	3·9467	·0013
230	3·9479	3·9492	3·9504	3·9517	3·9529	3·9542	3·9554	3·9567	3·9579	3·9592	·0013
231	3·9604	3·9617	3·9629	3·9642	3·9654	3·9667	3·9679	3·9692	3·9704	3·9716	·0012
232	23 3·9729	3·9741	3·9754	3·9766	3·9779	3·9791	3·9803	3·9816	3·9828	3·9841	·0012
233	3·9853	3·9866	3·9878	3·9890	3·9903	3·9915	3·9927	3·9940	3·9952	3·9965	·0012
234	3·9977	3·9989	4·0002	4·0014	4·0026	4·0039	4·0051	4·0063	4·0076	4·0088	·0012
235	23 4·0100	4·0113	4·0125	4·0137	4·0150	4·0162	4·0174	4·0186	4·0199	4·0211	·0012
236	4·0223	4·0236	4·0248	4·0260	4·0272	4·0284	4·0297	4·0309	4·0321	4·0334	·0012
237	4·0346	4·0358	4·0370	4·0383	4·0395	4·0407	4·0419	4·0431	4·0444	4·0456	·0012
238	23 4·0468	4·0480	4·0492	4·0505	4·0517	4·0529	4·0541	4·0553	4·0566	4·0578	·0012
239	4·0590	4·0602	4·0614	4·0626	4·0639	4·0651	4·0663	4·0675	4·0687	4·0699	·0012
240	4·0711	4·0724	4·0736	4·0748	4·0760	4·0772	4·0784	4·0796	4·0809	4·0821	·0012
241	23 4·0833	4·0845	4·0857	4·0869	4·0881	4·0893	4·0905	4·0917	4·0930	4·0042	·0012
242	4·0954	4·0966	4·0978	4·0990	4·1002	4·1014	4·1026	4·1038	4·1050	4·1062	·0012
243	4·1074	4·1087	4·1099	4·1111	4·1123	4·1135	4·1147	4·1159	4·1171	4·1183	·0012

TABLE III—*continued.*

$t = C\,(T_V - T_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
244	23 4·1195	4·1207	4·1219	4·1231	4·1243	4·1255	4·1267	4·1279	4·1291	4·0303	·0012
245	4·1315	4·1327	4·1339	4·1351	4·1363	4·1375	4·1387	4·1399	4·1411	4·1423	·0012
246	4·1435	4·1447	4·1459	4·1471	4·1483	4·1495	4·1506	4·1518	4·1530	4·1542	·0012
247	23 4·1554	4·1566	4·1578	4·1590	4·1602	4·1614	4·1626	4·1638	4·1649	4·1661	·0012
248	4·1673	4·1685	4·1697	4·1709	4·1721	4·1733	4·1744	4·1756	4·1768	4·1780	·0012
249	4·1792	4·1804	4·1815	4·1827	4·1839	4·1851	4·1863	4·1874	4·1886	4·1898	·0012
250	23 4·1910	4·1922	4·1933	4·1945	4·1957	4·1969	4·1980	4·1992	4·2004	4·2015	·0012
251	4·2027	4·2039	4·2051	4·2062	4·2074	4·2086	4·2097	4·2109	4·2121	4·2132	·0012
252	4·2144	4·2156	4·2167	4·2179	4·2190	4·2202	4·2214	4·2225	4·2237	4·2248	·0012
253	23 4·2260	4·2272	4·2283	4·2295	4·2306	4·2318	4·2329	4·2341	4·2352	4·2364	·0012
254	4·2375	4·2387	4·2398	4·2410	4·2421	4·2433	4·2444	4·2455	4·2467	4·2478	·0011
255	4·2490	4·2501	4·2513	4·2524	4·2535	4·2547	4·2558	4·2569	4·2581	4·2592	·0011
256	23 4·2603	4·2615	4·2626	4·2637	4·2648	4·2660	4·2671	4·2682	4·2693	4·2705	·0011
257	4·2716	4·2727	4·2738	4·2749	4·2760	4·2772	4·2783	4·2794	4·2805	4·2816	·0011
258	4·2827	4·2838	4·2849	4·2860	4·2871	4·2882	4·2893	4·2904	4·2915	4·2926	·0011
259	23 4·2937	4·2948	4·2959	4·2970	4·2981	4·2992	4·3003	4·3014	4·3025	4·3036	·0011
260	4·3046	4·3057	4·3068	4·3079	4·3090	4·3101	4·3111	4·3122	4·3133	4·3144	·0011
261	4·3154	4·3165	4·3176	4·3187	4·3197	4·3208	4·3219	4·3229	4·3240	4·3250	·0011
262	23 4·3261	4·3272	4·3282	4·3293	4·3303	4·3314	4·3325	4·3335	4·3346	4·3356	·0011
263	4·3367	4·3377	4·3388	4·3398	4·3409	4·3419	4·3429	4·3440	4·3450	4·3461	·0010
264	4·3471	4·3482	4·3492	4·3502	4·3513	4·3523	4·3533	4·3544	4·3554	4·3564	·0010
265	23 4·3574	4·3585	4·3595	4·3605	4·3615	4·3626	4·3636	4·3646	4·3656	4·3667	·0010
266	4·3677	4·3687	4·3697	4·3707	4·3717	4·3728	4·3738	4·3748	4·3758	4·3768	·0010
267	4·3778	4·3788	4·3798	4·3808	4·3818	4·3828	4·3838	4·3848	4·3858	4·3868	·0010
268	23 4·3878	4·3888	4·3898	4·3908	4·3918	4·3928	4·3938	4·3948	4·3958	4·3968	·0010
269	4·3977	4·3987	4·3997	4·4007	4·4017	4·4027	4·4036	4·4046	4·4056	4·4066	0010
270	4·4075	4·4085	4·4095	4·4105	4·4114	4·4124	4·4134	4·4143	4·4153	4·4163	·0010
271	23 4·4172	4·4182	4·4192	4·4201	4·4211	4·4220	4·4230	4·4240	4·4249	4·4259	·0010
272	4·4268	4·4278	4·4287	4·4297	4·4307	4·4316	4·4326	4·4335	4·4344	4·4354	·0010
273	4·4363	4·4373	4·4382	4·4392	4·4401	4·4411	4·4420	4·4429	4·4439	4·4448	·0009
274	23 4·4457	4·4467	4·4476	4·4485	4·4495	4·4504	4·4513	4·4523	4·4532	4·4541	·0009
275	4·4551	4·4560	4·4569	4·4578	4·4587	4·4597	4·4606	4·4615	4·4624	4·4633	·0009
276	4·4643	4·4652	4·4661	4·4670	4·4679	4·4688	4·4697	4·4706	4·4715	4·4725	·0009
277	23 4·4734	4·4743	4·4752	4·4761	4·4770	4·4779	4·4788	4·4797	4·4806	4·4815	·0009
278	4 4824	4·4833	4·4842	4·4850	4·4859	4·4868	4·4877	4·4886	4·4895	4·4904	·0009
279	4·4913	4·4922	4 4930	4·4939	4·4948	4·4957	4·4966	4·4975	4·4983	4·4992	·0009
280	23 4·5001	4·5010	4·5018	4·5027	4·5036	4·5045	4·5053	4·5062	4·5071	4·5080	·0009
281	4·5088	4·5097	4·5105	4·5114	4·5123	4·5131	4·5140	4·5148	4·5157	4·5166	·0009
282	4·5174	4·5183	4·5191	4·5200	4·5208	4·5217	4·5226	4·5234	4·5243	4·5251	·0009
283	23 4·5260	4·5268	4·5277	4·5285	4·5293	4·5302	4·5310	4·5319	4·5327	4·5336	·0008
284	4·5344	4·5352	4·5361	4·5369	4·5378	4·5386	4·5394	4·5403	4·5411	4·5419	·0008
285	4·5427	4 5436	4·5444	4·5452	4·5461	4·5469	4·5477	4·5485	4·5494	4·5502	·0008
286	23 4·5510	4·5518	4·5527	4·5535	4·5543	4·5551	4·5559	4·5567	4·5576	4·5584	·0008
287	4·5592	4·5600	4·5608	4·5616	4·5624	4·5632	4·5641	4·5648	4·5657	4·5665	·0008
288	4·5673	4·5681	4·5689	4·5697	4·5705	4·5713	4·5721	4·5729	4·5737	4·5745	·0008
289	23 4·5753	4·5761	4·5769	4·5777	4·5785	4·5793	4·5800	4·5808	4·5816	4·5824	·0008
290	4·5832										

Table IV.

Distance s in feet, between velocities V and v f/s;

$$s = C\ (S_V - S_v).$$

(From Supplement *Bashforth's Motion of Projectiles*, 1881.)

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
10	1066	1238	1409	1578	1745	1910	2074	2236	2397	2557	166
11	2715	2871	3026	3180	3333	3484	3633	3782	3929	4075	151
12	4220	4363	4506	4647	4787	4926	5064	5200	5336	5471	139
13	5604	5737	5866	5999	6129	6257	6385	6511	6637	6762	129
14	6886	7009	7132	7253	7373	7493	7612	7730	7847	7964	120
15	8079	8194	8309	8422	8535	8647	8758	8868	8978	9087	112
16	9196	9304	9411	9517	9623	9728	9833	9937	10040	10142	105
17	10244	10346	10447	10546	10645	10743	10841	10939	11037	11134	98
18	11230	11326	11421	11516	11610	11704	11797	11890	11982	12074	94
19	12165	12256	12346	12436	12525	12614	12703	12791	12878	12966	89
20	13052	13139	13224	13310	13395	13480	13564	13648	13731	13814	85
21	13896	13979	14060	14142	14223	14303	14384	14463	14543	14622	81
22	14701	14779	14857	14935	15013	15090	15167	15244	15319	15395	77
23	15470	15545	15620	15694	15768	15842	15916	15989	16061	16134	74
24	16206	16278	16350	16421	16492	16563	16633	16703	16773	16843	71
25	1 6912·1	6981·2	7050·0	7118·5	7186·7	7254·7	7322·4	7389·8	7457·0	7523·9	68·0
26	7590·6	7657·0	7723·2	7789·1	7854·7	7920·1	7985·3	8050·2	8114·8	8179·3	65·4
27	8243·5	8307·5	8371·2	8434·7	8498·0	8561·0	8623·9	8686·4	8748·8	8810·9	63·0
28	1 8872·8	8934·5	8996·0	9057·2	9118·3	9179·1	9239·7	9300·1	9360·3	9420·3	60·8
29	9480·0	9539·6	9598·9	9658·1	9717·0	9775·8	9834·3	9892·6	9950·8	*0008·7	58·7
30	2 0066·5	0124·0	0181·4	0238·5	0295·5	0352·3	0409·0	0465·4	0521·7	0577·7	56·8
31	2 0633·6	0689·3	0744·8	0800·1	0855·3	0910·2	0965·0	1019·6	1074·0	1128·3	55·0
32	1182·4	1236·3	1290·0	1343·5	1396·9	1450·2	1503·2	1556·1	1608·8	1661·4	53·2
33	1713·8	1766·0	1818·1	1870·0	1921·7	1973·3	2024·7	2076·0	2127·1	2178·1	51·6
34	2 2228·9	2279·6	2330·0	2380·4	2430·6	2480·6	2530·5	2580·2	2629·7	2679·1	50·0
35	2728·4	2777·5	2826·4	2875·2	2923·8	2972·3	3020·7	3068·8	3116·9	3164·7	48·5
36	3212·5	3260·1	3307·5	3354·8	3402·0	3449·0	3495·9	3542·6	3589·2	3635·6	47·0
37	2 3682·0	3728·1	3774·2	3820·0	3865·8	3911·4	3956·9	4002·2	4047·4	4092·5	45·6
38	4137·4	4182·2	4226·8	4271·4	4315·7	4360·0	4404·1	4448·1	4491·9	4535·7	44·3
39	4579·2	4622·7	4666·0	4709·2	4752·3	4795·2	4838·1	4880·8	4923·3	4965·7	42·9
40	2 5008·0	5050·2	5092·3	5134·2	5176·0	5217·6	5259·2	5300·6	5341·9	5383·0	41·7
41	5424·0	5464·9	5505·7	5546·4	5586·9	5627·3	5667·6	5707·8	5747·8	5787·8	40·4
42	5827·6	5867·3	5906·9	5946·4	5985·8	6025·0	6064·2	6103·3	6142·2	6181·0	39·3
43	2 6219·8	6258·4	6296·9	6335·3	6373·6	6411·8	6449·9	6487·9	6525·8	6536·6	38·2
44	6601·3	6638·9	6676·4	6713·7	6751·0	6788·2	6825·3	6862·3	6899·3	6936·1	37·2
45	6972·8	7009·4	7046·0	7082·4	7118·8	7155·0	7191·2	7227·3	7263·3	7299·2	36·3
46	2 7335·1	7370·8	7406·5	7442·1	7477·6	7513·0	7548·3	7583·6	7618·8	7653·9	35·4
47	7688·9	7723·8	7758·7	7793·5	7828·2	7862·8	7897·3	7931·8	7966·2	8000·5	34·6
48	8034·7	8068·9	8103·0	8137·0	8170·9	8204·8	8238·6	8272·3	8305·9	8339·5	33·9
49	2 8373·0	8406·5	8439·8	8473·1	8506·4	8539·5	8572·6	8605·6	8638·6	8671·5	33·2
50	8704·3	8737·1	8769·8	8802·4	8835·0	8867·5	8900·0	8932·3	8964·7	8996·9	32·5
51	9029·1	9061·2	9093·2	9125·2	9157·1	9189·0	9220·8	9252·5	9284·2	9315·8	31·9
52	2 9347·3	9378·8	9410·3	9441·6	9472·9	9504·2	9535·4	9566·5	9597·6	9628·7	31·3
53	9659·6	9690·6	9721·4	9752·2	9783·0	9813·7	9844·3	9874·9	9905·4	9935·9	30·7
54	9966·3	9996·7	*0027·0	0057·3*	*0087·5	*0117·7	0147·8*	*0177·8	*0207·8	0237·8*	30·2
55	3 0267·6	0297·5	0327·3	0357·0	0386·7	0416·3	0445·9	0475·4	0504·9	0534·3	29·6
56	0563·6	0592·9	0622·2	0651·4	0680·6	0709·7	0738·7	0787·7	0796·7	0825·6	29·1
57	0854·5	0883·3	0912·1	0940·9	0969·6	0998·2	1026·8	1055·4	1083·9	1112·4	28·6
58	3 1140·8	1169·2	1197·6	1226·0	1254·3	1282·5	1310·8	1339·0	1367·1	1395·2	28·3
59	1423·3	1451·3	1479·3	1507·3	1535·2	1563·0	1590·9	1618·7	1646·4	1674·2	27·9
60	1701·8	1729·5	1757·1	1784·6	1812·2	1839·6	1867·1	1894·5	1921·9	1949·2	27·5
61	3 1976·5	2003·7	2031·0	2058·1	2085·3	2112·4	2139·4	2166·4	2193·4	2220·4	27·1
62	2247·3	2274·2	2301·0	2327·8	2354·5	2381·3	2407·9	2434·6	2461·2	2487·7	26·7
63	2514·3	2540·8	2567·2	2593·6	2620·0	2646·3	2672·6	2698·9	2725·1	2751·3	26·3

TABLE IV—*continued.*

$$s = C\,(S_V - S_v).$$

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
64	3 2777·5	2803·6	2829·7	2855·7	2881·7	2907·7	2933·7	2959·6	2985·4	3011·2	26·0
65	3037·0	3062·8	3088·5	3114·2	3139·8	3165·4	3191·0	3216·5	3242·0	3267·4	25·6
66	3292·8	3318·2	3343·5	3368·8	3394·1	3419·3	3444·5	3469·6	3494·7	3519·8	25·2
67	3 3544·8	3569·8	3594·8	3619·8	3644·7	3669·5	3694·3	3719·1	3743·9	3768·6	24·8
68	3793·3	3818·0	3842·6	3867·2	3891·7	3916·2	3940·7	3965·2	3989·6	4014·0	24·5
69	4038·4	4062·7	4087·0	4111·3	4135·6	4159·8	4184·0	4208·1	4232·2	4256·3	24·2
70	3 4280·4	4304·5	4328·5	4352·4	4376·4	4400·3	4424·1	4448·0	4471·8	4495·5	23·9
71	4519·3	4543·0	4566·6	4590·2	4613·8	4637·4	4660·9	4684·4	4707·8	4731·3	23·5
72	4754·7	4777·9	4801·3	4824·6	4847·9	4871·1	4894·2	4917·4	4940·5	4963·6	23·2
73	3 4986·6	5009·6	5032·6	5055·5	5078·4	5101·3	5124·1	5146·9	5169·6	5192·4	22·8
74	5215·1	5237·7	5260·3	5282·9	5305·5	5328·0	5350·5	5373·0	5395·4	5417·8	22·5
75	5440·2	5462·5	5484·8	5507·1	5529·3	5551·5	5573·7	5595·8	5617·9	5640·0	22·2
76	3 5662·1	5684·1	5706·0	5728·0	5749·9	5771·7	5793·5	5815·3	5837·0	5858·7	21·8
77	5880·4	5902·0	5923·6	5945·1	5966·6	5988·1	6009·5	6030·9	6052·2	6073·6	21·5
78	6094·8	6116·1	6137·3	6158·4	6179·6	6200·7	6221·7	6242·7	6263·7	6284·6	21·1
79	3 6305·5	6326·4	6347·2	6368·0	6388·8	6409·5	6430·2	6450·8	6471·4	6492·0	20·7
80	6512·6	6533·1	6553·6	6574·0	6594·4	6614·8	6635·1	6655·4	6675·7	6695·9	20·4
81	6716·1	6736·3	6756·4	6776·5	6796·5	6816·3	6836·5	6856·4	6876·3	6896·1	20·0
82	3 6916·0	6935·7	6955·5	6975·1	6994·8	7014·4	7033·9	7053·4	7072·9	7092·3	19·6
83	7111·7	7131·0	7150·3	7169·6	7188·8	7207·9	7227·1	7246·1	7265·2	7284·1	19·1
84	7303·1	7322·0	7340·8	7359·6	7378·4	7397·1	7415·8	7434·4	7453·0	7471·5	18·7
85	3 7490·0	7508·5	7526·9	7545·3	7563·6	7581·8	7600·0	7618·2	7636·3	7654·4	18·2
86	7692·4	7690·5	7708·4	7726·4	7744·2	7762·0	7799·9	7797·6	7815·4	7833·0	17·8
87	7850·6	7868·2	7885·8	7903·3	7920·8	7938·2	7955·6	7973·0	7990·3	8007·6	17·4
88	3 8024·8	8042·0	8059·2	8076·3	8093·4	8110·4	8127·4	8144·4	8161·3	8178·2	17·0
89	8195·0	8211·9	8228·6	8245·4	8262·1	8278·7	8295·4	8312·0	8328·5	8345·0	16·6
90	8361·5	8377·9	8394·3	8410·7	8427·0	8143·3	8459·6	8475·8	8492·0	8508·2	16·3
91	3 8524·3	8540·4	8556·4	8572·4	8588·4	8604·3	8620·3	8636·1	8652·0	8667·8	15·9
92	8683·5	8699·3	8715·0	8730·7	8746·3	8761·9	8777·5	8793·0	8808·5	8824·0	15·6
93	8839·4	8854·8	8870·2	8885·5	8900·8	8916·1	8931·3	8946·5	8961·7	8976·8	15·3
94	3 8991·9	9007·0	9022·0	9037·0	9052·0	9066·9	9081·9	9096·7	9111·6	9126·4	15·0
95	9141·2	9156·0	9170·7	9185·4	9200·1	9214·7	9229·3	9243·9	9258·4	9272·9	14·6
96	9287·4	9301·9	9316·3	9330·7	9345·0	9359·4	9373·7	9387·9	9402·2	9416·4	14·3
97	3 9430·6	9444·7	9458·9	9473·0	9487·0	9501·1	9515·1	9529·1	9543·0	9557·0	14·0
98	9570·8	9584·7	9598·6	9612·4	9626·1	9639·9	9653·6	9667·3	9681·0	9694·6	13·7
99	9708·3	9721·9	9735·4	9749·0	9762·5	9775·9	9789·4	9802·8	9816·2	9829·6	13·5
100	3 9842·9	9856·3	9869·6	9882·9	9896·1	9909·3	9922·5	9935·3	9948·8	9961·9	13·2
101	9975·0	9988·1	*0001·1	0014·1*	*0027·1	*0040·0	0052·9*	*0065·8	*0078·7	0091·5*	12·9
102	4 0104·3	0117·1	0129·8	0142·5	0155·2	0167·8	0180·4	0192·9	0205·4	0217·8	12·6
103	4 0230·1	0242·4	0254·6	0266·8	0278·8	0290·8	0302·7	0314·5	0326·2	0337·8	11·9
104	0349·4	0360·8	0372·2	0383·4	0394·5	0405·6	0416·5	0427·3	0438·1	0448·7	11·0
105	0459·2	0469·6	0479·9	0490·0	0500·1	0510·1	0520·0	0529·8	0539·5	0549·2	9·9
106	4 0558·7	0568·2	0577·6	0586·9	0596·2	0605·4	0614·5	0623·6	0632·6	0641·6	9·2
107	0650·5	0659·3	0668·1	0676·9	0685·6	0694·2	0702·8	0711·4	0719·9	0728·4	8·6
108	0736·8	0745·2	0753·6	0761·9	0770·2	0778·4	0786·6	0794·8	0802·9	0811·0	8·2
109	4 0819·0	0827·1	0835·0	0843·0	0850·9	0858·9	0866·7	0874·6	0882·4	0890·2	7·9
110	0897·9	0905·7	0913·4	0921·1	0928·7	0936·4	0944·0	0951·5	0959·1	0966·6	7·6
111	0974·2	0981·6	0989·1	0996·6	1004·0	1011·4	1018·8	1026·2	1033·5	1040·9	7·4
112	4 1048·2	1055·5	1062·8	1070·0	1077·3	1084·5	1091·7	1099·0	1106·1	1113·3	7·2
113	1120·5	1127·6	1134·8	1141·9	1149·0	1156·1	1163·2	1170·2	1177·3	1184·4	7·1
114	1191·4	1198·4	1205·4	1212·4	1219·4	1226·4	1233·3	1240·3	1247·2	1254·1	6·9
115	4 1261·0	1267·9	1274·8	1281·7	1288·6	1295·4	1302·3	1309·1	1315·9	1322·7	6·8
116	1329·5	1336·3	1343·1	1349·8	1356·6	1363·3	1370·0	1376·7	1383·4	1390·1	6·7
117	1396·8	1403·5	1410·1	1416·8	1423·4	1430·0	1436·6	1443·2	1449·8	1456·4	6·6
118	4 1462·9	1469·5	1476·0	1482·6	1489·1	1495·6	1502·1	1508·6	1515·1	1521·5	6·5
119	1528·0	1534·4	1540·9	1547·3	1553·7	1560·1	1566·5	1572·9	1579·2	1585·6	6·4
120	1591·9	1598·3	1604·6	1610·9	1617·2	1623·5	1629·8	1636·1	1642·3	1648·6	6·3
121	4 1654·8	1661·1	1667·3	1673·5	1679·7	1685·9	1692·1	1698·2	1704·4	1710·5	6·2
122	1716·7	1722·8	1728·9	1735·0	1741·1	1747·2	1753·3	1759·4	1765·4	1771·5	6·1
123	1777·5	1783·6	1789·6	1795·6	1801·6	1807·6	1813·6	1819·6	1825·6	1831·5	6·0

TABLE IV—*continued.*

$s = C\,(S_V - S_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
124	4 1837·5	1843·4	1849·4	1855·3	1861·2	1867·1	1873·0	1878·9	1884·8	1890·6	5·9
125	1896·5	1902·3	1908·2	1914·0	1919·8	1925·6	1931·5	1937·3	1943·0	1948·9	5·8
126	1954·6	1960·4	1966·1	1971·9	1977·6	1983·3	1989·0	1994·8	2000·5	2006 2	5·7
127	4 2011 8	2017·5	2023·2	2028·9	2034·5	2040·2	2045·8	2051·4	2057·0	2062·7	5·6
128	2068·3	2073·9	2079·5	2085·0	2090·6	2096·2	2101·8	2107·3	2112·9	2118·4	5·6
129	2123·9	2129·4	2135·0	2140·5	2146·0	2151·5	2157·0	2162·4	2167·9	2173·4	5·5
130	4 2178·8	2184·3	2189·7	2195·1	2200·6	2206·0	2211·4	2216·8	2222·2	2227·6	5·4
131	2233·0	2238·4	2243·7	2249·1	2254·5	2259·8	2265·1	2270·5	2275·8	2281·1	5·3
132	2286·4	2291·8	2297·1	2302·4	2307·5	2312·9	2318·2	2323·5	2328·7	2334·0	5·3
133	4 2339·2	2344·5	2349·7	2355·0	2360·2	2365·4	2370·6	2375·8	2381·0	2386·2	5·2
134	2391·4	2396·6	2401·8	2406·9	2412·1	2417·3	2422·4	2427·6	2432·7	2437·8	5·2
135	2443·0	2448·1	2453·2	2458·3	2463·4	2468·5	2473·6	2478·7	2483·8	2488·9	5·1
136	4 2493·9	2499·0	2504·1	2509·1	2514·2	2519·2	2524·3	2529·3	2534·3	2539·4	5·0
137	2544·4	2549·4	2554·4	2559·4	2564·4	2569·2	2574·4	2579·4	2584·3	2589·3	5·0
138	2594·3	2599·2	2604·2	2609·1	2614·1	2619·0	2624·0	2628·9	2633·8	2638·8	4·9
139	4 2643·7	2648·6	2653·5	2658·4	2663·3	2668·2	2673·1	2678·0	2682·9	2687·8	4·9
140	2692·6	2697·5	2702·4	2707·2	2712·1	2717·0	2721·8	2726·7	2731·5	2736·3	4·9
141	2741·2	2746·0	2750·8	2755·7	2760·5	2765·3	2770·1	2774·9	2779·7	2784·5	4·8
142	4 2789·3	2794·1	2798·9	2803·7	2808·5	2813·2	2818·0	2822·8	2827·5	2832·3	4·8
143	2837·1	2841·8	2846·6	2851·3	2856·0	2860·8	2865·5	2870·2	2875·0	2879·7	4·7
144	2884·4	2889·1	2893·8	2898·6	2903·3	2908·0	2912·7	2917·4	2922·1	2926·7	4·7
145	4 2931·4	2936·1	2940·8	2945·5	2950·1	2954·8	2959·5	2964·1	2968·8	2973·5	4·7
146	2978·1	2982·8	2987·4	2992·1	2996·7	3001·3	3006·0	3010·6	3015·2	3019·9	4·6
147	3024·5	3029·1	3033·7	3038·4	3043·0	3047·6	3052·2	3056·8	3061·4	3066·0	4·6
148	4 3070·6	3075·2	3079·8	3084·4	3089·0	3093·5	3098·1	3102·7	3107·3	3111·8	4·6
149	3116·4	3121·0	3125·6	3130·1	3134·7	3139·2	3143·8	3148·3	3152·9	3157·4	4·6
150	3162·0	3166·5	3171·0	3175·6	3180·1	3184·6	3189·2	3193·7	3198·2	3202·7	4·5
151	4 3207·2	3211·8	3216·3	3220·8	3225·3	3229·8	3234·3	3238·8	3243·3	3247·8	4·5
152	3252·3	3256·8	3261·3	3265·8	3270·3	3274·8	3279·3	3283·8	3288·3	3292·8	4·5
153	3297·2	3301·7	3306·2	3310·6	3315·1	3319·6	3324·1	3328·5	3333·0	3337·5	4·5
154	4 3342·0	3346·4	3350·9	3355·3	3359·8	3364·3	3368·7	3373·2	3377·6	3382·1	4·5
155	3386·5	3391·0	3395·4	3399·9	3404·3	3408·7	3413·2	3417·6	3422·0	3426·5	4·4
156	3430·9	3435·3	3439·8	3444·2	3448·6	3453·0	3457·4	3461·9	3466·3	3470·7	4·4
157	4 3475·1	3479·5	3483·9	3488·3	3492·7	3497·1	3501·5	3505·9	3510·3	3514·7	4·4
158	3519·1	3523·5	3527·9	3532·3	3536·7	3541·1	3545·4	3549·8	3554·2	3558·6	4·4
159	3563·0	3567·3	3571·7	3576·1	3580·4	3584·8	3589·1	3593·5	3597·9	3602·2	4·4
160	4 3606·6	3610·9	3615·3	3619·6	3624·0	3628·3	3632·6	3637·0	3641·3	3645·7	4·3
161	3650·0	3654·3	3658·7	3663·0	3667·3	3671·6	3676·0	3680·3	3684·6	3688·9	4·3
162	3693·3	3697·6	3701·9	3706·1	3710·5	3714·8	3719·1	3723·4	3727·7	3732·0	4·3
163	4 3736·3	3740·6	3744·9	3749·2	3753·5	3757·8	3762·1	3766·4	3770·6	3774·9	4·3
164	3779·2	3783·5	3787·8	3792·0	3796·3	3800·6	3804·9	3809·1	3813·4	3817·6	4·3
165	3821·9	3826·2	3830·4	3834·7	3838·9	3843·2	3847·4	3851·7	3855·9	3860·2	4·3
166	4 3864·4	3868·7	3872·9	3877·2	3881·4	3885·6	3889·9	3894·1	3898·3	3902·5	4·2
167	3906·8	3911·0	3915·2	3919·5	3923·7	3927·9	3932·1	3936·3	3940·5	3944·7	4·2
168	3949·0	3953·2	3957·4	3961·6	3965·8	3970·0	3974·2	3978·4	3982·6	3986·7	4·2
169	4 3990·9	3995·1	3999·3	4003·5	4007·7	4011·9	4016·0	4020·2	4024·4	4028·6	4·2
170	4032·7	4036·9	4041·1	4045·2	4049·4	4053·6	4057·7	4061·9	4066·0	4070·2	4·2
171	4074·3	4078·5	4082·6	4086·8	4090·9	4095·1	4099·2	4103·3	4107·5	4111·6	4·1
172	4 4115·7	4119·9	4124·0	4128·1	4132·3	4136·4	4140·5	4144·6	4148·7	4152·2	4·1
173	4157·0	4161·1	4165·2	4169·3	4173·4	4177·5	4181·6	4185·7	4189·8	4193·9	4·1
174	4198·0	4202·1	4206·2	4210·3	4214·4	4218·5	4222·6	4226·7	4230·8	4234·8	4·1
175	4 4238·9	4243·0	4247·1	4251·2	4255·3	4259·3	4263·4	4267·5	4271·5	4275·6	4·1
176	4279·6	4283·7	4287 8	4291·8	4295·9	4300·0	4304·0	4308·0	4312·1	4316·1	4·1
177	4320·2	4324·2	4328·3	4332·3	4336·4	4340·4	4344·4	4348·5	4352·5	4356·5	4·0
178	4 4360·5	4364·6	4368·6	4372·6	4376·6	4380·7	4384·7	4388·7	4392·7	4396·7	4·0
179	4400·7	4404·7	4408·8	4412·8	4416·8	4420·8	4424·8	4428·8	4432·8	4436·8	4·0
180	4440·8	4444 7	4448·7	4452·7	4456·7	4460 7	4464·7	4468·7	4472·6	4476·6	4·0
181	4 4480·6	4484·6	4488·5	4492·5	4496·5	4500·5	4504·4	4508·4	4512·4	4516·3	4·0
182	4520·3	4524·2	4528·2	4532·2	4536·1	4540·1	4544·0	4548·0	4551·9	4555·9	4·0
183	4559·8	4563·7	4567·7	4571·6	4575·6	4579·5	4583·4	4587·4	4591·3	4595·2	3·9

TABLE IV—*continued.*

$s = C\,(S_V - S_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
184	4 4599·2	4603·1	4607·0	4610·9	4614·9	4618·8	4622·7	4626·6	4630·5	4634·4	3·9
185	4638·4	4642·3	4646·2	4650·1	4654·0	4657·9	4661·8	4665·7	4669·6	4673·5	3·9
186	4677·4	4681·3	4685·2	4689·1	4693·0	4696·9	4700·8	4704·6	4708·5	4712·4	3·9
187	4 4716·3	4720·2	4724·1	4727·9	4731·8	4735·7	4739·6	4743·4	4747·3	4751·2	3·9
188	4755·0	4758·9	4762·8	4766·7	4770·5	4774·4	4778·2	4782·1	4786 0	4789·8	3·9
189	4793·7	4797·5	4801·4	4805·2	4809·1	4812·9	4816·8	4820·6	4824·5	4828·3	3·8
190	4 4832·2	4836·0	4839·8	4843·7	4847·5	4851·4	4855·2	4859·0	4862·8	4866·7	3·8
191	4870·5	4874·3	4878·1	4882·0	4885·8	4889·6	4893·4	4897·3	4901·1	4904·9	3·8
192	4908·7	4912·5	4916·3	4920·1	4923·9	4927·7	4931·5	4935·3	4939·1	4942·9	3·8
193	4 4946·7	4950·5	4954·3	4958·1	4961·9	4965·7	4969·4	4973·2	4977·0	4980·7	3·8
194	4984·5	4988·3	4992·1	4995·8	4999·6	5003·4	5007·1	5010·9	5014·7	5018·4	3·8
195	5022·2	5025·9	5029·7	5033·4	5037·2	5040·9	5044·7	5048·4	5052·1	5055·9	3·7
196	4 5059·6	5063·4	5067·1	5070·8	5074·6	5078·3	5082·0	5085·7	5089·4	5093·1	3·7
197	5096·9	5100·6	5104·3	5108·0	5111·7	5115·4	5119·1	5122·8	5126·5	5130·2	3·7
198	5133·9	5137·5	5141·2	5144·9	5148·6	5152·3	5156·0	5159·6	5163·3	5166·9	3·7
199	4 5170·6	5174·3	5177·9	5181·6	5185·2	5188·9	5192·5	5196·2	5199·8	5203·4	3·6
200	5207·1	5210·7	5214·3	5218·0	5221·6	5225·2	5228 8	5232·5	5236·1	5239·7	3·6
201	5243·3	5246·9	5250·5	5254·1	5257·7	5261·3	5264·9	5268·5	5272·1	5275·7	3·6
202	4 5279·2	5282·8	5286·4	5290·0	5293·6	5297·2	5300·7	5304·3	5307·8	5311·4	3·6
203	5314·9	5318·5	5322·0	5325·6	5329·1	5332·7	5336·2	5339·7	5343·3	5346·8	3·5
204	5350·3	5353·8	5357·3	5360·9	5364·4	5367·9	5371·4	5374·9	5378·4	5381·9	3·5
205	4 5385·4	5388·9	5392·4	5395·9	5399·4	5402·9	5406·3	5409·8	5413·3	5416·7	3·5
206	5420·2	5423·7	5427·1	5430·6	5434·1	5437·5	5441·0	5444·[illegible]	5447·8	5451·3	3·5
207	5454·7	5458·1	5461·6	5465·0	5468·4	5471 9	5475·3	5478·7	5482·1	5485·5	3·4
208	4 5488·9	5492·3	5495·7	5499·1	5502·5	5505·9	5509·3	5512·7	5516·1	5519·4	3·4
209	5522·8	5526·2	5529·6	5532·9	5536·3	5539·7	5543·0	5546·4	5549·7	5553·1	3·4
210	5556·4	5559·8	5563·1	5566·4	5569·8	5573·1	5576·5	5579·8	5583·1	5586·4	3·3
211	4 5589·7	5593·0	5596·4	5599·7	5603·0	5606·3	5609·6	5612·9	5616·2	5619·5	3·3
212	5622·8	5626·1	5629·3	5632·6	5635·9	5639·2	5642·5	5645·7	5649·0	5652·3	3·3
213	5655·5	5658·8	5662·0	5665·3	5668·6	5671·8	5675·1	5678·3	5681·5	5684·8	3·2
214	4 5688·0	5691·2	5694·5	5697·7	5700·9	5704·2	5707·4	5710·6	5713·8	5717·0	3·2
215	5720·2	5723·4	5726·6	5729·9	5733·1	5736·3	5739·5	5742 6	5745·8	5749·0	3·2
216	5752·2	5755·4	5758·6	5761·8	5764·9	5768·1	5771·3	5774·4	5777·6	5780·8	3·2
217	4 5783·9	5787·1	5790·2	5793·4	5796·6	5799·7	5802·9	5806·0	5809·1	5812·2	3·1
218	5815·4	5818·5	5821·6	5824·8	5827·9	5831·0	5834·1	5837·3	5840·4	5843·5	3·1
219	5846·6	5849·7	5852·8	5855·9	5859·0	5862·1	5865·2	5868·3	5871·4	5874·4	3·1
220	4 5877·5	5880·6	5883·7	5886·8	5889·9	5893·0	5896·0	5899·1	5902·1	5905·2	3·1
221	5908·3	5911·3	5914·4	5917·4	5920·5	5923·6	5926·6	5929·6	5932·7	5935·7	3·0
222	5938·7	5941·8	5944·8	5947·8	5950·9	5953·9	5956·9	5959·9	5963·0	5966·0	3·0
223	4 5969·0	5972·0	5975·0	5978·0	5981·0	5984·0	5987·0	5990·0	5993·0	5996·0	3·0
224	5999·0	6002·0	6004·9	6007·9	6010·9	6013·9	6016·9	6019·8	6022·8	6025·8	3·0
225	6028·7	6031·7	6034·6	6037·6	6040·5	6043·5	6046·5	6049·4	6052·4	6055·3	3·0
226	4 6058·3	6061·2	6064·1	6067·1	6070·0	6072·9	6075·9	6078·8	6081·7	6084·7	2·9
227	6087·6	6090·5	6093·4	6096·3	6099·3	6102·2	6105·1	6108·0	6110·9	6113·8	2·9
228	6116·7	6119·6	6122·5	6125·4	6128·3	6131·2	6134·1	6137·0	6139·9	6142·8	2·9
229	4 6145·7	6148·6	6151·5	6154·4	6157·3	6160·2	6163·1	6166·0	6168·8	6171·7	2·
230	6174·6	6177·5	6180·4	6183·3	6186·2	6189·1	6191·9	6194·8	6197·7	6200·6	2·9
231	6203·5	6206·4	6209·3	6212·1	6215·0	6217·9	6220 8	6223·7	6226·6	6229·5	2 9
232	4 6232·3	6235·2	6238·1	6241·0	6243·9	6246·8	6249·7	6252·6	6255·4	6258·3	2·9
233	6261·2	6264·1	6267·0	6269·9	6272·8	6275·7	6278·6	6281·5	6284·3	6287·2	2·9
234	6290·1	6293·0	6295·9	6298·8	6301·7	6304·6	6307·5	6310·4	6313·3	6316·2	2·9
235	4 6319·0	6322·0	6324·9	6327·7	6330·6	6333·5	6336·4	6339·3	6342·2	6345·1	2·9
236	6348·0	6350·9	6353·8	6356·7	6359·6	6362·5	6365·4	6368·3	6371·2	6374·1	2·9
237	6377·0	6379·9	6382·8	6385·7	6388·6	6391·5	6394·4	6397·3	6400·2	6403·1	2·9
238	4 6406·0	6408·9	6411·8	6414·8	6417·7	6420·6	6423·5	6426·4	6429·3	6432·2	2·9
239	6435·1	6438·0	6440·9	6443·8	6446·8	6449·7	6452·6	6455·5	6458·4	6461·3	2·9
240	6464·2	6467·1	6470·1	6473·0	6475·9	6478·8	6481·7	6484·6	6487·6	6490·5	2·9
241	4 6493·4	6496·3	6499·2	6502·2	6505·1	6508·0	6510·9	6513·8	6516·8	6519·7	2·9
242	6522·6	6525·6	6528·5	6531·4	6534·3	6537·3	6540·2	6543·1	6546·1	6549·0	2·9
243	6551·9	6554·9	6557·8	6560·7	6563·7	6566·6	6569·5	6572·5	6575·4	6578·3	2·9

TABLE IV—*continued.*

$s = C(S_V - S_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f/s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
244	4 6581·3	6584·2	6587·2	6590·1	6593·0	6596·0	6598·9	6601·8	6604·8	6607·7	2·9
245	6610·6	6613·6	6616·5	6619·5	6622·4	6625·3	6628·3	6631·2	6634·2	6637·1	2·9
246	6640·1	6643·0	6645·9	6748·9	6651·8	6654·8	6657·7	6660·6	6663·6	6666·5	2·9
247	4 6669·5	6672·4	6675·4	6678 3	6681·3	6684·2	6687·2	6690·1	6693·0	6696·0	2·9
248	6698·9	6701·9	6704·8	6707·8	6710 7	6713·7	6716·6	6719·6	6722·5	6725·5	2·9
249	6728·4	6731·3	6734·3	6737·2	6740·2	6743·1	6746·1	6749·0	6752·0	6754·9	2·9
250	4 6757·8	6760·7	6763·7	6766·7	6769·6	6772·6	6775·5	6778·4	6781·4	6784·3	2·9
251	6787·3	6790·2	6793·1	6796·1	6799·0	6802·0	6804·9	6807·8	6810·8	6813·7	2·9
252	6816·6	6819·6	6822·5	6825·4	6828·4	6831·3	6834·2	6837·1	6840·1	6843·0	2·9
253	4 6845·9	6848·8	6851·8	6854·7	6857·6	6860·5	6863·5	6866·4	6869·3	6872·2	2·9
254	6875·1	6878·1	6881·0	6883·9	6886·8	6889·7	6892·6	6895·6	6898·5	6901·4	2·9
255	6904·3	6907·2	6910·1	6913·0	6915·9	6918·8	6921·7	6924·6	6927·5	6930·4	2·9
256	4 6933·3	6936·2	6939·1	6942·0	6944·9	6947·8	6950·6	6953·5	6956·4	6959·3	2·9
257	6962·2	6965·0	6967·9	6970·8	6973·7	6976·5	6979·4	6982·3	6985·1	6988·0	2·9
258	6990·9	6993·7	6996·6	6999·4	7002·3	7005·1	7008·0	7010·8	7013·7	7016·5	2·9
259	4 7019·4	7022·2	7025·0	7027·9	7030·7	7033·5	7036·4	7039·2	7042·0	7044·8	2·8
260	7047·7	7050·5	7053·3	7056·1	7058·9	7061·7	7064·5	7067·4	7070·2	7073·0	2·8
261	7075·8	7078·6	7081·4	7084·2	7087·0	7089·7	7092·5	7095·3	7098·1	7100·9	2·8
262	4 7103·7	7106·5	7109·2	7112·0	7114·8	7117·6	7120·3	7123·1	7125·9	7128·6	2·8
263	7131·4	7134·2	7136·9	7139·7	7142·4	7145·2	7147·9	7150·7	7153·4	7156·2	2·8
264	7158·9	7161·7	7164·4	7167·1	7169·9	7172·6	7175·4	7178·1	7180·8	7183·5	2·7
265	4 7186·3	7189·0	7191·7	7194·4	7197·1	7199·9	7202·6	7205·3	7208·0	7210·7	2·7
266	7213·4	7216·1	7218·8	7221·5	7224·2	7226·9	7229·6	7232·3	7235·0	7237·7	2·7
267	7240·4	7243·1	7245·8	7248·5	7251·2	7253·8	7256·5	7259·2	7261·9	7264·5	2·7
268	4 7267·2	7269·9	7272·5	7275·2	7277·9	7280·5	7283·2	7285·9	7288·5	7291·2	2·7
269	7293·8	7296·5	7299·1	7301·8	7304·4	7307·1	7309·7	7312·3	7315·0	7317·6	2·6
270	7320·2	7322·9	7325·5	7328·1	7330·8	7333·4	7336·0	7338·6	7341·2	7343·9	2·6
271	4 7346·5	7349·1	7351·7	7354·3	7356·9	7359·5	7362·1	7364·7	7367·3	7369·9	2·6
272	7372·5	7375·1	7377·7	7380·3	7382·9	7385·5	7388·1	7390·7	7393·3	7395·8	2·6
273	7398·4	7401·0	7403·6	7406·2	7408·7	7411·3	7413·9	7416·4	7419·0	7421·6	2·6
274	4 7424·1	7426·7	7429·3	7431·8	7434·4	7436·9	7439·5	7442·0	7444·6	7447·1	2·6
275	7449·7	7452·2	7454·8	7457·3	7459·8	7462·4	7464·9	7467·4	7470·0	7472·5	2·5
276	7475·0	7477·5	7480·1	7482·6	7485·1	7487·6	7490·1	7492·7	7495·2	7497·7	2·5
277	4 7500·2	7502·7	7505·2	7507·7	7510·2	7512·7	7515·2	7517·7	7520·2	7522·7	2·5
278	7525·2	7527·7	7530·1	7532·6	7535·1	7537·6	7540·1	7542·6	7545·0	7547·5	2·5
279	7550·0	7552·4	7554·9	7557·4	7559·9	7562·3	7564·8	7567·2	7569·7	7572·2	2·5
280	4 7574·6	7577·1	7579·5	7582·0	7584·4	7586·8	7589·3	7591·7	7594·2	7596·6	2·4
281	7599·0	7601·5	7603·9	7606·4	7608·8	7611·2	7613·6	7616·1	7618·5	7620·9	2·4
282	7623·3	7625·7	7628·2	7630·6	7633·0	7635·4	7637·8	7640·2	7642·6	7645·0	2·4
283	4 7647·4	7649·8	7652·2	7654·6	7657·0	7659·4	7661·8	7664·2	7666·6	7669·0	2·4
284	7671·3	7673·7	7676·1	7678·5	7680·9	7683·3	7685·6	7688·0	7690·4	7692·7	2·4
285	7695·1	7697·5	7699·8	7702·2	7706·9	7706·9	7709·3	7711·6	7714·0	7716·4	2·4
286	4 7718·7	7721·1	7723·4	7725·8	7728·1	7730·4	7732·8	7735·1	7737·5	7739·8	2·3
287	7742·1	7744·5	7746·8	7749·1	7751·5	7753·8	7756·1	7758·4	7760·8	7763·1	2·3
288	7765·4	7767·7	7770·0	7772·4	7774·7	7777·0	7779·3	7781·6	7783·9	7786·2	2·3
289	4 7788·5	7790·8	7793·1	7795·4	7797·7	7800·0	7802·3	7804·6	7806·9	7809·2	2·3
290	7811·5										

TABLE V.

Deviation δ in degrees, between velocities V and v f/s.

$$\delta = C\,(D_V - D_v).$$

(By W. D. Niven, F.R.S.)

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
40	0·0000	0·4838	0·9640	1·4407	1·9137	2·3830	2·8488	3·3110	3·7689	4·2240
41	4·6757	5·1240	5·5688	6·0101	6·4482	6·8828	7·3141	7·7421	8·1660	8·5874
42	9·0056	9·4207	9·8327	10·2410	10·6467	11·0496	11·4494	11·8462	12·2397	12·6306
43	13·0187	13·4039	13·7862	14·1652	14·5419	14·9159	15·2872	15·6557	16·0211	16·3843
44	16·7450	17·1030	17·4585	17·8110	18·1614	18·5094	18·8549	19·1980	19·5383	19·8766
45	20·2125	20·5460	20·8772	21·2054	21·5320	21·8565	22·1788	22·4989	22·8169	23·1327
46	23·4463	23·7578	24·0671	24·3736	24·6788	24·9821	25·2834	25·5827	25·8801	26·1756
47	26·4691	26·7607	27·0503	27·3376	27·6234	27·9075	28·1897	28·4702	28·7486	29·0254
48	29·3006	29·5739	29·8455	30·1151	30·3833	30·6498	30·9147	31·1779	31·4393	31·6993
49	31·9576	32·2143	32·4695	32·7227	32·9747	33·2253	33·4743	33·7219	33·9679	34·2125
50	34·4557	34·6973	34·9375	35·1761	35·4134	35·6493	35·8837	36·1167	36·3480	36·5783
51	36·8073	37·0349	37·2613	37·4862	37·7099	37·9323	38·1534	38·3731	38·5914	38·8086
52	39·0246	39·2394	39·4529	39·6651	39·8762	40·0860	40·2947	40·5022	44·7082	40·9135
53	41·1175	41·3204	41·5221	41·7225	41·9221	42·1205	42·3179	42·5142	42·7095	42·9037
54	43·0967	43·2887	43·4795	43·6690	43·8578	44·0456	44·2324	44·4182	44·6031	44·7870
55	44·9698	45·1516	45·3325	45·5122	45·6910	45·8689	46·0457	46·2217	46·3964	46·5705
56	46·7437	46·9160	47·0874	47·2581	47·4277	47·5965	47·7644	47·9314	48·0973	48·2625
57	48·4270	48·5906	48·7534	48·9153	49·0764	49·2368	49·3963	49·5551	49·7130	49·8701
58	50·0265	50·1822	50·3370	50·4909	50·6442	50·7968	50·9487	51·0999	51·2505	51·4002
59	51·5492	51·6977	51·8451	51·9917	52·1378	52·2832	52·4280	52·5721	52·7155	52·8583
60	53·0003	53·1417	53·2825	53·4224	53·5618	53·7005	53·8386	53·9761	54·1130	54·2492
61	54·3847	54·5196	54·6539	54·7875	54·9205	55·0529	55·1846	55·3158	55·4462	55·5761
62	55·7054	55·8341	55·9623	56·0899	56·2169	56·3433	56·4690	56·5942	56·7188	56·8428
63	56·9663	57·0891	57·2114	57·3330	57·4542	57·5749	57·6950	57·8146	57·9338	58·0523
64	58·1703	58·2878	58·4046	58·5209	58·6367	58·7521	58·8669	58·9832	59·0949	59·2081
65	59·3209	59·4332	59·5449	59·6562	59·7669	59·8772	59·9869	60·0961	60·2047	60·3130
66	60·4207	60·5280	60·6348	60·7411	60·8470	60·9523	61·0572	61·1616	61·2654	61·3688
67	61·4719	61·5744	61·6766	61·7783	61·8796	61·9804	62·0808	62·1807	62·2802	62·3793
68	62·4779	62·5761	62·6739	62·7711	62·8680	62·9646	63·0607	63·1565	63·2519	63·3468
69	63·4414	63·5356	63·6294	63·7227	63·8157	63·9084	64·0006	64·0924	64·1838	64·2749
70	64·3656	64·4559	64·5459	64·6356	64·7249	64·8137	64·9022	64·9903	65·0779	65·1652
71	65·2522	65·3388	65·4250	65·5107	65·5962	65·6813	65·7660	65·8504	65·9345	66·0182
72	66·1015	66·1845	66·2671	66·3494	66·4315	66·5128	66·5940	66·6749	66·7553	66·8355
73	66·9153	66·9949	67·0740	67·1529	67·2314	67·3096	67·3875	67·4649	67·5422	67·6190
74	67·6955	67·7717	67·8476	67·9231	67·9983	68·0733	68·1479	68·2223	68·2964	68·3702
75	68·4436	68·5168	68·5896	68·6620	68·7342	68·8062	68·8778	68·9492	69·0204	69·0912
76	69·1617	69·2318	69·3017	69·3712	69·4404	69·5094	69·5780	69·6464	69·7145	69·7823
77	69·8497	69·9169	69·9838	70·0503	70·1166	70·1826	70·2483	70·3137	70·3787	70·4436
78	70·5082	70·5725	70·6365	70·7004	70·7639	70·8271	70·8901	70·9527	71·0149	71·0770
79	71·1388	71·2004	71·2617	71·3228	71·3837	71·4442	71·5045	71·5646	71·6244	71·6839
80	71·7432	71·8023	71·8611	71·9196	71·9779	72·0359	72·0937	72·1513	72·2086	72·2656
81	72·3225	72·3791	72·4354	72·4915	72·5473	72·6030	72·6584	72·7135	72·7685	72·8232
82	72·8776	72·9317	72·9856	73·0393	73·0927	73·1458	73·1988	73·2514	73·3038	73·3560
83	73·4079	73·4596	73·5111	73·5622	73·6132	73·6639	73·7145	73·7648	73·8149	73·8647
84	73·9143	73·9636	74·0127	74·0615	74·1101	74·1585	74·2067	74·2546	74·3023	74·3498
85	74·3971	74·4441	74·4910	74·5376	74·5839	74·6301	74·6760	74·7217	74·7670	74·8123
86	74·8573	74·9022	74·9468	74·9912	75·0355	75·0795	75·1233	75·1669	75·2104	75·2536
87	75·2966	75·3395	75·3821	75·4246	75·4668	75·5089	75·5507	75·5924	75·6339	75·6752
88	75·7163	75·7572	75·7980	75·8385	75·8788	75·9190	75·9590	75·9983	76·0384	76·0778
89	76·1171	76·1562	76·1952	76·2339	76·2725	76·3109	76·3492	76·3873	76·4252	76·4629
90	76·5005	76·5379	76·5751	76·6121	76·6490	76·6857	76·7223	76·7588	76·7951	76·8312
91	76·8671	76·9029	76·9385	76·9739	77·0092	77·0444	77·0794	77·1142	77·1489	77·1835
92	77·2179	77·2522	77·2863	77·3203	77·3541	77·3878	77·4213	77·4547	77·4879	77·5210
93	77·5540	77·5868	77·6195	77·6520	77·6844	77·7167	77·7488	77·7807	77·8125	77·8442

TABLE V—*continued.*

$\delta = C\,(D_r - D_v).$

v.	0	1	2	3	4	5	6	7	8	9
f/s.	degs	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
94	77·9757	77·9071	77·9384	77·9695	78·0005	78·0314	78·0622	78·0929	78·1234	78·1538
95	78·1841	78·2142	78·2442	78·2741	78·3039	78 3335	78·3630	78·3924	78·4216	78·4508
96	78·4798	78·5087	78·5375	78·5622	78·5947	78·6231	78·6514	78·6796	78·7076	78·7356
97	78·7634	78·7911	78·8188	78·8463	78·8736	78·9009	78·9280	78 9551	78·9819	79·0087
98	79·0354	79·0621	79·0886	79·1150	79·1413	79·1675	79·1936	79·2195	79·2454	79·2712
99	79·2968	79·3224	79·3478	79·3731	79·3983	79·4234	79·4484	79·4734	79·4982	79·5230
100	79·5476	79·5722	79·5966	79·6210	79·6543	79·6695	79·6935	79·7175	79·7414	79·7652
101	79·7889	79·8124	79·8359	79·8593	79·8826	79·9058	79 9289	79·9519	79·9748	79·9976
102	80·0203	80·0430	80·0655	80·0879	80·1102	80·1324	80·1544	80·1763	80·1981	80·2197
103	80·2412	80·2625	80·2837	80·3048	80·3256	80·3462	80·3667	80·3869	80·4071	80·4270
104	80·4466	80·4661	80·4854	80·5045	80·5234	80 5420	80·5605	80·5787	80·5967	80·6145
105	80·6321	80·6495	80·6667	80·6835	80·7003	80·7169	80·7333	80·7495	80·7654	80·7813
106	80·7970	80·8126	80·8280	80·8432	80·8583	80·8733	80·8882	80·9029	80·9175	80·9319
107	80·9463	80·9606	80·9747	80·9886	81·0026	81·0164	81·0301	81·0437	81·0573	81·0707
108	81·0841	81·0973	81·1105	81·1236	81·1366	81·1495	81·1624	81·1751	81·1877	81·2003
109	81·2129	81·2253	81·2377	81·2501	81·2623	81·2745	81·2866	81·2986	81·3105	81·3224
110	81·3342	81·3460	81·3571	81·3695	81·3811	81·3927	81·4042	81·4156	81·4269	81·4332
111	81·4495	81·4607	81·4719	81·4829	81·4939	81·5049	81·5159	81·5268	81·5377	81·5485
112	81·5593	81·5700	81·5807	81·5913	81·6019	81·6124	81·6230	81·6334	81·6439	81·6543
113	81·6647	81·6750	81·6853	81·6955	81·7057	81·7159	81·7260	81·7361	81·7462	81·7562
114	81·7662	81·7761	81·7861	81·7960	81·8058	81·8156	81·8254	81·8351	81·8448	81·8545
115	81·8641	81·8737	81·8833	81·8929	81·9024	81·9119	81·9213	81·9307	81·9401	81·9495
116	81·9588	81·9681	81·9774	81·9866	81·9958	82·0049	82·0141	82·0232	82·0322	82·0413
117	82·0503	82·0592	82·0682	82·0771	82·0860	82·0948	82·1036	82·1124	82·1212	82·1299
118	82·1386	82·1473	82·1559	82·1645	82·1731	82·1817	82·1902	82·1988	82·2073	82·2157
119	82·2241	82·2325	82·2408	82·2492	82·2575	82·2657	82·2740	82·2822	82·2903	82·2985
120	82·3066	82·3147	82·3228	82·3309	82·3389	82·3469	82·3549	82·3629	82·3708	82·3787
121	82·3865	82·3944	82·4022	82·4100	82·4178	82·4255	62·4333	82·4410	82·4486	82·4563
122	82·4639	82 4715	82·4790	82·4865	82·4940	82·5015	82·5090	82·5164	82·5238	82·5312
123	82·5386	82·5459	82·5533	82·5606	82·5679	82·5751	82·5824	82·5896	82·5968	82·6040
124	82·6112	82·6183	82·6254	82·6324	82·6395	82·6465	82·6535	82·6605	82·6675	82·6744
125	82·6814	82·6883	82·6951	82·7019	82·7088	82·7156	82·7224	82·7291	82·7359	82·7427
126	82·7494	82·7561	82·7627	82·7694	82·7760	82·7826	82·7892	82·7957	82·8023	82·8088
127	82·8153	82·8218	82·8283	82 8348	82·8412	82·8477	82·8541	82·0604	82·8668	82·8731
128	82·8794	82·8857	82·8920	82·8983	82·9045	82·9107	82·9169	82·9231	82·9292	82·9354
129	82·9415	82·9477	82·9538	82·9599	82·9660	82·9720	82·9780	82·9840	82·9900	82·9950
130	83·0019	83·0079	83·0138	83·0197	83·0256	83·0315	83·0373	83·0432	83·0490	83·0548
131	83·0606	83·0664	83·0721	83·0779	83·0836	83·0893	83·0950	83·1007	83·1063	83·1119
132	83·1176	83·1232	83·1288	83·1344	83·1400	83·1455	83·1511	83·1566	83·1621	83·1676
133	83·1730	83·1785	83·1840	83·1894	83·1949	83·2003	83·2057	83·2110	83·2164	83·2217
134	83·2271	83·2324	83·2377	83·2430	83·2483	83·2536	83·2588	83·2641	83·2693	83·2745
135	83·2797	83·2849	83·2900	83·2951	83·3003	83·3054	83·3105	83·3156	83·3207	83·3257
136	83·3308	83·3359	83·3409	83·3459	83·3509	83·3560	83·3609	83·3659	83·3709	83·3759
137	83·3808	83·3857	83·3906	83·3955	83·4004	83·4053	83·4101	83·4150	83·4198	83·4247
138	83·4295	83·4343	83·4391	83·4438	83·4486	83·4533	83·4581	83·4628	83·4676	83·4723
139	83·4770	83·4817	83·4863	83·4910	83·4956	83·5003	83·5049	83·5095	83·5141	83·5187
140	83·5233	83·5279	83·5325	83·5371	83·5417	83·5462	83·5507	83·5553	83·5598	83·5642
141	83·5687	83·5732	83·5777	83·5821	83·5866	83·5910	83·5954	83·5999	83·6043	83·6087
142	83·6130	83·6174	83·6218	83·6261	83·6305	83·6348	83·6392	83·6435	83·6478	83·6522
143	83·6565	83·6607	83·6650	83·6693	83·6735	83·6778	83·6820	83·6862	83·6904	83·6946
144	83·6988	83·7030	83·7072	83·7114	83·7156	83·7197	83·7239	83·7280	83·7321	83·7362
145	83·7403	83·7444	83·7485	83·7526	83·7567	83·7608	83·7649	83·7689	83·7730	83·7770
146	83·7810	83·7850	83·7891	83·7930	83·7970	83·8010	83·8050	83·8090	83·8130	83·8170
14	83·8209	83·8249	83·8288	83·8327	83·8366	83·8106	83·8445	83·8484	83·8522	83·8561
148	83·8600	83·8639	83·8677	83·8715	83 8754	83·8792	83·8830	83·8869	83·8907	83·8945
149	83·8983	83·9021	83·9059	83·9096	83·9134	83·9172	83·9209	83·9247	83·9285	83·9322
150	83·9359	83·9396	83·9433	83·9470	83·9507	83·9544	83·9581	83·9617	83·9654	83·9691
151	83·9727	83·9764	83·9800	83·9837	83·9873	83·9909	83·9946	83·9982	84·0018	84·0054
152	84·0090	84·0126	84·0161	84·0197	84·0233	84·0269	84·0304	84·0340	84·0375	84·0410
153	84·0446	84·0481	84·0516	84·0551	81·0587	84·0622	84·0657	84·0692	84·0727	84·0762

TABLE V—*continued*.

$\delta = C\,(D_V - D_v)$.

v.	0	1	2	3	4	5	6	7	8	9
f/s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
154	84·0796	84·0831	84·0866	84·0900	84·0935	84·0969	84·1004	84·1038	84·1072	84·1106
155	84·1140	84·1174	84·1208	84·1242	84·1276	84·1310	84·1344	84·1378	84·1412	84·1445
156	84·1479	84·1513	84·1546	84·1579	84·1613	84·1646	85·1679	84·1713	84·1746	84·1779
157	84·1812	84·1845	84·1878	84·1911	84·1943	84·1976	84·2009	84·2041	84·2074	84·2107
158	84·2139	84·2172	84·2204	84·2237	84·2269	84·2301	84·2333	84·2366	84·2398	84·2430
159	84·2461	84·2493	84·2525	84·2557	84·2588	84·2620	84·2652	84·2683	84·2715	84·2746
160	84·2778	84·2809	84·2840	84·2871	84·2903	84·2933	84·2965	84·2996	84·3027	84·3058
161	84·3088	84·3119	84·3150	84·3180	84·3210	84·3242	84·3272	84·3302	84·3333	84·3363
162	84·3394	84·3424	84·3454	84·3484	84·3514	84·3544	84·3574	84·3604	84·3634	84·3664
163	84·3694	84·3724	84·3753	84·3783	84·3813	84·3843	84·3872	84·3902	84·3931	84·3960
164	84·3990	84·4019	84·4048	84·4078	84·4107	84·4136	84·4165	84·4194	84·4223	84·4252
165	84·4281	84·4310	84·4339	84·4367	85·4396	84·4425	84·4453	84·4182	84·4510	84·4539
166	84·4567	84·4595	84·4624	84·4652	84·4680	84·4709	84·4737	84·4765	84·4793	84·4821
167	84·4849	84·4877	84·4905	84·4933	84·4961	84·4988	84·5016	84·5044	84·5070	84·5099
168	84·5127	84·5154	84·5181	84·5209	84·5236	84·5263	84·5291	84·5318	84·5345	84·5372
169	84·5399	84·5426	84·5453	84·5480	84·5508	84·5534	84·5561	84·5588	84·5615	84·5641
170	84·5668	84·5695	84·5721	84·5748	84·5775	84·5801	84·5828	84·5854	84·5880	84·5907
171	84·5933	84·5959	84·5985	84·6012	84·6038	84·6064	84·6090	84·6116	84·6142	84·6168
172	84·6193	84·6219	84·6245	84·6271	84·6297	84·6322	84·6348	84·6373	84·6399	84·6424
173	84·6449	84·6475	84·6500	84·6525	84·6550	84·6575	84·6601	84·6626	84·6651	84·6676
174	84·6701	84·6726	84·6750	84·6776	84·6800	84·6825	84·6850	84·6875	84·6899	84·6924
175	84·6948	84·6973	84·6997	84·7022	84·7046	84·7071	84·7095	84·7119	84·7144	84·7168
176	84·7192	84·7216	84·7240	84·7264	84·7288	84·7312	84·7336	84·7360	84·7384	84·7408
177	84·7432	84·7455	84·7479	84·7503	84·7526	84·7550	84·7574	84·7597	84·7621	84·7645
178	84·7668	84·7692	84·7715	84·7739	84·7762	84·7785	84·7809	84·7832	84·7855	84·7878
179	84·7902	84·7925	84·7948	84·7972	84·7994	84·8017	84·8040	84·8063	84·8086	84·8109
180	84·8131	84·8154	84·8177	84·8199	84·8222	84·8244	84·8267	84·8289	84·8312	84·8334
181	84·8357	84·8379	84·8401	84·8424	84·8446	84·8468	84·8490	84·8513	84·8535	84·8557
182	84·8579	84·8601	84·8623	84·8645	84·8667	84·8689	84·8711	84·8732	84·8754	84·8776
183	84·8798	84·8819	84·8841	84·8863	84·8884	84·8906	84·8927	84·8949	84·8970	84·8992
184	84·9013	84·9035	84·9056	84·9077	84·9099	84·9120	84·9141	84·9162	84·9184	84·9205
185	84·9226	84·9476	84·9268	84·9289	84·9310	84·9331	84·9351	84·9372	84·9393	84·9414
186	84·9435	84·9456	84·9476	84·9497	84·9518	84·9538	84·9559	84·9580	84·9600	84·9621
187	84·9641	84·9662	84·9682	84·9702	84·9723	84·9743	84·9763	84·9784	84·9804	84·9824
188	84·9845	84·9865	84·9885	84·9905	84·9925	84·9946	84·9966	84·9986	85·0006	85·0026
189	85·0045	85·0065	85·0085	85·0105	85·0125	85·0145	85·0165	85·0185	85·0204	85·0224
190	85·0244	85·0263	85·0283	85·0303	85·0322	85·0342	85·0361	85·0380	85·0400	85·0419
191	85·0438	85·0458	85·0477	85·0496	85·0515	85·0535	85·0554	85·0573	85·0592	85·0611
192	85·0630	85·0650	85·0669	85·0687	85·0706	85·0725	85·0744	85·0763	85·0782	85·0801
193	85·0820	85·0838	85·0857	85·0876	85·0895	85·0913	85·0932	85·0951	85·0969	85·0988
194	85·1006	85·1025	85·1043	85·1062	85·1080	85·1099	85·1117	85·1136	85·1154	85·1172
195	85·1190	85·1208	85·1227	85·1245	85·1263	85·1281	85·1299	85·1317	85·1335	85·1353
196	85·1371	85·1389	85·1407	85·1425	85·1443	85·1460	85·1478	85·1496	85·1514	85·1531
197	85·1549	85·1567	85·1584	85·1602	85·1619	85·1637	85·1654	85·1672	85·1689	85·1707
198	85·1724	85·1741	85·1759	85·1776	85·1793	85·1810	85·1827	85·1844	85·1862	85·1879
199	85·1896	85·1913	85·1930	85·1947	85·1964	85·1981	85·1998	85·2014	85·2031	85·2048
200	85·2065	85·2081	85·2098	85·2115	85·2131	85·2148	85·2165	85·2181	85·2198	85·2214
201	85·2231	85·2247	85·2264	85·2280	85·2296	85·2313	85·2329	85·2346	85·2362	85·2378
202	85·2394	85·2411	85·2427	85·2443	85·2459	85·2476	85·2492	85·2507	85·2524	85·2540
203	85·2556	85·2572	85·2588	85·2604	85·2620	85·2635	85·2651	85·2667	85·2682	85·2698
204	85·2714	85·2729	85·2745	85·2760	85·2776	85·2791	85·2807	85·2822	85·2838	85·2853
205	85·2868	85·2884	85·2899	85·2915	85·2930	85·2945	85·2960	85·2975	85·2990	85·3005
206	85·3020	85·3035	85·3051	85·3066	85·3081	85·3095	85·3110	85·3125	85·3140	85·3155
207	85·3170	85·3184	85·3199	85·3214	85·3229	85·3244	85·3258	85·3273	85·3287	85·3302
208	85·3316	85·3331	85·3345	85·3360	85·3373	85·3388	85·3403	85·3417	85·3431	85·3446
209	85·3460	85·3474	85·3488	85·3503	85·3517	85·3531	85·3545	85·3559	85·3573	85·3587
210	85·3601	85·3615	85·3629	85·3643	85·3657	85·3671	85·3685	85·3698	85·3712	85·3726
211	85·3740	85·3754	85·3767	85·3781	85·3795	85·3808	85·3822	85·3836	85·3849	85·3863
212	85·3876	85·3890	85·3903	85·3917	85·3930	85·3943	85·3957	85·3970	85·3983	85·3996
213	85·4010	85·4023	85·4036	85·4049	85·4063	85·4076	85·4089	85·4102	85·4115	85·4128

TABLE V—*continued*.

$$\delta = C\,(D_V - D_v).$$

v.	0	1	2	3	4	5	6	7	8	9
f/s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
214	85·4141	85·4154	85·4167	85·4180	85·4913	85·4206	85·4219	85·4232	85·4245	85·4258
215	85·4271	85·4284	85·4297	85·4309	85·4322	85·4335	85·4348	85·4360	85·4373	85·4385
216	85·4398	85·4411	85·4423	85·4436	85·4448	85·4461	85·4473	85·4485	85·4498	85·4510
217	85·4523	85·4535	85·4547	85·4560	85·4572	85·4584	85·4597	85·4609	85·4621	85·4633
218	85·4645	85·4658	85·4670	85·4682	85·4694	85·4706	85·4718	85·4730	85·4742	85·4754
219	85·4766	85·4778	85·4790	85·4802	85·4814	85·4825	85·4837	85·4849	85·4861	85·4873
220	85·4885	85·4896	85·4908	85·4920	85·4932	85·4943	85·4955	85·4967	85·4978	85·4990
221	85·5001	85·5013	85·5024	85·5036	85·5047	85·5059	85·5070	85·5082	85·5093	85·5105
222	85·5116	85·5128	85·5139	85·5150	85·5162	85·5173	85·5184	85·5195	85·5207	85·5218
223	85·5229	85·5240	85·5251	85·5262	85·5273	85·5285	85·5296	85·5307	85·5318	85·5329
224	85·5340	85·5351	85·5362	85·5373	85·5384	85·5394	85·5405	85·5416	85·5427	85·5438
225	85·5449	85·5460	85·5470	85·5481	85·5492	85·5502	85·5513	85·5524	85·5534	85·5545
226	85·5556	85·5566	85·5577	85·5588	85·5598	85·5609	85·5619	85·5630	85·5640	85·5651
227	85·5661	85·5672	85·5682	85·5693	85·5703	85·5713	85·5724	85·5734	85·5744	85·5755
228	85·5765	85·5775	85·5785	85·5796	85·5806	85·5816	85·5826	85·5836	85·5846	85·5856
229	85·5866	85·5876	85·5886	85·5896	85·5906	85·5916	85·5926	85·5936	85·5946	85·5956
230	85·5966	85·5976	85·5986	85·5996	85·6006	85·6015	85·6025	85·6035	85·6045	85·6055
231	85·6064	85·6074	85·6084	85·6094	85·6103	85·6113	85·6123	85·6132	85·6142	85·6151
232	85·6161	85·6171	85·6180	85·6190	85·6199	85·6209	85·6218	85·6228	85·6237	85·6247
233	85·6256	85·6265	85·6275	85·6284	85·6294	85·6303	85·6312	85·6321	85·6331	85·5340
234	85·6349	85·6358	85·6367	85·6377	85·6386	85·6395	85·6404	85·6413	85·6422	85·6431
235	85·6441	85·6450	85·6459	85·6468	85·6477	85·6486	85·6495	85·6504	85·6513	85·6522
236	85·6531	85·6540	85·6549	85·6558	85·6566	85·6575	85·6584	85·6593	85·6602	85·6611
237	85·6619	85·6628	85·6637	85·6646	85·6654	85·6663	85·6672	85·6680	85·6689	85·6698
238	85·6706	85·6715	85·6724	85·6732	85·6741	85·6749	85·6758	85·6766	85·6775	85·6783
239	85·6792	85·6800	85·6809	85·6817	85·6826	85·6834	85·6843	85·6851	85·6859	85·6868
240	85·6876	85·6885	85·6893	85·6901	85·6909	85·6918	85·6926	85·6934	85·6942	85·6951
241	85·6959	85·6967	85·6975	85·6984	85·6992	85·7000	85·7008	85·7016	85·7024	85·7032
242	85·7041	85·7049	85·7057	85·7065	85·7073	85·7081	85·7089	85·7097	85·7105	85·7113
243	85·7121	85·7128	85·7136	85·7144	85·7152	85·7160	85·7168	85·7176	85·7184	85·7192
244	85·7200	85·7207	85·7215	85·7223	85·7231	85·7239	85·7246	85·7254	85·7262	85·7270
245	85·7277	85·7285	85·7293	85·7301	85·7308	85·7316	85·7324	85·7331	85·7339	85·7346
246	85·7354	85·7362	85·7369	85·7377	85·7384	85·7392	85·7399	85·7407	85·7414	85·7422
247	85·7429	85·7436	85·7444	85·7451	85·7459	85·7466	85·7474	85·7481	85·7488	85·7496
248	85·7503	85·7510	85·7517	85·7525	85·7532	85·7539	85·7547	85·7554	85·7561	85·7568
249	85·7575	85·7583	85·7590	85·7597	85·7604	85·7611	85·7618	85·7626	85·7633	85·7640

Table VI.

Inclination, or I(v) Table.

$$\tan\phi - \tan\theta = C(I_V - I_v).$$

v.	0	1	2	3	4	5	6	7	8	9	Δ
50	0·00000	0·00421	0·00839	0·01255	0·01669	0·02080	0·02489	0·02896	0·03300	0·03702	410
51	·04101	·04497	·04891	·05283	·05673	·06061	·06446	·06829	·07210	·07589	387
52	·07966	·08340	·08712	·09082	·09450	·09816	·10180	·10542	·10902	·11260	365
53	·11616	·11969	·12321	·12671	·13019	·13365	·13709	·14051	·14391	·14729	345
54	·15066	·15400	·15732	·16063	·16392	·16719	·17045	·17369	·17691	·18011	326
55	·18330	·18647	·18962	·19275	·19587	·19897	·20205	·20512	·20817	·21121	309
56	·21423	·21713	·22021	·22318	·22613	·22907	·23200	·23491	·23781	·24069	293
57	·24356	·24641	·24925	·25207	·25488	·25767	·26045	·26321	·26596	·26870	279
58	·27142	·27413	·27683	·27951	·28218	·28484	·28749	·29012	·29274	·29535	265
59	·29795	·30054	·30311	·30567	·30822	·31076	·31328	·31579	·31829	·32078	253
60	·32325	·32571	·32816	·33060	·33303	·33545	·33786	·34026	·34265	·34503	242
61	·34740	·34975	·35209	·35442	·35674	·35905	·36135	·36364	·36592	·36818	230
62	·37043	·37267	·37490	·37712	·37933	·38153	·38373	·38592	·38810	·39027	220
63	·39243	·39458	·39672	·39885	·40097	·40308	·40517	·40725	·40932	·41138	210
64	·41343	·41548	·41752	·41955	·42157	·42358	·42558	·42757	·42955	·43152	201
65	·43349	·43545	·43740	·43934	·44127	·44319	·44510	·44700	·44890	·45079	192
66	·45267	·45454	·45640	·45825	·46009	·46193	·46376	·46558	·46739	·46920	183
67	·47100	·47279	·47457	·47634	·47811	·47987	·48162	·48336	·48509	·48682	175
68	·48854	·49025	·49196	·49366	·49535	·49703	·49871	·50038	·50204	·50370	168
69	·50535	·50699	·50863	·51026	·51188	·51350	·51511	·51671	·51830	·51989	161
70	·52147	·52304	·52461	·52617	·52773	·52928	·53082	·53236	·53389	·53541	155
71	·53693	·53844	·53994	·54144	·54293	·54442	·54590	·54737	·54884	·55030	148
72	·55175	·55320	·55464	·55607	·55750	·55892	·56034	·56175	·56315	·56455	142
73	·56594	·56733	·56871	·57008	·57145	·57281	·57417	57552	·57687	·57821	136
74	·57954	·58087	·58219	·58351	·58482	·58613	·58743	·58873	·59002	·59131	131
75	·59259	·59387	·59514	·59641	·59767	·59893	·60018	·60142	·60266	·60389	125
76	·60512	·60634	·60756	·60877	·60998	·61118	·61238	·61357	·61476	·61594	120
77	·61712	·61829	·61946	·62062	·62178	·62293	·62408	·62522	·62635	·62748	115
78	·62860	·62972	·63084	·63195	·63306	·63416	·63526	·63635	·63744	·63852	110
79	·63960	·64067	·64174	·64281	·64387	·64492	·64597	·64702	·64806	·64910	105
80	·65013	·65116	·65219	·65321	·65423	·65524	·65625	·65725	·65825	·65924	101
81	·66023	·66122	·66220	·66318	·66415	·66512	·66609	·66705	·66801	·66896	97
82	·66991	·67086	·67180	·67274	·67367	·67460	·67522	·67644	·67735	·67826	93
83	·67916	·68006	·68096	·68185	·68274	·68362	·68450	·68537	·68624	·68711	88
84	·68797	·68883	·68969	·69054	·69139	·69223	·69307	·69391	·69474	·69557	84
85	·69639	·69721	·69803	·69884	·69965	·70045	·70125	·70205	·70284	·70363	80

Table VI—*continued.*

$$\tan\phi - \tan\theta = C(I_r - I_c).$$

r.	0	1	2	3	4	5	6	7	8	9	Δ
86	0·70442	0·70520	0·70598	0·70676	0·70753	0·70830	0·70907	0·70983	0·71059	0·71134	77
87	·71209	·71284	·71358	·71432	·71506	·71579	·71652	·71725	·71797	·71869	73
88	·71941	·72012	·72083	·72154	·72224	·72294	·72364	·72434	·72503	·72572	70
89	·72641	·72709	·72777	·72845	·72912	·72979	·73046	·73112	·73178	·73244	67
90	·73310	·73375	·73440	·73505	·73569	·73633	·73697	·73761	·73824	·73887	64
91	·73950	·74012	·74074	·74136	·74198	·74259	·74320	·74381	·74442	·74502	61
92	·74562	·74622	·74681	·74740	·74799	·74858	·74916	·74974	·75032	·75090	59
93	·75148	·75205	·75262	·75319	·75376	·75432	·75488	·75544	·75600	·75655	56
94	·75710	·75765	·75820	·75874	·75928	·75982	·76036	·76089	·76142	·76195	54
95	·76248	·76301	·76353	·76405	·76457	·76509	·76560	·76611	·76662	·76713	52
96	·76764	·76814	·76864	·76914	·76964	·77014	·77063	·77112	·77161	·77210	50
97	·77259	·77307	·77355	·77403	·77451	·77499	·77546	·77593	·77640	·77687	48
98	·77734	·77780	·77826	·77872	·77918	·77964	·78010	·78055	·78100	·78145	46
99	·78190	·78235	·78279	·78323	·78367	·78411	·78455	·78499	·78542	·78585	44
100	·78628	·78671	·78714	·78757	·78799	·78841	·78883	·78925	·78967	·79008	42
101	·79049	·79090	·79131	·79172	·79212	·79252	·79292	·79332	·79372	·79412	40
102	·79452	·79492	·79531	·79570	·79609	·79648	·79686	·79724	·79762	·79800	39
103	·79837	·79874	·79911	·79948	·79984	·80020	·80056	·80091	·80126	·80161	36
104	·80195	·80229	·80262	·80295	·80328	·80360	·80392	·80424	·80455	·80486	32
105	·80517	·80547	·80577	·80606	·80635	·80664	·80692	·80720	·80748	·80776	29
106	·80803	·80830	·80856	·80882	·80908	·80934	·80960	·80986	·81011	·81036	26
107	·81061	·81086	·81111	·81135	·81159	·81183	·81207	·81231	·81255	·81278	24
108	·81301	·81324	·81347	·81370	·81393	·81415	·81437	·81459	·81481	·81503	22
109	·81525	·81547	·81568	·81589	·81610	·81631	·81652	·81673	·81694	·81715	21
110	·81736	·81756	·81776	·81796	·81816	·81836	·81856	·81876	·81896	·81916	20
111	·81936	·81956	·81975	·81994	·82013	·82032	·82051	·82070	·82089	·82108	19
112	·82127	·82146	·82165	·82184	·82203	·82221	·82239	·82257	·82275	·82293	18
113	·82311	·82329	·82347	·82365	·82383	·82401	·82419	·82437	·82454	·82471	18
114	·82488	·82506	·82523	·82540	·82557	·82574	·82591	·82608	·82625	·82642	17
115	·82659	·82676	·82693	·82710	·82727	·82744	·82760	·82776	·82792	·82808	17
116	·82824	·82840	·82856	·82872	·82888	·82904	·82920	·82936	·82952	·82968	16
117	·82983	·82999	·83014	·83030	·83045	·83061	·83076	·83092	·83107	·83122	15
118	·83137	·83152	·83167	·83182	·83197	·83212	·83227	·83242	·83257	·83272	15
119	·83286	·83301	·83315	·83330	·83344	·83359	·83373	·83388	·83402	·83416	14
120	·83430	·83444	·83458	·83472	·83486	·83500	·83514	·83528	·83542	·83556	14
121	·83569	·83583	·83596	·83610	·83623	·83637	·83650	·83664	·83677	·83691	14
122	·83704	·83717	·83730	·83743	·83756	·83769	·83782	·83795	·83808	·83821	13
123	·83834	·83847	·83860	·83873	·83885	·83898	·83910	·83923	·83935	·83948	13
124	·83960	·83973	·83985	·83998	·84010	·84022	·84034	·84046	·84058	·84070	12
125	·84082	·84094	·84106	·84118	·84130	·84142	·84154	·84166	·84178	·84190	12
126	·84201	·84213	·84224	·84236	·84247	·84259	·84270	·84282	·84293	·84305	12
127	·84316	·84328	·84339	·84351	·84362	·84373	·84384	·84395	·84406	·84417	11

TABLE VI—*continued.*

$$\tan \phi - \tan \theta = C(I_V - I_v).$$

v.	0	1	2	3	4	5	6	7	8	9	Δ
128	0·84428	0·84439	0·84450	0·84461	0·84472	0·84483	0·84494	0·84505	0·84515	0·84526	11
129	·84536	·84547	·84557	·84568	·84578	·84589	·84599	·84610	·84620	·84631	11
130	·84641	·84652	·84662	·84673	·84683	·84693	·84703	·84713	·84723	·84733	10
131	·84743	·84753	·84763	·84773	·84783	·84793	·84803	·848129	·848228	·848327	100
132	·848425	·848523	·848621	·848718	·848815	·848912	·849009	·849105	·849201	·849297	97
133	·849393	·849488	·849583	·849678	·849773	·849867	·849961	·850055	·850149	·850242	94
134	·850335	·850428	·850521	·850613	·850705	·850797	·850888	·850979	·851070	·851161	92
135	·851252	·851342	·851432	·851522	·851612	·851701	·851790	·851879	·851968	·852057	89
136	·852145	·852233	·852321	·852409	·852496	·852583	·852670	·852757	·852844	·852930	87
137	·853016	·853102	·853188	·853274	·853359	·853444	·853529	·853614	·853698	·853782	85
138	·853866	·853950	·854034	·854118	·854201	·854284	·854367	·854450	·854532	·854614	83
139	·854696	·854778	·854860	·854941	·855022	·855103	·855184	·855265	·855346	·855426	81
140	·855506	·855586	·855666	·855746	·855825	·855904	·855983	·856062	·856141	·856219	79
141	·856297	·856375	·856453	·856531	·856609	·856686	·856763	·856840	·856917	·856994	77
142	·857071	·857148	·857224	·857300	·857376	·857452	·857528	·857603	·857678	·857753	76
143	·857828	·857903	·857978	·858052	·858126	·858200	·858274	·858348	·858422	·858495	74
144	·858568	·858641	·858714	·858787	·858860	·858933	·859005	·859077	·859149	·859221	73
145	·859293	·859365	·859437	·859508	·859579	·859650	·859721	·859792	·859863	·859933	71
146	·860003	·860073	·860143	·860213	·860283	·860353	·860422	·860491	·860560	·860629	70
147	·860698	·860767	·860836	·860904	·860972	·861040	·861108	·861176	·861244	·861312	68
148	·861380	·861448	·861515	·861582	·861649	·861716	·861783	·861850	·861917	·861983	67
149	·862049	·862115	·862181	·862247	·862313	·862379	·862445	·862510	·862575	·862640	66
150	·862705	·862770	·862835	·862900	·862965	·863029	·863093	·863157	·863221	·863285	64
151	·863349	·863413	·863477	·863541	·863604	·863667	·863730	·863793	·863856	·863919	63
152	·863982	·864045	·864108	·864170	·864232	·864294	·864356	·864418	·864480	·864542	62
153	·864604	·864666	·864728	·864789	·864850	·864911	·864972	·865033	·865094	·865155	61
154	·865216	·865277	·865337	·865397	·865457	·865517	·865577	·865637	·865697	·865757	60
155	·865817	·865877	·865936	·865995	·866054	·866113	·866172	·866231	·866290	·866349	59
156	·866408	·866467	·866525	·866583	·866641	·866699	·866757	·866815	·866873	·866931	58
157	·866989	·867047	·867104	·867161	·867218	·867275	·867332	·867389	·867446	·867503	57
158	·867560	·867617	·867674	·867730	·867786	·867842	·867898	·867954	·868010	·868066	56
159	·868122	·868178	·868234	·868290	·868345	·868400	·868455	·868510	·868565	·868620	55
160	·868675	·868730	·868785	·868840	·868894	·868948	·869002	·869056	·869110	·869164	54
161	·869218	·869272	·869326	·869380	·869434	·869487	·869540	·869593	·869646	·869699	53
162	·869752	·869805	·869858	·869911	·869964	·870017	·870069	·870121	·870173	·870225	53
163	·870277	·870329	·870381	·870433	·870485	·870537	·870589	·870641	·870692	·870743	52
164	·870794	·870845	·870896	·870947	·870998	·871049	·871100	·871151	·871202	·871252	51
165	·871302	·871352	·871402	·871452	·871502	·871552	·871602	·871652	·871702	·871752	50
166	·871802	·871852	·871902	·871951	·872000	·872049	·872098	·872147	·872196	·872245	49
167	·872294	·872343	·872392	·872441	·872490	·872538	·872586	·872634	·872682	·872730	48
168	·872778	·872826	·872874	·872922	·872970	·873018	·873066	·873113	·873160	·873207	48
169	·873254	·873301	·873348	·873395	·873442	·873489	·873536	·873583	·873629	·873676	47

TABLE VI—*continued.*

$$\tan\phi - \tan\theta = C(I^r - I_v)$$

r.	0	1	2	3	4	5	6	7	8	9	Δ
170	0·873722	0·873769	0·873815	0·873861	0·873907	0·873953	0·873999	0·874045	0·874091	0·874137	46
171	·874183	·874229	·874275	·874321	·874366	·874411	·874456	·874501	·874546	·874591	45
172	·874636	·874681	·874726	·874771	·874816	·874861	·874906	·874950	·874994	·875038	45
173	·875082	·875126	·875170	·875214	·875258	·875302	·875346	·875390	·875434	·875478	44
174	·875521	·875565	·875608	·875652	·875695	·875738	·875781	·875824	·875867	·875910	43
175	·875953	·875996	·876039	·876082	·876125	·876168	·876210	·876252	·876294	·876336	43
176	·876378	·876420	·876462	·876504	·876546	·876588	·876630	·876672	·876714	·876756	42
177	·876797	·876839	·876880	·876922	·876963	·877004	·877045	·877086	·877127	·877168	41
178	·877209	·877250	·877291	·877332	·877373	·877414	·877455	·877495	·877535	·877575	41
179	·877615	·877655	·877695	·877735	·877775	·877815	·877855	·877895	·877935	·877975	40
180	·878015	·878055	·878095	·878135	·878174	·878213	·878252	·878291	·878330	·878369	39
181	·878408	·878447	·878486	·878525	·878564	·878603	·878642	·878681	·878719	·878758	39
182	·878796	·878835	·878873	·878912	·878950	·878988	·879026	·879064	·879102	·879140	38
183	·879178	·879216	·879254	·879292	·879330	·879368	·879406	·879443	·879480	·879517	38
184	·879554	·879591	·879628	·879665	·879702	·879739	·879776	·879813	·879850	·879887	37
185	·879924	·879961	·879998	·880035	·880072	·880109	·880145	·880181	·880217	·880253	37
186	·880289	·880325	·880361	·880397	·880433	·880469	·880505	·880541	·880577	·880613	36
187	·880649	·880685	·880721	·880757	·880793	·880829	·880864	·880899	·880934	·880969	36
188	·881004	·881039	·881074	·881109	·881144	·881179	·881214	·881249	·881284	·881319	35
189	·881354	·881389	·881424	·881459	·881494	·881529	·881563	·881597	·881631	·881665	35
190	·881699	·881733	·881767	·881801	·881835	·881869	·881903	·881937	·881971	·882005	34
191	·882039	·882073	·882107	·882141	·882175	·882209	·882242	·882275	·882308	·882341	34
192	·882374	·882407	·882440	·882473	·882506	·882539	·882572	·882605	·882638	·882671	33
193	·882704	·882737	·882770	·882803	·882836	·882869	·882901	·882933	·882965	·882997	33
194	·883029	·883061	·883093	·883125	·883157	·883189	·883221	·883253	·883285	·883317	32
195	·883349	·883381	·883413	·883445	·883477	·883509	·883541	·883572	·883603	·883634	32
196	·883665	·883697	·883728	·883759	·883790	·883821	·883852	·883883	·883914	·883945	31
197	·883976	·884007	·884038	·884069	·884100	·884131	·884161	·884191	·884221	·884251	31
198	·884281	·884311	·884341	·884371	·884401	·884431	·884461	·884491	·884521	·884551	30
199	·884581	·884611	·884641	·884671	·884701	·884731	·884760	·884789	·884818	·884847	30
200	·884876	·884905	·884934	·884963	·884992	·885021	·885050	·885079	·885108	·885137	29
201	·885166	·885195	·885224	·885253	·885282	·885311	·885339	·885367	·885395	·885423	29
202	·885451	·885479	·885507	·885535	·885563	·885591	·885619	·885647	·885675	·885703	28
203	·885731	·885759	·885787	·885815	·885843	·885871	·885898	·885925	·885952	·885979	28
204	·886006	·886033	·886060	·886087	·886114	·886141	·886168	·886195	·886222	·886249	27
205	·886276	·886303	·886330	·886357	·886384	·886411	·886437	·886463	·886489	·886515	27
206	·886541	·886567	·886593	·886619	·886645	·886671	·886697	·886723	·886749	·886775	26
207	·886801	·886827	·886853	·886879	·886905	·886931	·886957	·886982	·887007	·887032	26
208	·887057	·887083	·887108	·887133	·887158	·887183	·887208	·887233	·887258	·887283	25
209	·887308	·887333	·887358	·887383	·887408	·887433	·887457	·887482	·887506	·887531	25
210	·887555	·887580	·887604	·887629	·887653	·887677	·887701	·887725	·887749	·887773	24
211	·887797	·887821	·887845	·887869	·887893	·887917	·887941	·887965	·887988	·888012	24

TABLE VI—*continued.*

$\tan\phi - \tan\theta = C(I_V - I_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Δ
212	0·888035	0·888059	0·888082	0·888106	0·888129	0·888153	0·888176	0·888200	0·888223	0·888246	23
213	·888269	·888292	·888315	·888338	·888361	·888384	·888407	·888430	·888453	·888476	23
214	·888499	·888522	·888545	·888568	·888590	·888613	·888635	·888658	·888680	·888703	23
215	·888725	·888748	·888770	·888793	·888815	·888837	·888859	·888881	·888903	·888925	22
216	·888947	·888969	·888991	·889013	·889035	·889057	·889079	·889101	·889122	·889144	22
217	·889165	·889187	·889208	·889230	·889251	·889273	·889294	·889316	·889337	·889358	21
218	·889379	·889401	·889422	·889433	·889464	·889485	·889506	·889527	·889548	·889569	21
219	·889590	·889611	·889632	·889653	·889674	·889695	·889715	·889736	·889756	·889777	21
220	·889797	·889818	·889838	·889859	·889879	·889900	·889920	·889941	·889961	·889981	20
221	·890001	·890021	·890041	·890061	·890081	·890101	·890121	·890141	·890161	·890181	20
222	·890201	·890221	·890241	·890261	·890281	·890301	·890320	·890340	·890359	·890379	20
223	·890398	·890418	·890437	·890457	·890476	·890496	·890515	·890534	·890553	·890572	19
224	·890591	·890610	·890629	·890648	·890677	·890686	·890705	·890724	·890743	·890762	19
225	·890781	·890800	·890819	·890838	·890857	·890876	·890894	·890913	·890931	·890954	19
226	·890968	·890986	·891005	·891023	·891042	·891060	·891079	·891097	·891116	·891134	19
227	·891152	·891171	·891189	·891207	·891225	·891243	·891261	·891279	·891297	·891315	18
228	·891333	·891351	·891369	·891387	·891405	·891423	·891441	·891459	·891477	·891495	18
229	·891512	·891530	·891548	·891566	·891584	·891601	·891619	·891636	·891654	·891671	18
230	·891689	·891706	·891724	·891741	·891759	·891776	·891794	·891811	·891829	·891846	18
231	·891864	·891881	·891899	·891916	·891934	·891951	·891969	·891986	·892003	·892020	17
232	·892037	·892055	·892072	·892090	·892107	·892124	·892141	·892158	·892175	·892192	17
233	·892209	·892227	·892244	·892261	·892278	·892295	·892312	·892329	·892346	·892363	17
234	·892380	·892397	·892414	·892431	·892448	·892465	·892482	·892499	·892516	·892533	17
235	·892549	·892566	·892583	·892600	·892617	·892634	·892651	·892668	·892684	·892701	17
236	·892717	·892734	·892751	·892768	·892785	·892802	·892818	·892835	·892851	·892868	17
237	·892884	·892901	·892918	·892935	·892951	·892968	·892984	·893001	·893017	·893034	17
238	·893050	·893067	·893083	·893100	·893116	·893133	·893149	·893166	·893182	·893199	17
239	·893215	·893232	·893248	·893265	·893281	·893298	·893314	·893330	·893346	·893362	16
240	·893378	·893395	·893411	·893428	·893444	·893460	·893476	·893492	·893508	·893524	16
241	·893540	·893557	·893573	·893589	·893605	·893621	·893637	·893653	·893669	·893685	16
242	·893701	·893717	·893733	·893749	·893765	·893781	·893797	·893813	·893829	·893845	16
243	·893861	·893877	·893893	·893909	·893925	·893941	·893957	·893973	·893989	·894005	16
244	·894020	·894036	·894052	·894068	·894084	·894100	·894116	·894132	·894147	·894163	16
245	·894178	·894194	·894210	·894226	·894242	·894258	·894273	·894289	·894304	·894320	16
246	·894335	·894351	·894367	·894383	·894398	·894414	·894429	·894445	·894460	·894476	16
247	·894491	·894507	·894522	·894538	·894553	·894569	·894584	·894600	·894615	·894631	16
248	·894646	·894662	·894677	·894693	·894708	·894724	·894739	·894755	·894770	·894785	15
249	·894800	·894816	·894831	·894847	·894862	·894877	·894892	·894907	·894922	·894937	15
250	·894952	·894968	·894983	·894998	·895013	·895028	·895043	·895058	·895073	·895088	15
251	·895103	·895118	·895133	·895148	·895163	·895178	·895193	·895208	·895223	·895238	15
252	·895253	·895268	·895283	·895298	·895313	·895328	·895343	·895358	·895372	·895387	15
253	·895401	·895416	·895431	·895446	·895461	·895476	·895490	·895505	·895519	·895534	15

TABLE VI—*continued.*

$\tan \phi - \tan \theta = C(I_V - I_v)$.

v.	0	1	2	3	4	5	6	7	8	9	Δ
254	0·895548	0·895563	0·895577	0·895592	0·895606	0·895621	0·895635	0·895650	0·895664	0·895679	15
255	·895693	·895708	·895722	·895737	·895751	·895766	·895780	·895794	·895808	·895822	14
256	·895836	·895851	·895865	·895879	·895893	·895907	·895921	·895935	·895949	·895963	14
257	·895977	·895991	·896005	·896019	·896033	·896047	·896061	·896075	·896089	·896103	14
258	·896116	·896130	·896144	·896158	·896172	·896186	·896199	·896213	·896226	·896240	14
259	·896253	·896267	·896280	·896294	·896307	·896321	·896334	·896348	·896361	·896375	14
260	·896388	·896402	·896415	·896429	·896442	·896456	·896469	·896482	·896495	·896508	13
261	·896521	·896535	·896548	·896561	·896574	·896587	·896600	·896613	·896626	·896639	13
262	·896652	·896665	·896678	·896691	·896704	·896717	·896730	·896743	·896756	·896769	13
263	·896781	·896794	·896807	·896820	·896833	·896846	·896859	·896872	·896884	·896897	13
264	·896909	·896922	·896935	·896948	·896960	·896973	·896985	·896998	·897010	·897023	13
265	·897035	·897048	·897060	·897073	·897085	·897098	·897110	·897123	·897135	·897147	12
266	·897159	·897172	·897184	·897197	·897209	·897221	·897233	·897245	·897257	·897269	12
267	·897281	·897293	·897305	·897317	·897329	·897341	·897353	·897365	·897377	·897389	12
268	·897401	·897413	·897425	·897437	·897449	·897461	·897473	·897485	·897497	·897509	12
269	·897520	·897532	·897544	·897556	·897568	·897580	·897591	·897603	·897614	·897626	12
270	·897637	·897649	·897660	·897672	·897683	·897695	·897706	·897718	·897729	·897741	12
271	·897752	·897764	·897775	·897787	·897798	·897810	·897821	·897833	·897844	·897855	11
272	·897866	·897878	·897889	·897901	·897912	·897923	·897934	·897945	·897956	·897967	11
273	·897978	·897990	·898001	·898012	·898023	·898034	·898045	·898056	·898067	·898078	11
274	·898089	·898100	·898111	·898122	·898133	·898144	·898155	·898166	·898177	·898188	11
275	·898198	·898209	·898220	·898231	·898242	·898253	·898264	·898275	·898285	·898296	11
276	·898306	·898317	·898328	·898339	·898349	·898360	·898370	·898381	·898391	·898402	11
277	·898412	·898423	·898433	·898444	·898454	·898465	·898475	·898486	·898496	·898506	10
278	·898516	·898527	·898537	·898548	·898558	·898569	·898579	·898589	·898599	·898609	10
279	·898619	·898630	·898640	·898650	·898660	·898670	·898680	·898690	·898700	·898710	10
280	·898720	·898730	·898740	·898750	·898760	·898770	·898780	·898790	·898800	·898810	10
281	·898819										

Table VII.

Altitude or $A(v)$ Table.

v.	0	1	2	3	4	5	6	7	8	9	Δ
50	0·00	0·11	0·33	0·68	1·16	1·76	2·49	3·34	4·32	5·43	0·67
51	6·66	8·01	9·50	11·13	12·89	14·78	16·79	18·91	21·13	23·45	1·92
52	25·87	28·41	31·08	33·88	36·80	39·83	42·96	46·19	49·52	52·95	3·06
53	56·47	60·01	63·83	67·66	71·60	75·64	79·78	83·03	88·38	92·83	4·09
54	97·39	102·03	106·76	111·57	116·47	121·46	126·53	131·68	136·93	142·26	5·03
55	147·67	153·17	158·76	164·43	170·19	176·04	181·97	187·98	194·09	200·28	5·88
56	206·49	212·83	219·24	225·72	232·27	238·89	245·58	252·35	259·19	266·09	6·66
57	273·07	280·12	287·24	294·43	301·69	309·02	316·42	323·90	331·45	339·06	7·37
58	346·75	354·50	362·32	370·20	378·13	386·13	394·19	402·32	410·50	418·74	8·03
59	427·05	435·42	443·85	452·34	460·89	469·50	478·18	486·91	495·71	504·57	8·64
60	513·49	522·47	531·48	540·56	549·69	558·87	568·10	577·39	586·72	596·10	9·21
61	605·56	615·03	624·56	634·15	643·78	653·47	663·21	673·00	682·84	692·73	9·71
62	702·69	712·68	722·72	732·80	742·92	753·08	763·28	773·52	783·81	794·14	10·18
63	804·51	814·92	825·37	835·86	846·40	856·97	867·59	878·25	888·95	899·69	10·60
64	910·48	921·31	932·17	942·05	953·99	964·95	975·95	986·98	998·04	*009·14	10·98
65	1 020·28	031·45	042·65	053·89	065·16	076·47	087·81	099·19	110·60	122·04	11·32
66	133·52	145·02	156·55	168·11	179·70	191·32	202·96	214·63	226·33	238·06	11·63
67	249·82	261·61	273·42	285·26	297·13	309·03	320·95	332·91	344·89	356·90	11·91
68	1 368·94	381·00	393·09	405·20	417·34	429·50	441·69	453·90	466·13	478·39	12·17
69	490·67	502·98	515·31	527·67	540·05	552·45	564·88	577·33	589·81	602·31	12·42
70	614·84	627·39	639·95	652·53	665·14	677·76	690·40	703·06	715·73	728·42	12·63
71	1 741·14	753·87	766·62	779·39	792·17	804·97	817·80	830·64	843·50	856·38	12·81
72	869·27	882·16	895·07	907·99	920·92	933·86	946·82	959·72	972·77	985·76	12·95
73	998·77	*011·79	*024·82	*037·86	*050·91	*063·98	*077·06	*090·15	*103·26	*116·37	13·07
74	2 129·50	142·64	155·79	168·94	182·11	195·29	208·48	221·67	234·88	248·10	13·18
75	261·33	274·56	287·81	301·07	314·34	327·61	340·90	354·20	367·51	380·82	13·28
76	394·15	407·48	420·82	434·15	447·49	460·82	474·16	487·50	500·84	514·18	13·34
77	2 527·52	540·86	554·20	567·54	580·89	594·24	607·59	620·93	634·28	647·63	13·35
78	660·98	674·33	687·68	701·03	714·38	727·73	741·07	754·42	767·76	781·10	13·35
79	794·44	807·79	821·13	834·47	847·81	861·15	874·49	887·83	901·16	914·49	13·34
80	2 927·82	941·16	954·49	967·82	981·15	994·47	*007·79	*021·11	*034·42	*047·73	13·32
81	3 061·03	074·33	087·62	100·91	114·19	127·47	140·75	154·02	167·29	180·56	13·28
82	193·83	207·08	220·32	233·55	246·77	259·97	273·16	286·34	299·51	312·66	13·20
83	3 325·80	338·93	352·04	365·14	378·24	391·32	404·38	417·43	430·48	443·51	13·07
84	456·52	469·51	482·49	495·45	508·40	521·33	534·25	547·15	560·04	572·91	12·93
85	585·77	598·61	611·44	624·25	637·05	649·83	662·60	675·35	688·09	700·81	12·78
86	3 713·52	726·21	738·89	751·55	764·19	776·82	789·43	802·03	814·61	827·18	12·62
87	839·73	852·26	864·78	877·29	889·78	902·25	914·71	927·15	939·57	951·98	12·47
88	964·38	976·76	989·12	*001·47	*013·80	*026·11	*038·41	*050·69	*062·96	*075·21	12·31
89	4 087·45	099·67	111·87	124·06	136·23	148·38	160·52	172·64	184·75	196·84	12·15
90	208·92	220·98	233·02	245·05	257·06	269·05	281·03	292·99	304·94	316·87	11·99
91	328·79	340·69	352·57	364·44	376·29	388·12	399·94	411·74	423·53	435·30	11·83

TABLE VII—*continued.*

ALTITUDE OR $A(v)$ TABLE.

v.	0	1	2	3	4	5	6	7	8	9	Δ
92	4 447·05	458·79	470·51	482·21	493·90	505·57	517·23	528·87	540·50	552·11	11·67
93	563·70	575·28	586·84	598·38	609·91	621·42	632·92	644·40	655·86	667·31	11·50
94	678·74	690·16	701·56	712·94	724·31	735·66	746·99	758·31	769·62	780·91	11·34
95	4 792·18	803·44	814·68	825·90	837·11	848·30	859·48	870·64	881·79	892·92	11·19
96	904·03	915·13	926·21	937·28	948·33	959·36	970·38	981·38	992·37	*003·34	11·03
97	5 014·30	025·24	036·17	047·08	057·97	068·85	079·71	090·56	101·39	112·20	10·87
98	5 123·00	133·78	144·55	155·30	166·04	176·76	187·47	198·16	208·84	219·50	10·71
99	230·14	240·77	251·38	261·98	272·56	283·13	293·68	304·22	314·74	325·25	10·56
100	335·74	346·23	356·70	367·15	377·58	388·00	398·40	408·78	419·14	429·49	10·41
101	5 439·82	450·15	460·46	470·74	481·00	491·24	501·46	511·66	521·84	531·99	10·23
102	542·12	552·23	562·32	572·39	582·44	592·46	602·46	612·44	622·40	632·34	10·01
103	642·26	652·12	661·91	671·62	681·27	691·84	700·32	709·73	719·08	728·35	9·53
104	5 737·54	746·66	755·70	764·66	773·56	782·38	791·12	799·79	808·39	816·91	8·78
105	825·35	833·60	841·79	849·92	857·99	866·00	873·95	881·84	889·67	897·44	7·98
106	905·16	912·82	920·41	927·94	935·42	942·84	950·19	957·49	964·73	971·91	7·39
107	5 979·03	986·15	993·25	*000·31	*007·34	*014·34	*021·30	*028·24	*035·15	*042·02	6·98
108	6 048·86	055·67	062·45	069·20	075·91	082·59	089·25	095·87	102·46	109·02	6·67
109	115·55	122·07	128·57	135·05	141·51	147·95	154·37	160·77	167·15	173·51	6·43
110	6 179·84	186·15	192·44	198·71	204·96	211·19	217·40	223·59	229·75	235·89	6·22
111	242·01	248·13	254·24	260·34	266·42	272·49	278·55	284·60	290·63	296·61	6·07
112	302·66	308·65	314·63	320·60	326·55	332·49	338·42	344·33	350·23	356·12	5·93
113	6 361·99	367·86	373·73	379·59	385·44	391·28	397·11	402·93	408·74	414·55	5·84
114	420·35	426·14	431·92	437·69	443·45	449·20	454·94	460·67	466·39	472·11	5·75
115	477·82	483·52	489·21	494·89	500·56	506·22	511·87	517·52	523·16	528·79	5·66
116	6 534·41	540·02	545·62	551·21	556·80	562·38	567·95	573·51	579·06	584·60	5·57
117	590·14	595·67	601·19	606·70	612·20	617·69	623·17	628·65	634·12	639·58	5·49
118	645·03	650·47	655·91	661·34	666·76	672·17	677·57	682·96	688·35	693·73	5·41
119	6 699·10	704·46	709·81	715·16	720·50	725·83	731·15	736·46	741·77	747·07	5·33
120	752·36	757·64	762·91	768·18	773·44	778·69	783·93	789·17	794·40	799·62	5·25
121	804·83	810·03	815·23	820·42	825·60	830·77	835·94	841·10	846·25	851·39	5·17
122	6 856·52	861·65	866·77	871·88	876·98	882·08	887·17	892·25	897·32	902·39	5·09
123	907·45	912·51	917·56	922·60	927·63	932·66	937·68	942·69	947·69	952·69	5·02
124	957·68	962·66	967·64	972·61	977·57	982·52	987·47	992·41	997·34	*002·26	4·95
125	7 007·18	012·09	017·00	021·90	026·79	031·68	036·56	041·43	046·30	051·16	4·88
126	056·01	060·86	065·70	070·53	075·36	080·18	084·99	089·80	094·60	099·39	4·82
127	104·18	108·96	113·74	118·51	123·28	128·04	132·79	137·54	142·28	147·02	4·76
128	7 151·75	156·47	161·19	165·90	170·61	175·31	180·01	184·70	189·38	194·06	4·70
129	198·73	203·40	208·06	212·71	217·36	222·00	226·64	231·27	235·90	240·52	4·64
130	245·13	249·74	254·34	258·94	263·53	268·12	272·70	277·27	281·84	286·40	4·58
131	7 290·96	295·51	300·06	304·60	309·14	313·68	318·21	322·74	327·26	331·77	4·53
132	336·28	340·79	345·29	349·78	354·27	358·75	363·23	367·70	372·17	376·63	4·48
133	381·09	385·54	389·99	394·44	398·88	403·32	407·75	412·17	416·59	421·00	4·43

TABLE VII—*continued.*

ALTITUDE OR $A(v)$ TABLE.

v.	0	1	2	3	4	5	6	7	8	9	Δ
134	7 425·41	429·81	434·21	438·60	442·99	447·38	451·71	456·14	460·51	464·88	4·38
135	469·24	473·60	477·95	482·30	486·65	490·99	495·33	499·66	503·99	508·31	4·34
136	512·63	516·95	521·26	525·57	529·87	534·17	538·46	542·75	547·04	551·32	4·30
137	7 555·60	559·88	564·15	568·42	572·68	576·94	581·20	585·45	589·70	593·95	4·26
138	598·19	602·43	606·66	610·89	615·12	619·34	623·56	627·78	631·99	636·20	4·22
139	640·40	644·60	648·80	652·99	657·18	661·37	665·55	669·73	673·91	678·08	4·19
140	7 682·25	686·42	690·58	694·74	698·90	703·05	707·20	711·35	715·50	719·64	4·15
141	723·78	727·92	732·05	736·18	740·31	744·43	748·55	752·67	756·78	760 89	4·12
142	765·00	769·11	773·21	777·31	781·41	785·50	789·59	793·68	797·76	801·84	4·09
143	7 805·92	809·99	814·06	818·13	822·20	826·26	830·30	834·38	838·44	842·49	4·06
144	846·54	850·59	854·64	858·68	862·72	866·76	870 80	874·83	878·86	882·89	4·04
145	886·91	890·93	894·95	898·97	902·99	907·00	911·01	915·02	919·03	923·03	4·01
146	7 927·03	931·03	935·03	939·02	943·01	947·00	950·99	954·98	958·96	962·94	3·99
147	966·92	970·90	974·88	978·86	982·83	986·80	990·77	994·74	998·70	*002·66	3·97
148	8 006·62	010·58	014·54	018·49	022·44	026·39	030·33	034·27	038·21	042 15	3·95
149	8 046·09	050·02	053·93	057·88	061·81	065·74	069·66	073·58	077·50	081·42	3·93
150	085·34	089·26	093·17	097·08	100·99	104·90	108·81	112·72	116·62	120·52	3·91
151	124·42	128·32	132·22	136·12	140·02	143·92	147·81	151·70	155·59	159·48	3·90
152	8 163·37	167·26	171·15	175·04	178·92	182·80	186·68	190·56	194·44	198·32	3·88
153	202·20	206·08	209·96	213·83	217·70	221·57	225·44	229·31	233·18	237·05	3·87
154	240·92	244·78	248·64	252·50	256·36	260·22	264·08	267·94	271·79	275·64	3·86
155	8 279·49	283·34	287·19	291·04	294·88	298·72	302·56	306·40	310·24	314·08	3·84
156	317·92	321·76	325·59	329·42	333·25	337·08	340·91	344·74	348·57	352·40	3·83
157	356·22	360·04	363·86	367·68	371·50	375·32	379·14	382·96	386·77	390·58	3·82
158	8 394·39	398·20	402·01	405·82	409·63	413·43	417·23	421·03	424·83	428·63	3·80
159	432·43	436·23	440 03	443·83	447·63	451·42	455·21	459·00	462·79	466·58	3·79
160	470·36	474·14	477·92	481·70	485·48	489·26	493·04	496·82	500·59	504·36	3·78
161	8 508·13	511·90	515·67	519·44	523·20	526·96	530·72	534·48	538·24	542·00	3·76
162	545·76	549·52	553·27	557·02	560·77	564·52	568·27	572·02	575·76	579·50	3·75
163	583·24	586·98	590·72	594·46	598·20	601·94	605·67	609·40	613·13	616·86	3·74
164	8 620·59	624·32	628·05	631·77	635·49	639·21	642·93	646·65	650·37	654·09	3·72
165	657·80	661·51	665·22	668·93	672·64	676·35	680·06	683·77	687·48	691·19	3·71
166	694·89	698·59	702·29	705·99	709·69	713·39	717·09	720·78	724·47	728·16	3·70
167	8 731·85	735·54	739·23	742·91	746·59	750·27	753·95	757·63	761·31	764·98	3·68
168	768·65	772·32	775·99	779·66	783·33	787·00	790·66	794·32	797·98	801·64	3·67
169	805·30	808·96	812·61	816·26	819·91	823·56	827·21	830·86	834·51	838·16	3·65
170	8 841·80	845·44	849·08	852·72	856·36	860·00	863·63	867·26	870·89	874·52	3·64
171	878·15	881·78	885·41	889·03	892·65	896·27	899·89	903·51	907·13	910·75	3·62
172	914·36	917·97	921·58	925·19	928·80	932·41	936·02	939·63	943·23	946·83	3·61
173	8 950·43	954·03	957·63	961·23	964·83	968·42	972·01	975·60	979·19	982·78	3·60
174	986·37	989·96	993·55	997·13	*000·71	*004·29	*007·87	*011·45	*015·03	*018·61	3·58
175	9 022·18	025·75	029·32	032·89	036·46	040·03	043·59	047·15	050·71	054·27	3·57

Table VII—*continued.*

Altitude or $A(v)$ Table.

v.	0	1	2	3	4	5	6	7	8	9	Δ
176	9 057·83	061·39	064·95	068·51	072·06	075·61	079·16	082·71	086·26	089·81	3·55
177	093·36	096·90	100·44	103·98	107·52	111·06	114·60	118·14	121·67	125·20	3·54
178	128·73	132·26	135·79	139·32	142·85	146·38	149·91	153·43	156·95	160·47	3·53
179	9 163·99	167·51	171·03	174·54	178·05	181·56	185·07	188·58	192·09	195·60	3·51
180	199·10	202·60	206·10	209·60	213·10	216·60	220·10	223·59	227·08	230·57	3·50
181	234·06	237·55	241·04	244·53	248·02	251·51	255·00	258·48	261·96	265·44	3·49
182	9 268·92	272·40	275·88	279·36	282·83	286·30	289·77	293·24	296·71	300·18	3·47
183	303·65	307·11	310·57	314·03	317·49	320·95	324·41	327·87	331·33	334·79	3·46
184	338·24	341·69	345·14	348·59	352·04	355·49	358·94	362·38	365·82	369·26	3·45
185	9 372·70	376·14	379·58	383·02	386·45	389·88	393·31	396·74	400·17	403·60	3·43
186	407·03	410·46	413·89	417·32	420·75	424·18	427·60	431·02	434·44	437·86	3·42
187	441·28	444·70	448·12	451·54	454·96	458·37	461·78	465·19	468·60	472·01	3·41
188	9 475·42	478·83	482·24	485·64	489·04	492·44	495·84	499·24	502·64	506·04	3·40
189	509·44	512·84	516·23	519·62	523·01	526·40	529·79	533·18	536·57	539·96	3·39
190	543·35	546·73	550·11	553·49	556·87	560·25	563·63	567·01	570·39	573·77	3·38
191	9 577·15	580·53	583·90	587·27	590·64	594·01	597·38	600·75	604·12	607·48	3·37
192	610·84	614·20	617·56	620·92	624·28	627·64	630·99	634·34	637·69	641·04	3·36
193	644·39	647·74	651·08	654·42	657·76	661·10	664·44	667·78	671·12	674·45	3·34
194	9 677·79	681·12	684·45	687·78	691·11	694·44	697·77	701·09	704·41	707·73	3·33
195	711·05	714·37	717·69	721·01	724·32	727·63	730·94	734·25	737·56	740·87	3·31
196	744·17	747·47	750·77	754·07	757·37	760·67	763·96	767·25	770·54	773·83	3·29
197	9 777·11	780·39	783·63	786·95	790·22	793·49	796·76	800·03	803·30	806·56	3·27
198	809·82	813·08	816·34	819·59	822·84	826·09	829·34	832·58	835·82	839·06	3·25
199	842·30	845·54	848·77	852·00	855·23	858·46	861·68	864·90	868·12	871·34	3·23
200	9 874·56	877·77	880·98	884·19	887·40	890·61	893·81	897·01	900·21	903·41	3·21
201	906·61	909·80	912·99	916·18	919·37	922·55	925·73	928·91	932·09	935·27	3·18
202	938·44	941·61	944·78	947·94	951·10	954·26	957·41	960·56	963·71	966·86	3·16
203	9 970·01	973·15	976·29	979·43	982·57	985·70	988·85	991·96	995·09	998·21	3·13
204	10 001·33	004·45	007·57	010·68	013·79	016·90	020·01	023·11	026·21	029·31	3·11
205	032·40	035·49	038·58	041·67	044·76	047·84	050·92	054·00	057·08	060·15	3·08
206	10 063·22	066·29	069·36	072·42	075·48	078·54	081·60	084·65	087·70	090·75	3·06
207	093·80	096·84	099·88	102·92	105·96	109·00	112·03	115·06	118·09	121·12	3·03
208	124·14	127·16	130·18	133·20	136·21	139·22	142·23	145·24	148·24	151·24	3·01
209	10 154·24	157·23	160·22	163·21	166·20	169·18	172·16	175·14	178·12	181·09	2·98
210	184·06	187·03	190·00	192·96	195·92	198·88	201·84	204·80	207·75	210·70	2·96
211	213·65	216·59	219·53	222·47	225·41	228·34	231·27	234·20	237·13	240·05	2·93
212	10 242·97	245·89	248·88	251·73	254·64	257·55	260·46	263·37	266·27	269·17	2·91
213	272·07	274·97	277·87	280·76	283·65	286·54	289·43	292·31	295·19	298·07	2·89
214	300·95	303·83	306·70	309·57	312·44	315·31	318·17	321·03	323·89	326·75	2·87
215	10 329·61	334·26	335·31	338·16	341·01	343·86	346·70	349·54	352·38	355·22	2·84
216	358·05	360·88	363·71	366·54	369·37	372·19	375·01	377·83	380·65	383·47	2·82
217	386·28	389·09	391·89	394·69	397·49	400·29	403·09	405·88	408·67	411·46	2·80

TABLE VII—*continued.*

ALTITUDE OR $A(v)$ TABLE.

v.	0	1	2	3	4	5	6	7	8	9	Δ
218	10 414·25	417·04	419·82	422·60	425·38	428·16	430·94	433·71	436·48	439·25	2·78
219	442·02	444·78	447·54	450·30	453·06	455·82	458·58	461·33	464·08	466·83	2·76
220	469·58	472·32	475·06	477·80	480·54	483·28	486·02	488·75	491·48	494·21	2·74
221	10 496·94	499·67	502·39	505·11	507·83	510·55	513·26	515·97	518·68	521·39	2·72
222	524·10	526·80	529·50	532·20	534·90	537·59	540·28	542·97	545·66	548·34	2·69
223	551·02	553·70	556·38	559·06	561·74	564·41	567·08	569·75	572·42	575·09	2·67
224	10 577·75	580·41	583·07	585·73	588·39	591·04	593·69	596·34	598·99	601·64	2·65
225	604·28	606·92	609·56	612·20	614·83	617·46	620·09	622·72	625·35	627·97	2·63
226	630·59	633·21	635·83	638·45	641·06	643·67	646·28	648·89	651·50	654·11	2·61
227	10 656·71	659·31	661·91	664·51	667·11	669·71	672·31	674·91	677·50	680·09	2·60
228	682·68	685·27	687·86	690·45	693·03	695·61	698·19	700·77	703·35	705·93	2·58
229	708·51	711·09	713·67	716·25	718·83	721·41	723·99	726·57	729·14	731·71	2·58
230	10 734·28	736·85	739·42	741·99	744·56	747·13	749·70	752·27	754·84	757·41	2·57
231	759·98	762·55	765·12	767·69	770·26	772·83	775·40	777·97	780·55	783·12	2·57
232	785·70	788·27	790·85	793·42	796·00	798·57	801·15	803·72	806·30	808·87	2·58
233	10 811·45	814·02	816·60	819·17	821·75	824·32	826·90	829·48	832·06	834·64	2·58
234	837·22	839·80	842·38	844·96	847·54	850·12	852·70	855·28	857·86	860·44	2·58
235	863·02	865·60	868·18	870·76	873·34	875·92	878·51	881·09	883·68	886·26	2·58
236	10 888·85	891·13	894·02	896·60	899·19	901·77	904·36	906·95	909·54	912·13	2·59
237	914·72	917·31	919·90	922·49	925·08	927·67	930·26	932·85	935·44	938·03	2·59
238	940·63	943·22	945·81	948·40	950·99	953·58	956·18	958·78	961·38	963·98	2·60
239	10 966·58	969·18	971·78	974·38	976·98	979·58	982·18	984·78	987·38	989·98	2·60
240	992·58	995·18	997·78	*000·38	*002·98	*005·58	*008·19	*010·80	*013·41	*016·02	2·61
241	11 018·63	021·24	023·85	026·46	029·07	031·68	034·29	036·90	039·51	042·12	2·61
242	11 044·73	047·34	049·95	052·56	055·17	057·79	060·41	063·03	065·65	068·27	2·62
243	070·89	073·51	076·13	078·75	081·37	083·99	086·61	089·23	091·86	094·48	2·62
244	097·11	099·73	102·35	104·98	107·60	110·23	112·85	115·48	118·10	120·73	2·62
245	11 123·35	125·98	128·60	131·23	133·85	136·48	139·10	141·73	144·35	146·98	2·63
246	149·61	152·23	154·86	157·49	160·12	162·73	165·38	168·01	170·64	173·27	2·63
247	175·90	178·53	181·16	183·79	186·42	189·05	191·68	194·31	196·94	199·57	2·63
248	11 202·21	204·84	207·47	210·10	212·74	215·37	218·01	220·64	223·28	225·91	2·63
249	228·55	231·18	233·82	236·45	239·09	241·72	244·36	246·99	249·62	252·25	2·63
250	254·88	257·52	260·15	262·78	265·41	268·04	270·68	273·31	275·94	278·57	2·63
251	11 281·10	283·84	286·47	289·10	291·73	294·36	296·99	299·62	302·25	304·88	2·63
252	307·51	310·14	312·77	315·39	318·02	320·64	323·27	325·89	328·52	331·14	2·63
253	333·76	336·38	339·00	341·62	344·24	346·86	349·48	352·10	354·72	357·34	2·62
254	11 359·95	362·56	365·17	367·78	370·39	373·00	375·61	378·22	380·83	383·44	2·61
255	386·04	388·64	391·24	393·84	396·44	399·04	401·64	404·24	406·84	409·44	2·60
256	412·03	414·62	417·21	419·80	422·39	424·97	427·55	430·13	432·71	435·29	2·58
257	11 437·87	440·45	443·02	445·59	448·16	450·73	453·30	455·87	458·44	461·00	2·57
258	463·56	466·12	468·68	471·24	473·80	476·36	478·91	481·46	484·01	486·56	2·56
259	489·11	491·65	494·19	496·73	499·27	501·81	504·34	506·87	509·40	511·93	2·54

TABLE VII—*continued*

ALTITUDE OR $A(v)$ TABLE.

v.	0	1	2	3	4	5	6	7	8	9	Δ
260	11 514·46	516·19	519·52	522·04	524·56	527·08	529·60	532·12	534·64	537·16	2·52
261	539·67	542·18	544·69	547·20	549·70	552·20	554·70	557·20	559 70	562·20	2·50
262	564·69	567·18	569·67	572·16	574·65	577·13	579·61	582·09	584·57	587·05	2·48
263	11 589·53	592·01	594·49	596·96	599·43	601·90	604·37	606·84	609·31	611·77	2·47
264	614·23	616·69	619·15	621·61	624·06	626·51	628·96	631·41	633·86	636·31	2·45
265	638·75	641·19	643·63	646·07	648·51	650·95	653·39	655·83	658·27	660·70	2·44
266	11 663·13	665·66	667·99	670·41	672·83	675·25	677·67	680·09	682·51	684·92	2·42
267	687·33	689·74	692·15	694·56	696·96	699·36	701·76	704·16	706·56	708·96	2·40
268	711·36	713·16	716·15	718·54	720·93	723·32	725·71	728·10	730·49	732·87	2·39
269	11 735·25	737·63	740·01	742·39	744·76	747·13	749·50	751·87	754·24	756·61	2·37
270	758·97	761·33	763·69	766·05	768·41	770·77	773·12	775·47	777·82	780·17	2·36
271	782·52	784·87	787·22	789·57	791·91	794·25	796·59	798·93	801·27	803·61	2·34
272	11 805·94	808·27	810·60	812·93	815·26	817·59	819·92	822·24	824·56	826·88	2·33
273	829·20	831·52	833·84	836·15	838·46	840·77	843·08	845·39	847·69	849·99	2·31
274	852·29	854·59	856·89	859·19	861·48	863·77	866·06	868·35	870·64	872·93	2·29
275	11 875·21	877·49	879·77	882·05	884·33	886·61	888·89	891·16	893·43	895·70	2·28
276	897·97	900·24	902·51	904·77	907·03	909·29	911·55	913·81	916·07	918·32	2·26
277	920·57	922·82	925·07	927·32	929·57	931·81	934·05	936·29	938·53	940·77	2·24
278	11 943·01	945·25	947·48	949·71	951·94	954·17	956·40	958·63	960·85	963·07	2·23
279	965·29	967·51	969·73	971·95	974·16	976·37	978·58	980·79	983·00	985·21	2·21
280	987·41	989·61	991·81	994·01	996·21	998·41	*000·61	*002·80	*004·99	*007·18	2·20
281	12 009·37										

TABLE VIII.

Table for $i\ (\theta) = \int \sec^3 \theta\, d\, \theta = \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \log (\sec \theta + \tan \theta)$, and for $\tan \theta$.

θ	$i(\theta)$	$\tan\theta$	θ	$i(\theta)$	$\tan\theta$	θ	$i(\theta)$	$\tan\theta$	θ	$i(\theta)$	$\tan\theta$	θ	$i(\theta)$	$\tan\theta$
0	·00000	·00000	15	·27112	·26795	30	·60799	·57735	45	1·14779	1·00000	60	2·39053	1·73205
1	·01746	·01746	16	·29063	·28675	31	·63527	·60086	46	1·19849	1·03553	61	2·53670	1·80405
2	·03493	·03492	17	·31043	·30573	32	·66343	·62487	47	1·25201	1·07237	62	2·69777	1·88073
3	·05243	·05241	18	·33055	·32492	33	·69253	·64941	48	1·30863	1·11061	63	2·87491	1·96261
4	·06998	·06993	19	·35101	·34433	34	·72263	·67451	49	1·36863	1·15037	64	3·07205	2·05030
5	·08760	·08749	20	·37185	·36397	35	·75382	·70021	50	1·43236	1·19175	65	3·29041	2·14451
6	·10530	·10510	21	·39309	·38386	36	·78617	·72654	51	1·50019	1·23490	66	3·53533	2·24604
7	·12309	·12278	22	·41477	·40403	37	·81977	·75355	52	1·57257	1·27994	67	3·81077	2·35585
8	·14100	·14054	23	·43690	·42447	38	·85473	·78129	53	1·64995	1·32704	68	4·12257	2·47509
9	·15904	·15838	24	·45953	·44523	39	·89114	·80978	54	1·73291	1·37638	69	4·47736	2·60509
10	·17724	·17633	25	·48269	·46631	40	·92914	·83910	55	1·82207	1·42815	70	4·88420	2·74748
11	·19560	·19438	26	·50643	·48773	41	·96884	·86929	56	1·91815	1·48256	71	5·35416	2·90421
12	·21415	·21256	27	·53078	·50953	42	1·01039	·90040	57	2·02199	1·53987	72	5·90112	3·07768
13	·23290	·23087	28	·55580	·53171	43	1·05395	·93252	58	2·13456	1·60033	73	6·54404	3·27085
14	·25189	·24933	29	·58151	·55431	44	1·09968	·96569	59	2·25697	1·66428	74	7·30713	3·48741
												75	8·21871	3·73205

Table IX.

Ballistic Table for Spherical Shot.

(Recalculated by Mr. Hadcock, R.A., from Bashforth's data, and extended to low velocities.)

For lower velocities this table is provisional, pending the results of further experiments.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
300	1·2232	0·0000	366·91	0·00	7·5191	0·0000
310	1·1505	1·2232	356·67	366·91	6·8454	7·5191
320	1·0824	2·3737	346·37	723·58	6·2387	14·3645
330	1·0217	3·4561	337·22	1069·95	5·7113	20·6032
340	0·9647	4·4778	328·01	1407·17	5·2335	26·3145
350	0·9137	5·4425	319·78	1735·18	4·8148	31·5480
360	0·8653	6·3562	311·51	2054·96	4·4333	36·3628
370	0·8218	7·2215	304·07	2366·47	4·0967	40·7961
380	0·7805	8·0433	296·60	2670·54	3·7884	44·8928
390	0·7432	8·8238	289·84	2967·14	3·5147	48·6812
400	0·7076	9·5670	283·05	3256·98	3·2629	52·1959
410	0·6753	10·2746	276·88	3540·03	3·0380	55·4588
420	0·6445	10·9499	270·69	3816·91	2·8303	58·4968
430	0·6151	11·5944	264·51	4087·60	2·6385	61·3271
440	0·5763	12·2095	253·59	4352·11	2·4159	63·9656
450	0·5508	12·7858	247·86	4605·70	2·2575	66·3815
460	0·5265	13·3366	242·20	4853·56	2·1111	68·6390
470	0·5035	13·8631	236·64	5095·76	1·9758	70·7501
480	0·4816	14·3666	231·18	5332·40	1·8506	72·7259
490	0·4609	14·8482	225·84	5563·58	1·7349	74·5765
500	0·4413	15·3091	220·63	5789·42	1·6277	76·3114
510	0·4227	15·7504	215·55	6010·05	1·5285	77·9391
520	0·4050	16·1731	210·61	6225·60	1·4366	79·4676
530	0·3883	16·5781	205·80	6436·21	1·3513	80·9042
540	0.3725	16·9664	201·14	6642·01	1·2722	82·2555
550	0·3575	17·3389	196 61	6843·15	1·1988	83·5277
560	0·3429	17·6964	192·01	7039·76	1·1293	84·7265
570	0·3291	18·0393	187·57	7231·77	1·0648	85·8558
580	0·3157	18·3684	183·11	7419·34	1·0039	86·9206
590	0 3028	18·6841	178·64	7602·45	0·9465	87·9245
600	0·2903	18·9869	174·19	7781·09	0·8925	88·8710
610	0·2786	19·2773	169·95	7955·28	0·8424	89·7635
620	0·2673	19·5558	165·75	8125·23	0·7953	90·6059
630	0·2567	19·8231	161·74	8[illegible]0·98	0·7516	91·4012
640	0·2467	20·0798	157·92	8452·72	0·7111	92·1528
650	0·2371	20·3265	154·14	8610·64	0·6729	92·8639
660	0·2281	20·5636	150·53	8764·78	0·6374	93·5368
670	0·2195	20·7917	147·09	8915·31	0·6044	94·1742
680	0·2115	21·0112	143·80	9062·40	0·5736	94·7786
690	0·2038	21·2227	140·65	9206·20	0·5449	95·3522
700	0·1966	21·4265	137·63	9346·85	0·5180	95·8971
710	0·1898	21·6231	134·73	9484·48	0·4930	96·4151
720	0·1832	21·8129	131·88	9619·21	0·4692	96·9081
730	0·1770	21·9961	129·22	9751·09	0·4472	97·3773
740	0·1711	22·1731	126·59	9880·31	0·4264	97·8245
750	0·1653	22·3442	123·99	10006·90	0·4066	98·2509
760	0·1600	22·5095	121·57	10130·89	0·3882	98·6575
770	0·1547	22·6695	119·12	10252·46	0·3706	99·0457
780	0·1496	22·8242	116·72	10371·58	0·3539	99·4163
790	0·1447	22·9738	114·30	10488·30	0·3378	99·7702
800	0·1399	23·1185	111·89	10602·60	0·3225	100·1080
810	0·1352	23·2584	109·50	10714·49	0·3078	100·4305
820	0·1306	24·3936	107·07	10823·99	0·2937	100·7383
830	0·1201	23·5242	104·68	10931·06	0·2803	101 0320

TABLE IX—*continued.*

Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
840	0·1218	23·6503	102·33	11035·74	0·2675	101·3123
850	0·1177	23·7721	100·01	11138·07	0·2553	101·5798
860	0·1137	23·8898	97·76	11238·08	0·2438	101·8351
870	0·1098	24·0035	95·53	11335·84	0·2328	102·0789
880	0·1062	24·1133	93·44	11431·37	0·2225	102·3117
890	0·1026	24·2195	91·35	11524·81	0·2127	102·5342
900	0·0993	24·3221	89·33	11616·16	0·2034	102·7469
910	0·0959	24·4214	87·32	11705·49	0·1945	102·9503
920	0·0928	24·5173	85·37	11792·81	0·1860	103·1448
930	0·0898	24·6101	83·48	11878·18	0·1780	103·3308
940	0·0869	24·6999	81·65	11961·66	0·1704	103·5088
950	0·0840	24·7868	79·83	12043·31	0·1631	103·6792
960	0·0813	24·8708	78·01	12123·14	0·1561	103·8423
970	0·0785	24·9521	76·19	12201·15	0·1493	103·9984
980	0·0759	25·0306	74·43	12277·34	0·1429	104·1477
990	0·0734	25·1065	72·67	12351·77	0·1368	104·2906
1000	0·0709	25·1799	70·87	12424·44	0·1307	104·4274
1010	0·0684	25·2508	69·08	12495·31	0·1249	104·5581
1020	0·0660	25·3192	67·31	12564·39	0·1193	104·6830
1030	0·0636	25·3852	65·55	12631·70	0·1140	104·8023
1040	0·0614	25·4488	63·81	12697·25	0·1088	104·9163
1050	0·0591	25·5102	62·08	12761·06	0·1039	105·0251
1060	0·0570	25·5693	60·42	12823·14	0·0992	105·1290
1070	0·0550	25·6263	58·82	12883·56	0·0948	105·2282
1080	0·0531	25·6813	57·31	12942·38	0·0906	105·3230
1090	0·0513	25·7344	55·89	12999·69	0·0868	105·4136
1100	0·0496	25·7857	54·59	13055·58	0·0832	105·5004
1110	0·0481	25·8353	53·36	13110·17	0·0799	105·5836
1120	0·0466	25·8834	52·21	13163·53	0·0768	105·6635
1130	0·0453	25·9300	51·15	13215·74	0·0739	105·7403
1140	0·0440	25·9753	50·16	13266·89	0·0712	105·8142
1150	0·0428	26·0193	49·23	13317·05	0·0687	105·8854
1160	0·0417	26·0621	48·35	13366·28	0·0663	105·9541
1170	0·0406	26·1038	47·53	13414·63	0·0640	106·0204
1180	0·0396	26·1444	46·73	13462·16	0·0619	106·0844
1190	0·0386	26·1840	45·97	13508·89	0·0599	106·1463
1200	0·0377	26·2226	45·27	13554·86	0·0580	106·2062
1210	0·0369	26·2603	44·61	13600·13	0·0562	106·2642
1220	0·0361	26·2972	44·00	13644·74	0·0545	106·3204
1230	0·0353	26·3333	43·43	13688·74	0·0529	106·3749
1240	0·0346	26·3686	42·87	13732·17	0·0514	106·4278
1250	0·0339	26·4032	42·36	13775·04	0·0500	106·4792
1260	0·0332	26·4371	41·85	13817·40	0·0486	106·5292
1270	0·0326	26·4703	41·39	13859·25	0·0473	106·5778
1280	0·0320	26·5029	40·94	13900·64	0·0461	106·6251
1290	0·0314	26·5349	40·49	13941·58	0·0449	106·6712
1300	0·0308	26·5663	40·04	13982·07	0·0437	106·7131
1310	0·0302	26·5971	39·59	14022·11	0·0425	106·7558
1320	0·0297	26·6273	39·18	14061·70	0·0415	106·8023
1330	0·0291	26·6570	38·75	14100·88	0·0404	106·8438
1340	0·0286	26·6861	38·33	14139·63	0·0394	106·8842
1350	0·0281	26·7147	37·92	14177·96	0·0384	106·9236
1360	0·0276	26·7428	37·52	14215·88	0·0374	106·9620
1370	0·0271	26·7704	37·15	14253·40	0·0365	106·9994
1380	0·0267	26·7975	36·80	14290·55	0·0356	107·0359
1390	0·0262	26·8242	36·45	14327·35	0·0348	107·0715
1400	0·0258	26·8504	36·11	14363·80	0·0340	107·1063
1410	0·0254	26·8762	35·77	14399·91	0·0332	107·1403
1420	0·0250	26·9016	35·48	14435·68	0·0324	107·1735
1430	0·0246	26·9266	35·16	14471·16	0·0317	107·2059

TABLE IX—*continued.*

Ballistic Table for Spherical Shot.

v	ΔT	T	ΔS	S	ΔD	D
f/s.						
1440	0·0242	26·9512	34·85	14506·32	0·0310	107·2376
1450	0·0238	26·9754	34·54	14541·17	0·0303	107·2686
1460	0·0235	26·9992	34·24	14575·71	0·0296	107·2989
1470	0·0231	27·0227	33·98	14609·95	0·0290	107·3285
1480	0·0228	27·0458	33·69	14643·93	0·0284	107·3575
1490	0·0224	27·0686	33·41	14677·62	0·0278	107·3859
1500	0·0221	27·0910	33·14	14711·03	0·0272	107·4137
1510	0·0218	27·1131	32·85	14744·17	0·0266	107·4409
1520	0·0214	27·1349	32·59	14777·02	0·0260	107·4675
1530	0·0211	27·1563	32·34	14809·61	0·0255	107·4935
1540	0·0208	27·1774	32·06	14841·95	0·0249	107·5190
1550	0·0205	27·1982	31·82	14874·01	0·0244	107·5439
1560	0·0202	27·2187	31·58	14905·83	0·0239	107·5683
1570	0·0200	27·2389	31·33	14937·41	0·0234	107·5922
1580	0·0197	27·2589	31·10	14968·74	0·0230	107·6156
1590	0·0194	27·2786	30·86	14999·84	0·0225	107·6386
1600	0·0191	27·2980	30·64	15030·70	0·0221	107·6611
1610	0·0189	27·3171	30·42	15061·34	0·0216	107·6832
1620	0·0186	27·3360	30·19	15091·76	0·0212	107·7048
1630	0·0184	27·3546	29·99	15121·95	0·0208	107·7260
1640	0·0182	27·3730	29·79	15151·94	0·0204	107·7468
1650	0·0179	27·3912	29·60	15181·73	0·0201	107·7672
1660	0·0177	27·4091	29·38	15211·33	0·0197	107·7873
1670	0·0175	27·4268	29·20	15240·71	0·0193	107·8070
1680	0·0173	27·4443	29·02	15269·91	0·0190	107·8263
1690	0·0171	27·4616	28·84	15298·93	0·0186	107·8453
1700	0·0168	27·4787	28·64	15327·77	0·0183	107·8639
1710	0·0167	27·4955	28·47	15356·41	0·0180	107·8822
1720	0·0165	27·5122	28·31	15384·88	0·0176	107·9002
1730	0·0163	27·5287	28·13	15413·19	0·0173	107·9178
1740	0·0161	27·5450	27·97	15441·32	0·0170	107·9351
1750	0·0159	27·5611	27·81	15469·29	0·0168	107·9521
1760	0·0157	27·5770	27·64	15497·10	0·0165	107·9689
1770	0·0155	27·5927	27·49	15524·74	0·0162	107·9854
1780	0·0154	27·6082	27·33	15552·23	0·0159	108·0016
1790	0·0152	27·6236	27·16	15579·56	0·0156	108·0175
1800	0·0150	27·6388	27·03	15606·72	0·0154	108·0331
1810	0·0148	27·6538	26·87	15633·75	0·0151	108·0485
1820	0·0147	27·6686	26·72	15660·62	0·0149	108·0636
1830	0·0145	27·6833	26·54	15687·34	0·0146	108·0785
1840	0·0143	27·6978	26·40	15713·88	0·0144	108·0931
1850	0·0142	27·7121	26·25	15740·28	0·0141	108·1075
1860	0·0140	27·7263	26·09	15766·53	0·0139	108·1216
1870	0·0139	27·7403	25·93	15792·62	0·0137	108·1355
1880	0·0137	27·7542	25·79	15818·55	0·0135	108·1492
1890	0·0136	27·7679	25·64	15844·34	0·0132	108·1627
1900	0·0134	27·7815	25·48	15869·98	0·0130	108·1759

TABLE X.

Distance yds. / v. ft.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500
550	0325	0658	0998	1346	1702										
560	0313	0633	0961	1296	1639	1993									
570	0302	0611	0927	1250	1581	1922	2273								
580	0291	0589	0894	1206	1527	1856	2194	2540							
590	0281	0569	0864	1166	1476	1794	2120	2454	2796						
600	0271	0549	0834	1128	1428	1736	2051	2374	2704	3042					
610	0262	0531	0807	1091	1382	1680	1986	2300	2620	2948					
620	0253	0513	0781	1056	1338	1627	1924	2228	2539	2857	3175				
630	0245	0496	0755	1022	1295	1576	1863	2158	2460	2769	3077				
640	0237	0480	0731	0989	1254	1526	1805	2091	2384	2684	2983	3300			
650	0229	0465	0707	0957	1214	1478	1748	2026	2311	2602	2892	3199			
660	0222	0450	0685	0927	1176	1432	1694	1964	2240	2523	2805	3102	3402		
670	0215	0436	0664	0898	1140	1387	1642	1904	2172	2446	2721	3009	3300		
680	0208	0423	0644	0871	1105	1345	1592	1846	2106	2372	2641	2919	3202	3493	
690	0202	0410	0623	0840	1072	1304	1545	1790	2042	2302	2564	2833	3108	3389	
700	0196	0398	0606	0820	1040	1266	1499	1738	1983	2234	2490	2751	3018	3291	3569
710	0192	0389	0592	0801	1015	1235	1460	1691	1929	2172	2419	2672	2932	3197	3467
720	0188	0380	0578	0782	0990	1204	1423	1647	1877	2111	2352	2598	2850	3107	3370
730	0184	0371	0564	0763	0966	1174	1387	1604	1826	2054	2289	2529	2773	3022	3278
740	0179	0363	0551	0744	0942	1144	1351	1562	1778	1999	2227	2460	2698	2941	3190
750	0175	0354	0538	0726	0919	1116	1316	1521	1731	1946	2165	2391	2623	2861	3106
760	0171	0346	0525	0709	0896	1088	1283	1483	1687	1895	2108	2329	2555	2788	3026
770	0167	0338	0513	0692	0874	1061	1252	1446	1644	1846	2055	2271	2491	2718	2950
780	0163	0330	0501	0675	0853	1035	1221	1410	1603	1800	2005	2215	2430	2651	2877
790	0159	0322	0488	0658	0832	1010	1191	1376	1565	1757	1956	2161	2371	2586	2807
800	0155	0314	0476	0642	0812	0985	1162	1343	1527	1715	1908	2108	2313	2523	2739
810	0151	0306	0464	0626	0792	0961	1134	1311	1490	1674	1863	2058	2258	2463	2674
820	0147	0299	0453	0611	0773	0938	1107	1280	1455	1635	1820	2010	2206	2406	2612
830	0144	0291	0442	0597	0755	0916	1081	1250	1422	1597	1778	1964	2155	2351	2552
840	0141	0284	0432	0583	0737	0895	1056	1220	1389	1560	1737	1918	2105	2297	2494
850	0138	0278	0422	0569	0719	0874	1032	1196	1357	1525	1698	1875	2058	2245	2438
860	0134	0271	0412	0556	0703	0854	1008	1166	1327	1491	1660	1834	2013	2196	2384
870	0131	0265	0402	0543	0686	0834	0985	1140	1298	1459	1624	1794	1969	2148	2332
880	0127	0258	0392	0530	0671	0815	0963	1114	1269	1427	1589	1755	1926	2102	2282
890	0124	0252	0383	0518	0656	0797	0942	1090	1242	1398	1556	1719	1886	2058	2234
900	0121	0246	0374	0506	0641	0780	0922	1067	1216	1369	1524	1684	1848	2016	2189
910	0118	0240	0365	0495	0627	0763	0902	1044	1190	1340	1493	1650	1811	1976	2145
920	0116	0235	0357	0484	0613	0746	0882	1022	1165	1312	1463	1616	1775	1937	2103
930	0113	0230	0349	0473	0600	0730	0864	1001	1141	1285	1433	1584	1740	1899	2062

Distance yds. / v. ft.	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100
550																
560																
570																
580																
590																
600																
610																
620																
630																
640																
650																
660																
670																
680																
690																
700																
710	3745															
720	3640															
730	3542	3814	4093													
740	3448	3712	3983													
750	3357	3615	3879	4149	4426											
760	3271	3521	3778	4039	4308											
770	3188	3431	3681	3935	4196											
780	3109	3346	3589	3837	4090											
790	3033	3264	3501	3743	3990											
800	2960	3186	3418	3655	3896	4146	4403	4666	4935	5211						
810	2890	3111	3338	3570	3807	4050	4299	4554	4815	5080						
820	2823	3039	3261	3488	3720	3957	4199	4446	4699	4956	5219					
830	2758	2970	3187	3409	3636	3867	4103	4343	4588	4838	5093					
840	2695	2903	3115	3332	3554	3780	4010	4244	4483	4727	4975	5228				
850	2635	2838	3045	3258	3475	3695	3921	4150	4384	4622	4865	5118				
860	2577	2776	2978	3186	3398	3614	3835	4060	4290	4523	4761	5014	5273			
870	2521	2715	2913	3116	3324	3536	3753	3973	4200	4429	4662	4915	5169			
880	2467	2656	2850	3049	3253	3460	3674	3891	4113	4339	4570	4818	5067	5326		
890	2415	2600	2790	2985	3184	3387	3596	3811	4028	4253	4484	4724	4968	5220		
900	2365	2547	2733	2923	3118	3317	3522	3732	3949	4171	4399	4633	4872	5118	5369	
910	2318	2496	2678	2864	3054	3251	3454	3661	3874	4092	4316	4545	4779	5018	5263	
920	2273	2447	2625	2807	2993	3188	3388	3592	3802	4016	4235	4459	4688	4922	5161	5406
930	2229	2399	2574	2752	2934	3127	3324	3525	3731	3942	4157	4376	4600	4829	5062	5301

Distance yds.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300
v. ft.																																											
940	0111	0225	0342	0463	0587	0715	0846	0980	1118	1259	1404	1553	1705	1862	2022	2186	2353	2525	2699	2878	3068	3262	3459	3662	3869	4080	4295	4515	4739	4967	5200	5473											
950	0108	0220	0335	0453	0575	0700	0828	0960	1095	1234	1376	1523	1672	1826	1983	2144	2308	2477	2648	2824	3011	3201	3395	3595	3798	4005	4216	4432	4652	4875	5103	5368											
960	0106	0215	0328	0444	0563	0686	0811	0941	1073	1209	1349	1494	1641	1792	1946	2104	2265	2431	2600	2772	2955	3142	3333	3529	3728	3932	4139	4351	4567	4787	5011	5267	5508										
970	0103	0210	0321	0434	0551	0672	0795	0922	1052	1186	1324	1466	1611	1759	1911	2066	2225	2387	2553	2722	2901	3085	3272	3464	3660	3861	4065	4272	4485	4702	4923	5170	5413										
980	0101	0206	0314	0425	0540	0658	0779	0903	1031	1163	1300	1439	1582	1727	1877	2029	2186	2345	2508	2674	2849	3029	3213	3401	3594	3791	3992	4197	4407	4621	4839	5079	5311	5562									
990	0099	0201	0307	0417	0529	0645	0764	0886	1012	1142	1276	1413	1553	1697	1844	1994	2148	2305	2465	2628	2798	2974	3155	3339	3529	3723	3921	4124	4332	4543	4760	4992	5223	5464									
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Distance yds.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
v. ft.																																																		
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1590	00388	00806	01251	01724	02226	02758	03318	03907	04524	05170	05850	06564	07314	08098	08918	09772	1066	1158	1255	1354	1457	1564	1674	1788	1904	2024	2147	2273	2403	2537	2670	2808	2949	3096	3247	3402	3562	3727	3895	4069	4248	4432	4622	4817	5019	5226	5440	5660	5887	6121
1600	00383	00795	01235	01703	02200	02725	03279	03861	04471	05110	05780	06485	07225	08000	08811	09658	1054	1146	1242	1341	1444	1550	1660	1773	1889	2008	2130	2255	2384	2516	2652	2791	2933	3080	3230	3384	3542	3704	3869	4039	4214	4394	4580	4773	4973	5179	5393	5614	5843	6081
1610	00378	00785	01220	01682	02174	02692	03239	03814	04417	05050	05714	06412	07145	07913	08716	09555	1043	1134	1230	1328	1430	1535	1644	1757	1872	1990	2112	2236	2364	2495	2631	2769	2910	3056	3205	3358	3515	3676	3840	4009	4183	4362	4548	4740	4939	5144	5357	5576	5804	6041
1620	00374	00776	01206	01662	02148	02660	03201	03770	04366	04992	05650	06342	07068	07829	08625	09456	1032	1123	1218	1315	1416	1521	1629	1741	1856	1973	2094	2217	2345	2475	2610	2747	2887	3032	3181	3333	3489	3649	3812	3980	4153	4332	4516	4707	4905	5109	5321	5540	5767	6002
1630	00369	00766	01192	01642	02122	02628	03162	03725	04315	04935	05587	06273	06992	07745	08534	09356	1021	1111	1205	1302	1403	1506	1614	1725	1839	1956	2076	2198	2325	2455	2589	2725	2865	3009	3156	3307	3462	3621	3784	3951	4123	4301	4485	4675	4872	5075	5286	5503	5729	5963
1640	00365	00757	01178	01622	02096	02597	03125	03682	04266	04880	05526	06206	06918	07665	08447	09263	1011	1100	1193	1289	1389	1491	1598	1709	1822	1938	2057	2180	2306	2435	2568	2703	2842	2985	3131	3282	3436	3594	3756	3922	4093	4270	4453	4642	4838	5040	5250	5467	5691	5924
1650	00361	00748	01164	01602	02070	02567	03090	03641	04219	04827	05466	06139	06844	07584	08359	09168	1001	1089	1181	1276	1375	1477	1583	1693	1806	1921	2040	2161	2286	2415	2547	2682	2820	2962	3107	3256	3409	3567	3728	3893	4064	4240	4422	4610	4805	5006	5215	5430	5653	5885
1660	00358	00740	01151	01583	02044	02536	03053	03599	04172	04775	05409	06077	06777	07510	08279	09082	09918	1079	1170	1264	1362	1463	1568	1677	1789	1904	2022	2142	2267	2395	2526	2660	2798	2938	3083	3231	3384	3540	3701	3865	4035	4210	4391	4579	4773	4973	5180	5394	5616	5846
1670	00354	00732	01137	01564	02018	02505	03018	03558	04125	04723	05352	06015	06710	07437	08199	08995	09825	1069	1159	1252	1349	1449	1553	1661	1773	1887	2004	2124	2248	2375	2505	2639	2775	2915	3059	3207	3358	3514	3673	3837	4006	4180	4361	4547	4740	4939	5146	5359	5579	5807
1680	00351	00725	01124	01545	01993	02476	02984	03518	04079	04671	05293	05948	06635	07354	08107	08894	09715	1057	1147	1240	1336	1436	1539	1647	1757	1870	1987	2106	2230	2356	2486	2618	2754	2893	3036	3182	3333	3488	3646	3809	3977	4150	4330	4516	4708	4906	5111	5323	5543	5769
1690	00347	00717	01111	01526	01969	02447	02950	03479	04034	04619	05235	05885	06567	07280	08026	08806	09621	1047	1136	1228	1324	1422	1525	1632	1742	1854	1970	2089	2211	2337	2466	2598	2732	2870	3012	3158	3308	3461	3619	3781	3948	4121	4300	4484	4675	4872	5077	5288	5506	5731
1700	00344	00710	01098	01508	01945	02418	02916	03439	03988	04567	05178	05822	06498	07206	07946	08720	09528	1037	1125	1216	1311	1409	1511	1617	1726	1838	1953	2071	2193	2318	2446	2577	2711	2848	2989	3134	3283	3436	3593	3754	3920	4092	4270	4454	4644	4840	5043	5253	5470	5694
1710	00340	00702	01085	01490	01921	02388	02880	03398	03941	04515	05120	05758	06428	07130	07864	08631	09432	1027	1114	1204	1298	1396	1497	1602	1710	1821	1936	2053	2174	2299	2426	2556	2689	2826	2967	3111	3259	3411	3567	3727	3892	4063	4240	4423	4612	4808	5010	5219	5434	5657

Distance yds. / v. ft.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
1720	00337	00695	01072	01473	01898	02359	02845	03357	03895	04463	05062	05694	06358	07054	07782	08542
1730	00333	00687	01060	01456	01876	02331	02811	03317	03849	04411	05004	05630	06288	06978	07700	08455
1740	00330	00680	01048	01439	01855	02304	02778	03278	03804	04359	04947	05567	06219	06903	07619	08368
1750	00326	00672	01036	01422	01835	02278	02745	03239	03759	04308	04891	05506	06153	06832	07543	08286
1760	00323	00665	01024	01406	01815	02252	02713	03200	03714	04258	04835	05445	06087	06761	07467	08205
1770	00320	00657	01012	01390	01795	02226	02681	03162	03670	04209	04780	05383	06019	06687	07387	08119
1780	00317	00650	01001	01375	01775	02200	02650	03125	03627	04160	04725	05322	05951	06613	07307	08033
1790	00314	00643	00990	01360	01755	02174	02619	03088	03585	04112	04672	05263	05887	06543	07231	07951
1800	00311	00636	00979	01345	01735	02149	02588	03052	03543	04065	04619	05205	05823	06473	07155	07869
1810	00307	00628	00968	01330	01715	02124	02558	03017	03503	04020	04568	05148	05760	06404	07081	07791
1820	00304	00621	00958	01316	01696	02100	02528	02982	03464	03975	04517	05091	05697	06336	07008	07713
1830	00300	00614	00947	01301	01677	02076	02500	02949	03425	03930	04467	05035	05635	06267	06933	07631
1840	00297	00607	00937	01287	01658	02053	02472	02916	03387	03887	04417	04979	05573	06199	06858	07550
1850	00293	00600	00926	01273	01639	02030	02444	02884	03349	03844	04368	04924	05512	06133	06786	07473
1860	00290	00593	00916	01259	01621	02007	02417	02852	03312	03801	04320	04870	05452	06067	06715	07396
1870	00287	00586	00906	01245	01603	01984	02389	02819	03274	03758	04272	04816	05393	06002	06644	07319
1880	00284	00580	00896	01231	01585	01962	02362	02786	03236	03715	04224	04763	05334	05937	06573	07242
1890	00281	00574	00886	01217	01567	01940	02335	02755	03200	03673	04178	04712	05278	05876	06507	07170
1900	00278	00568	00876	01203	01550	01918	02309	02724	03164	03633	04132	04662	05223	05816	06441	07098
1910	00275	00562	00866	01189	01532	01895	02282	02692	03128	03593	04088	04614	05171	05760	06380	07032
1920	00272	00556	00856	01176	01514	01873	02255	02661	03092	03553	04044	04566	05119	05704	06320	06967
1930	00269	00550	00846	01162	01496	01852	02228	02629	03056	03513	04001	04519	05067	05648	06259	06902
1940	00266	00544	00837	01149	01479	01830	02202	02598	03021	03474	03958	04472	05016	05592	06199	06837
1950	00263	00538	00827	01135	01462	01808	02175	02567	02986	03435	03916	04426	04966	05537	06138	06772
1960	00260	00533	00818	01122	01445	01786	02149	02536	02951	03397	03874	04381	04916	05482	06078	06707
1970	00257	00527	00809	01109	01428	01764	02123	02505	02917	03360	03835	04338	04867	05431	06018	06643
1980	00255	00522	00800	01096	01411	01743	02098	02475	02884	03325	03796	04295	04818	05380	05959	06580
1990	00252	00516	00791	01083	01394	01722	02073	02446	02853	03290	03758	04253	04770	05323	05901	06517
2000	00250	00511	00782	01070	01377	01702	02049	02419	02822	03256	03720	04211	04723	05267	05844	06454
2010	00247	00506	00774	01058	01361	01683	02026	02394	02793	03222	03682	04168	04677	05217	05788	06392
2020	00245	00501	00766	01046	01346	01664	02004	02369	02764	03190	03644	04125	04632	05168	05733	06331
2030	00242	00496	00758	01034	01331	01646	01983	02346	02738	03160	03609	04085	04588	05119	05679	06271
2040	00240	00491	00750	01023	01316	01628	01963	02323	02712	03130	03575	04046	04544	05070	05625	06213
2050	00237	00486	00742	01012	01302	01611	01943	02300	02686	03100	03542	04011	04507	05030	05580	06159
2060	00235	00481	00735	01001	01288	01594	01923	02278	02660	03070	03509	03976	04470	04990	05536	06109
2070	00233	00476	00727	00991	01274	01577	01903	02255	02633	03040	03475	03938	04427	04943	05484	06052
2080	00231	00472	00720	00981	01260	01561	01884	02232	02607	03010	03441	03900	04385	04896	05432	05995
2090	00229	00467	00712	00971	01247	01545	01865	02209	02580	02980	03407	03862	04343	04850	05381	05939
2100	00227	00463	00705	00961	01234	01530	01846	02186	02554	02950	03374	03825	04302	04804	05331	05884

Distance yds. / v. ft.	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100	3200	3300
1720	09336	1017	1103	1193	1286	1383	1483	1587	1694	1805	1919	2036	2156	2280	2406	2535	2668
1730	09243	1007	1092	1182	1274	1370	1470	1573	1679	1789	1902	2019	2138	2261	2386	2515	2647
1740	09150	09967	1082	1171	1263	1358	1457	1559	1664	1773	1886	2002	2121	2243	2367	2495	2627
1750	09063	09874	1072	1160	1251	1346	1444	1546	1650	1759	1871	1986	2104	2225	2348	2476	2607
1760	08976	09781	1062	1149	1240	1334	1432	1533	1637	1745	1856	1970	2087	2207	2330	2457	2588
1770	08884	09682	1051	1138	1228	1321	1418	1519	1622	1729	1839	1953	2069	2189	2311	2438	2568
1780	08792	09584	1041	1127	1216	1309	1405	1505	1608	1714	1823	1936	2052	2171	2293	2419	2548
1790	08704	09490	1031	1116	1205	1297	1393	1492	1594	1699	1807	1920	2035	2154	2275	2400	2528
1800	08616	09397	1021	1106	1194	1286	1381	1479	1580	1684	1792	1904	2019	2137	2258	2382	2509
1810	08533	09309	1011	1096	1183	1274	1368	1466	1566	1670	1777	1888	2002	2120	2240	2363	2489
1820	08451	09221	1002	1086	1172	1262	1356	1453	1553	1656	1763	1873	1986	2103	2222	2344	2470
1830	08363	09127	09923	1075	1160	1249	1342	1439	1538	1641	1748	1857	1970	2086	2204	2326	2451
1840	08275	09034	09826	1065	1149	1237	1329	1425	1524	1627	1733	1842	1954	2069	2187	2308	2432
1850	08192	08945	09731	1055	1138	1226	1317	1412	1510	1612	1718	1826	1938	2052	2170	2290	2414
1860	08110	08857	09637	1045	1128	1215	1306	1400	1497	1598	1703	1811	1922	2036	2153	2273	2396
1870	08027	08768	09542	1035	1117	1203	1293	1387	1483	1584	1688	1796	1906	2020	2136	2256	2378
1880	07944	08679	09448	1025	1107	1192	1281	1374	1470	1570	1674	1781	1891	2004	2120	2239	2360
1890	07866	08594	09358	1015	1097	1181	1269	1361	1457	1556	1659	1766	1875	1988	2103	2221	2342
1900	07788	08511	09268	1006	1087	1171	1258	1349	1444	1543	1645	1751	1860	1972	2087	2204	2324
1910	07717	08434	09185	09970	1078	1162	1249	1339	1433	1531	1632	1737	1845	1956	2070	2187	2306
1920	07646	08358	09103	09882	1069	1153	1239	1329	1423	1519	1619	1724	1831	1941	2054	2170	2289
1930	07576	08282	09021	09794	1059	1142	1228	1318	1410	1507	1606	1710	1816	1926	2038	2154	2272
1940	07506	08207	08940	09706	1050	1132	1217	1306	1398	1494	1593	1696	1802	1911	2023	2138	2256
1950	07436	08131	08858	09618	1040	1122	1206	1294	1386	1481	1579	1682	1787	1896	2007	2122	2239
1960	07366	08056	08777	09530	1031	1112	1196	1283	1374	1468	1566	1668	1773	1881	1992	2106	2223
1970	07296	07981	08696	09443	1021	1102	1185	1272	1362	1456	1553	1655	1759	1867	1977	2091	2207
1980	07226	07907	08616	09356	1012	1092	1174	1261	1350	1444	1541	1642	1746	1853	1963	2076	2191
1990	07157	07832	08535	09269	1002	1081	1163	1249	1338	1431	1528	1629	1732	1839	1948	2060	2174
2000	07089	07757	08455	09183	09930	1071	1152	1237	1326	1419	1516	1616	1719	1825	1933	2044	2158
2010	07022	07684	08375	09097	09837	1061	1141	1226	1314	1407	1503	1603	1705	1811	1918	2028	2141
2020	06955	07611	08296	09011	09745	1051	1131	1215	1303	1395	1491	1590	1692	1797	1903	2012	2125
2030	06889	07539	08218	08926	09655	1041	1121	1204	1292	1383	1479	1577	1679	1783	1889	1997	2110
2040	06824	07468	08140	08841	09565	1032	1111	1194	1281	1372	1467	1565	1666	1770	1875	1983	2095
2050	06767	07404	08064	08767	09480	1023	1101	1184	1270	1361	1455	1553	1653	1756	1861	1969	2080
2060	06710	07340	07988	08683	09395	1014	1092	1174	1260	1350	1444	1541	1641	1744	1848	1955	2066
2070	06647	07270	07916	08600	09310	1005	1082	1164	1249	1339	1432	1529	1628	1731	1834	1941	2051
2080	06584	07200	07845	08520	09225	09960	1073	1154	1239	1328	1421	1517	1616	1718	1821	1927	2037
2090	06523	07135	07775	08444	09145	09875	1064	1145	1229	1318	1410	1505	1603	1705	1808	1914	2023
2100	06463	07070	07705	08370	09065	09790	1056	1136	1220	1308	1399	1493	1591	1692	1795	1901	2009

Distance yds. / v. ft.	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
1720	2805	2945	3089	3236	3387	3542	3701	3865	4035	4211	4393	4581	4776	4977	5185	5399	5620
1730	2784	2923	3066	3213	3363	3517	3675	3838	4007	4182	4363	4550	4744	4944	5151	5364	5583
1740	2763	2902	3044	3190	3339	3492	3649	3811	3979	4153	4333	4519	4712	4911	5117	5329	5547
1750	2742	2880	3021	3166	3315	3467	3624	3785	3953	4126	4305	4490	4682	4880	5084	5295	5511
1760	2722	2859	2999	3143	3291	3443	3599	3760	3927	4100	4278	4462	4652	4849	5052	5261	5476
1770	2701	2837	2976	3119	3267	3418	3574	3734	3901	4073	4250	4433	4622	4817	5019	5227	5441
1780	2680	2815	2954	3096	3243	3394	3549	3709	3875	4046	4222	4404	4592	4786	4986	5193	5406
1790	2659	2793	2931	3073	3219	3370	3524	3684	3849	4019	4194	4375	4562	4755	4954	5159	5371
1800	2639	2772	2909	3050	3196	3346	3500	3659	3823	3992	4166	4346	4532	4724	4922	5126	5337
1810	2619	2751	2888	3028	3173	3322	3476	3634	3797	3965	4138	4317	4502	4693	4890	5093	5303
1820	2599	2731	2867	3007	3151	3299	3452	3609	3771	3938	4110	4288	4472	4662	4858	5061	5270
1830	2579	2710	2846	2985	3129	3276	3428	3584	3746	3912	4084	4261	4444	4633	4828	5029	5237
1840	2559	2690	2825	2964	3107	3254	3405	3560	3721	3887	4058	4234	4416	4604	4798	4998	5204
1850	2540	2670	2804	2942	3085	3231	3382	3536	3696	3861	4032	4207	4388	4575	4768	4966	5171
1860	2522	2651	2184	2921	3063	3209	3359	3512	3671	3836	4006	4181	4361	4547	4738	4935	5138
1870	2503	2631	2764	2900	3041	3186	3336	3488	3647	3811	3980	4154	4333	4518	4708	4904	5105
1880	2484	2612	2744	2880	3020	3164	3313	3465	3623	3786	3954	4127	4306	4490	4679	4873	5073
1890	2465	2593	2724	2860	3000	3143	3291	3442	3599	3761	3928	4100	4278	4461	4649	4842	5041
1900	2447	2574	2705	2840	2979	3122	3269	3420	3576	3737	3903	4074	4251	4433	4620	4812	5010
1910	2429	2555	2686	2820	2959	3101	3247	3397	3553	3713	3878	4048	4224	4405	4591	4782	4979
1920	2411	2537	2667	2801	2939	3080	3225	3375	3530	3689	3853	4023	4198	4378	4563	4753	4949
1930	2394	2519	2648	2782	2919	3059	3204	3353	3507	3665	3829	3998	4172	4351	4535	4724	4919
1940	2377	2502	2629	2763	2899	3039	3183	3331	3484	3641	3805	3973	4146	4324	4507	4695	4889
1950	2360	2484	2611	2744	2879	3018	3162	3309	3461	3618	3781	3948	4120	4297	4480	4667	4859
1960	2343	2466	2593	2725	2859	2998	3141	3288	3439	3595	3757	3923	4095	4271	4453	4639	4830
1970	2326	2448	2575	2706	2840	2978	3121	3267	3418	3573	3734	3900	4070	4245	4426	4611	4801
1980	2309	2431	2557	2687	2821	2959	3101	3247	3397	3552	3712	3877	4046	4220	4399	4583	4772
1990	2292	2413	2539	2668	2802	2939	3081	3226	3376	3531	3690	3855	4024	4197	4374	4556	4744
2000	2275	2396	2521	2650	2783	2920	3061	3206	3355	3509	3669	3834	4002	4174	4350	4530	4716
2010	2258	2379	2503	2632	2764	2901	3041	3185	3333	3485	3645	3809	3976	4153	4323	4503	4688
2020	2242	2362	2486	2614	2746	2882	3022	3165	3311	3463	3621	3784	3951	4122	4297	4476	4661
2030	2226	2346	2469	2596	2728	2863	3003	3145	3290	3441	3598	3760	3926	4096	4271	4449	4634
2040	2211	2330	2452	2579	2710	2845	2984	3125	3269	3419	3575	3736	3901	4071	4245	4423	4607
2050	2196	2314	2436	2563	2693	2827	2965	3105	3249	3398	3554	3714	3878	4047	4220	4397	4580
2060	2181	2299	2420	2546	2676	2809	2946	3086	3229	3378	3533	3692	3855	4023	4195	4371	4553
2070	2166	2283	2404	2539	2658	2790	2927	3066	3209	3357	3513	3670	3833	4000	4171	4346	4527
2080	2151	2268	2388	2512	2640	2772	2908	3047	3189	3337	3491	3649	3811	3977	4147	4321	4501
2090	2135	2251	2371	2494	2622	2753	2889	3027	3169	3317	3470	3628	3789	3954	4123	4296	4475
2100	2120	2235	2354	2477	2604	2735	2870	3008	3150	3297	3450	3607	3767	3931	4099	4271	4449

TABLE X—*continued.*

Distance yds.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500
v. ft.																									
2110	00225	00458	00698	00951	01221	01514	01827	02165	02529	02922	03342	03790	04263	04762	05284	05832	06406	07008	07638	08297	08986	09706	1047	1126	1210
2120	00223	00454	00691	00941	01208	01498	01809	02144	02505	02894	03311	03755	04225	04720	05238	05781	06350	06946	07571	08225	08908	09623	1038	1117	1200
2130	00221	00449	00684	00931	01196	01483	01792	02124	02481	02867	03280	03720	04186	04677	05191	05731	06295	06886	07505	08153	08830	09539	1029	1107	1190
2140	00219	00445	00677	00922	01184	01469	01775	02104	02458	02840	03249	03685	04147	04634	05145	05681	06241	06827	07440	08082	08753	09456	1020	1098	1180
2150	00217	00441	00670	00913	01172	01454	01757	02083	02434	02813	03219	03652	04110	04592	05099	05630	06185	06766	07375	08011	08677	09376	1011	1089	1170
2160	00215	00437	00664	00904	01160	01440	01740	02063	02411	02787	03190	03619	04073	04551	05053	05579	06129	06706	07310	07941	08602	09296	1003	1080	1161
2170	00213	00433	00657	00895	01149	01426	01723	02043	02388	02761	03161	03586	04036	04510	05007	05528	06074	06646	07245	07871	08527	09216	09945	1071	1151
2180	00211	00429	00651	00887	01138	01413	01707	02023	02365	02735	03132	03553	03999	04469	04962	05478	06019	06586	07180	07802	08453	09137	09860	1062	1142
2190	00209	00425	00645	00878	01127	01399	01690	02004	02343	02710	03103	03521	03963	04428	04917	05429	05965	06528	07117	07733	08380	09060	09779	1053	1133
2200	00207	00421	00639	00870	01116	01386	01674	01985	02321	02685	03075	03489	03927	04388	04872	05380	05912	06470	07054	07665	08307	08984	09698	1045	1124
2210	00205	00417	00633	00862	01106	01373	01659	01968	02301	02662	03048	03458	03892	04349	04828	05331	05858	06411	06990	07597	08235	08908	09617	1036	1115
2220	00203	00413	00627	00854	01096	01361	01645	01951	02282	02640	03022	03428	03858	04310	04785	05283	05805	06352	06926	07530	08164	08832	09536	1028	1106
2230	00201	00409	00621	00846	01086	01348	01630	01934	02263	02619	02997	03399	03823	04269	04739	05232	05749	06292	06863	07463	08094	08757	09457	1019	1097
2240	00199	00406	00616	00839	01076	01336	01616	01918	02245	02598	02973	03370	03789	04229	04693	05181	05694	06233	06800	07397	08024	08684	09378	1011	1088
2250	00197	00402	00610	00831	01066	01324	01602	01902	02226	02577	02948	03341	03755	04190	04649	05132	05640	06175	06738	07331	07955	08611	09303	1003	1079
2260	00195	00399	00605	00824	01056	01312	01588	01886	02208	02556	02924	03312	03721	04151	04605	05083	05586	06117	06676	07266	07886	08539	09225	09950	1071
2270	00193	00395	00599	00816	01047	01301	01574	01870	02189	02535	02899	03284	03689	04115	04565	05039	05538	06064	06618	07202	07818	08467	09148	09865	1062
2280	00192	00392	00594	00809	01038	01290	01561	01854	02171	02514	02875	03257	03658	04080	04525	04995	05490	06011	06560	07140	07750	08395	09071	09780	1053
2290	00190	00388	00589	00802	01029	01278	01547	01838	02153	02493	02851	03228	03627	04046	04487	04953	05444	05961	06505	07080	07685	08323	08995	09700	1044
2300	00189	00385	00584	00795	01020	01267	01534	01823	02135	02472	02827	03202	03597	04012	04450	04912	05399	05911	06451	07021	07621	08252	08919	09620	1036
2310	00187	00381	00579	00788	01012	01257	01522	01809	02118	02451	02803	03175	03566	03978	04412	04871	05354	05863	06399	06964	07559	08190	08845	09540	1027
2320	00186	00378	00574	00782	01004	01247	01511	01796	02102	02430	02779	03148	03536	03944	04375	04830	05310	05815	06347	06908	07498	08118	08771	09460	1019
2330	00184	00375	00569	00775	00996	01237	01499	01782	02085	02410	02756	03122	03507	03912	04340	04791	05265	05766	06296	06853	07439	08055	08705	09385	1011
2340	00183	00372	00565	00769	00988	01227	01488	01769	02069	02390	02733	03096	03478	03880	04305	04753	05220	05717	06245	06798	07380	07993	08639	09311	1004
2350	00181	00369	00560	00763	00980	01217	01477	01756	02053	02371	02710	03069	03447	03845	04266	04710	05173	05667	06192	06743	07320	07927	08568	09240	09960
2360	00180	00366	00556	00757	00972	01208	01466	01743	02038	02352	02687	03042	03416	03811	04228	04668	05127	05618	06140	06688	07260	07862	08498	09169	09880
2370	00178	00363	00551	00751	00964	01198	01455	01730	02023	02334	02665	03015	03385	03775	04187	04623	05080	05568	06087	06633	07202	07800	08432	09098	09800
2380	00177	00360	00547	00745	00956	01189	01444	01718	02009	02316	02643	02989	03354	03739	04146	04578	05034	05519	06035	06578	07144	07738	08366	09028	09730
2390	00175	00357	00542	00739	00949	01180	01434	01706	01994	02298	02619	02959	03319	03699	04103	04531	04987	05469	05983	06523	07085	07675	08299	08958	09650
2400	00174	00354	00538	00733	00942	01172	01425	01695	01980	02280	02595	02930	03285	03660	04060	04485	04940	05420	05932	06468	07026	07612	08232	08888	09580
2410	00172	00351	00534	00727	00935	01163	01414	01682	01965	02262	02574	02905	03257	03628	04025	04446	04897	05374	05881	06413	06968	07552	08168	08820	09509
2420	00171	00348	00530	00722	00928	01155	01404	01670	01950	02244	02553	02881	03229	03597	03990	04408	04854	05328	05830	06358	06911	07492	08105	08753	09438
2430	00169	00345	00526	00716	00921	01146	01393	01657	01934	02226	02532	02857	03201	03566	03955	04369	04811	05281	05779	06304	06854	07432	08042	08686	09367
2440	00168	00342	00522	00711	00914	01138	01383	01644	01919	02208	02511	02833	03174	03535	03920	04331	04769	05235	05729	06251	06798	07372	07979	08620	09297
2450	00166	00339	00518	00705	00907	01129	01372	01631	01904	02190	02490	02809	03147	03505	03887	04294	04728	05190	05680	06199	06743	07313	07916	08553	09225
2460	00165	00337	00514	00700	00900	01121	01362	01619	01889	02172	02469	02785	03120	03475	03854	04257	04687	05145	05632	06147	06688	07255	07854	08486	09153
2470	00164	00334	00510	00695	00893	01112	01351	01606	01873	02154	02448	02761	03093	03445	03820	04220	04646	05101	05584	06096	06633	07197	07792	08421	09080
2480	00163	00332	00506	00690	00887	01103	01340	01593	01858	02136	02427	02737	03066	03415	03787	04183	04606	05057	05537	06045	06578	07139	07731	08357	09017
2490	00162	00329	00502	00685	00880	01094	01329	01580	01843	02118	02406	02713	03039	03385	03754	04146	04565	05012	05490	05995	06525	07083	07673	08296	08953

Distance yds.	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
v. ft.																									
2110	1297	1387	1481	1578	1679	1781	1887	1994	2105	2219	2338	2460	2587	2717	2852	2989	3131	3277	3429	3585	3744	3908	4075	4247	4424
2120	1286	1376	1469	1566	1666	1768	1873	1980	2090	2204	2322	2444	2570	2700	2834	2971	3112	3258	3408	3563	3722	3885	4052	4223	4399
2130	1275	1365	1457	1554	1653	1754	1859	1965	2075	2188	2306	2427	2553	2682	2816	2953	3094	3239	3389	3543	3700	3863	4029	4149	4374
2140	1265	1354	1446	1542	1641	1741	1845	1951	2060	2173	2290	2411	2536	2665	2798	2935	3076	3221	3370	3523	3679	3841	4006	4175	4349
2150	1255	1343	1435	1530	1629	1728	1831	1937	2045	2158	2274	2395	2519	2648	2780	2917	3057	3202	3350	3503	3658	3819	3983	4151	4324
2160	1245	1333	1424	1519	1617	1716	1818	1923	2031	2143	2259	2379	2503	2631	2763	2899	3039	3183	3331	3483	3638	3798	3961	4128	4300
2170	1235	1322	1413	1507	1605	1703	1804	1908	2015	2127	2242	2362	2486	2614	2746	2881	3021	3164	3312	3463	3618	3777	3939	4105	4276
2180	1225	1312	1402	1496	1593	1690	1790	1893	2000	2111	2226	2345	2469	2597	2729	2864	3003	3146	3293	3444	3598	3756	3917	4082	4252
2190	1215	1302	1391	1485	1581	1678	1777	1880	1986	2097	2211	2329	2453	2580	2712	2846	2985	3127	3274	3424	3578	3735	3895	4059	4228
2200	1206	1292	1381	1474	1570	1666	1765	1867	1973	2083	2196	2314	2437	2564	2695	2829	2967	3109	3255	3405	3558	3714	3873	4036	4204
2210	1196	1282	1370	1463	1558	1654	1752	1854	1960	2069	2182	2299	2421	2548	2678	2812	2949	3091	3237	3386	3538	3693	3851	4014	4181
2220	1187	1272	1360	1452	1547	1642	1740	1842	1947	2056	2168	2285	2406	2532	2662	2795	2932	3074	3219	3367	3518	3672	3830	3992	4158
2230	1178	1262	1350	1441	1536	1630	1728	1829	1934	2042	2154	2270	2391	2516	2646	2778	2915	3056	3200	3347	3497	3651	3798	3970	4135
2240	1169	1253	1340	1431	1525	1619	1716	1817	1921	2029	2140	2256	2376	2501	2630	2762	2898	3038	3181	3327	3477	3630	3787	3948	4113
2250	1160	1243	1330	1420	1514	1607	1704	1804	1908	2015	2126	2242	2361	2486	2614	2746	2881	3021	3163	3309	3458	3610	3766	3926	4091
2260	1151	1234	1320	1410	1503	1596	1692	1792	1895	2002	2113	2228	2347	2471	2599	2730	2865	3004	3146	3291	3439	3590	3745	3905	4069
2270	1141	1224	1310	1399	1492	1585	1681	1780	1883	1989	2100	2214	2333	2456	2584	2714	2849	2987	3128	3273	3410	3571	3725	3884	4047
2280	1132	1215	1300	1389	1481	1574	1670	1769	1871	1977	2087	2201	2319	2442	2569	2699	2833	2970	3111	3255	3402	3552	3706	3864	4026
2290	1123	1205	1290	1379	1470	1563	1658	1757	1859	1964	2074	2187	2305	2427	2554	2683	2817	2954	3094	3238	3384	3534	3687	3844	4005
2300	1114	1196	1281	1369	1460	1552	1647	1745	1847	1952	2061	2174	2291	2413	2539	2668	2801	2938	3078	3221	3367	3516	3668	3824	3984
2310	1105	1186	1271	1359	1450	1541	1636	1733	1835	1940	2048	2161	2277	2398	2524	2652	2784	2921	3061	3203	3349	3497	3649	3804	3963
2320	1096	1177	1261	1349	1440	1531	1625	1722	1823	1928	2036	2148	2264	2384	2509	2637	2769	2905	3044	3186	3331	3479	3630	3784	3942
2330	1088	1168	1252	1339	1429	1520	1614	1710	1811	1915	2023	2134	2250	2369	2494	2621	2753	2888	3027	3168	3313	3460	3611	3764	3922
2340	1080	1160	1243	1329	1419	1510	1603	1699	1799	1903	2010	2121	2236	2355	2479	2606	2737	2872	3010	3151	3295	3442	3592	3745	3902
2350	1071	1151	1233	1319	1409	1499	1592	1687	1787	1890	1997	2107	2222	2340	2464	2590	2721	2855	2993	3133	3277	3423	3573	3725	3882
2360	1063	1142	1224	1310	1399	1489	1581	1676	1775	1878	1984	2094	2208	2326	2449	2575	2705	2839	2976	3116	3259	3405	3554	3706	3862
2370	1055	1133	1215	1300	1389	1478	1570	1665	1763	1866	1971	2081	2194	2312	2434	2559	2689	2822	2959	3098	3241	3386	3535	3686	3842
2380	1047	1125	1206	1291	1379	1468	1559	1654	1752	1854	1959	2068	2181	2298	2419	2544	2673	2806	2942	3081	3223	3368	3516	3667	3822
2390	1039	1116	1197	1281	1369	1458	1549	1643	1741	1842	1947	2055	2168	2284	2405	2529	2658	2790	2926	3064	3206	3350	3498	3648	3803
2400	1031	1108	1188	1272	1359	1448	1539	1633	1730	1831	1935	2043	2155	2271	2391	2515	2643	2775	2910	3048	3189	3333	3480	3630	3784
2410	1023	1100	1179	1263	1349	1438	1529	1622	1719	1819	1923	2030	2142	2257	2377	2500	2628	2759	2894	3031	3172	3315	3462	3611	3765
2420	1016	1092	1171	1254	1340	1428	1519	1612	1708	1808	1911	2018	2129	2244	2363	2486	2613	2744	2878	3015	3155	3298	3444	3593	3746
2430	1008	1084	1162	1245	1330	1418	1508	1601	1697	1796	1899	2005	2116	2230	2349	2471	2598	2728	2862	2998	3138	3280	3426	3574	3727
2440	1001	1076	1154	1236	1321	1408	1498	1591	1686	1785	1887	1993	2103	2217	2335	2457	2583	2713	2846	2982	3121	3263	3408	3556	3708
2450	09932	1067	1145	1227	1311	1398	1487	1580	1675	1773	1875	1980	2090	2203	2321	2442	2568	2697	2830	2965	3104	3245	3390	3537	3689
2460	09855	1059	1137	1218	1302	1388	1477	1569	1664	1762	1863	1968	2077	2190	2307	2428	2553	2682	2814	2949	3087	3228	3372	3519	3670
2470	09783	1051	1129	1209	1293	1378	1467	1558	1653	1750	1851	1955	2064	2177	2294	2414	2539	2667	2798	2933	3070	3201	3354	3501	3651
2480	09712	1044	1121	1201	1284	1369	1457	1548	1642	1739	1839	1943	2051	2164	2281	2401	2525	2652	2783	2917	3054	3194	3337	3483	3633
2490	09644	1037	1113	1193	1275	1360	1447	1538	1631	1728	1827	1931	2039	2151	2268	2387	2511	2637	2768	2901	3038	3177	3330	3465	3615

Distance yds.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000	2100	2200	2300	2400	2500	2600	2700
v. ft.																											
2500	00161	00327	00498	00680	00874	01086	01319	01568	01828	02100	02386	02690	03013	03356	03721	04110	04525	04968	05443	05946	06473	07028	07615	08235	08889	09577	1030
2510	00159	00324	00494	00675	00868	01078	01308	01555	01812	02082	02365	02666	02986	03326	03688	04075	04487	04928	05399	05898	06422	06973	07557	08174	08825	09510	1023
2520	00158	00322	00491	00670	00862	01071	01298	01542	01797	02064	02345	02643	02960	03297	03656	04040	04450	04888	05355	05850	06371	06919	07500	08114	08762	09444	1016
2530	00157	00319	00487	00665	00856	01063	01288	01530	01782	02047	02325	02621	02935	03269	03625	04006	04412	04847	05311	05803	06321	06866	07443	08054	08698	09376	1009
2540	00156	00317	00484	00661	00850	01056	01279	01518	01768	02030	02306	02599	02911	03242	03595	03972	04375	04806	05267	05756	06271	06813	07387	07994	08634	09308	1002
2550	00155	00315	00480	00656	00844	01048	01270	01507	01755	02015	02288	02579	02888	03217	03567	03941	04340	04768	05224	05710	06222	06761	07332	07935	08572	09242	09948
2560	00154	00313	00477	00652	00838	01041	01262	01496	01742	02000	02271	02559	02866	03192	03540	03910	04306	04730	05182	05664	06173	06710	07277	07877	08510	09176	09876
2570	00153	00311	00474	00647	00832	01033	01252	01485	01729	01985	02253	02539	02843	03166	03510	03877	04270	04691	05140	05619	06124	06658	07221	07817	08446	09107	09803
2580	00152	00309	00471	00643	00826	01026	01243	01474	01716	01970	02236	02519	02820	03140	03481	03845	04234	04652	05099	05574	06076	06606	07166	07758	08382	09039	09730
2590	00151	00307	00468	00639	00821	01020	01235	01463	01703	01955	02219	02500	02798	03116	03454	03814	04199	04613	05058	05530	06028	06555	07112	07700	08321	08969	09656
2600	00150	00305	00465	00635	00816	01014	01227	01453	01691	01941	02203	02481	02777	03092	03427	03783	04164	04574	05017	05486	05981	06504	07058	07643	08260	08900	09583
2610	00149	00303	00462	00631	00811	01007	01219	01443	01679	01927	02187	02463	02756	03069	03401	03754	04133	04540	04978	05443	05935	06455	07005	07586	08198	08839	09517
2620	00148	00301	00459	00627	00806	01001	01211	01433	01667	01913	02171	02445	02736	03046	03375	03725	04102	04506	04939	05401	05890	06406	06952	07529	08137	08778	09452
2630	00147	00299	00456	00623	00801	00994	01202	01423	01655	01899	02155	02427	02715	03023	03350	03698	04071	04472	04902	05360	05846	06359	06902	07476	08080	08717	09387
2640	00146	00297	00453	00619	00796	00988	01194	01413	01643	01885	02139	02409	02695	03000	03325	03671	04041	04439	04865	05320	05803	06313	06853	07423	08024	08657	09322
2650	00145	00295	00450	00615	00791	00981	01186	01403	01631	01871	02123	02391	02675	02977	03299	03643	04010	04405	04828	05280	05760	06266	06803	07369	07967	08596	09256
2660	00144	00293	00447	00611	00786	00975	01178	01393	01619	01857	02107	02373	02655	02955	03274	03615	03980	04372	04792	05241	05718	06221	06753	07316	07910	08535	09191
2670	00143	00291	00444	00607	00781	00968	01170	01383	01607	01843	02091	02355	02635	02932	03249	03587	03950	04339	04757	05203	05676	06176	06704	07263	07853	08473	09125
2680	00142	00289	00442	00604	00776	00962	01162	01373	01595	01829	02075	02337	02615	02910	03224	03560	03920	04307	04722	05165	05635	06131	06656	07211	07796	08412	09060
2690	00141	00287	00439	00600	00771	00955	01153	01362	01583	01815	02059	02319	02595	02888	03200	03533	03890	04274	04687	05127	05594	06087	06609	07160	07740	08352	08995
2700	00141	00286	00437	00597	00766	00949	01145	01352	01571	01801	02044	02302	02576	02867	03176	03506	03860	04242	04653	05090	05554	06044	06562	07109	07685	08292	08930
2710	00140	00284	00434	00593	00761	00942	01136	01341	01558	01787	02028	02284	02556	02845	03152	03480	03831	04210	04617	05053	05514	06001	06516	07059	07632	08235	08869
2720	00139	00282	00431	00589	00756	00936	01128	01331	01546	01773	02013	02267	02537	02824	03129	03454	03803	04178	04582	05016	05474	05958	06470	07010	07579	08178	08808
2730	00138	00280	00428	00585	00751	00929	01119	01321	01534	01759	01997	02249	02518	02803	03106	03429	03775	04148	04549	04980	05435	05916	06424	06960	07525	08120	08746
2740	00137	00278	00426	00581	00746	00923	01111	01311	01522	01745	01981	02232	02499	02782	03083	03404	03748	04118	04516	04944	05396	05874	06379	06911	07472	08063	08684
2750	00136	00276	00423	00577	00741	00916	01103	01301	01510	01731	01965	02214	02479	02760	03060	03379	03721	04088	04483	04908	05357	05831	06332	06861	07418	08005	08622
2760	00135	00275	00421	00574	00736	00910	01095	01291	01498	01717	01949	02196	02459	02739	03037	03355	03694	04058	04450	04872	05318	05789	06286	06811	07365	07948	08561
2770	00134	00273	00418	00570	00731	00904	01087	01281	01486	01703	01933	02179	02440	02718	03014	03330	03666	04028	04417	04836	05279	05747	06241	06762	07312	07891	08500
2780	00134	00272	00416	00567	00727	00898	01079	01271	01474	01689	01917	02161	02421	02697	02991	03304	03638	03997	04383	04799	05239	05704	06195	06713	07260	07835	08440
2790	00133	00270	00413	00563	00722	00892	01071	01261	01462	01675	01902	02144	02402	02676	02968	03278	03610	03966	04350	04762	05200	05662	06150	06665	07209	07781	08382
2800	00133	00269	00411	00560	00718	00886	01064	01252	01451	01662	01887	02127	02383	02655	02944	03252	03581	03935	04316	04726	05160	05619	06104	06617	07158	07727	08325
2810	00132	00267	00408	00556	00713	00880	01056	01242	01439	01649	01872	02110	02364	02634	02921	03227	03554	03905	04283	04690	05121	05576	06056	06566	07102	07667	08260
2820	00131	00266	00406	00553	00709	00874	01049	01233	01428	01636	01857	02093	02345	02613	02898	03202	03527	03875	04250	04654	05082	05534	06011	06515	07047	07607	08196
2830	00130	00264	00403	00549	00704	00867	01040	01223	01416	01623	01842	02076	02326	02592	02875	03177	03499	03845	04217	04618	05043	05492	05966	06467	06995	07552	08137
2840	00130	00263	00401	00546	00699	00861	01032	01213	01405	01610	01827	02059	02307	02571	02852	03152	03472	03815	04184	04582	05004	05450	05921	06419	06944	07497	08079
2850	00129	00261	00398	00542	00694	00854	01024	01203	01394	01597	01812	02043	02289	02551	02830	03127	03444	03785	04151	04546	04965	05408	05876	06371	06892	07441	08019
2860	00128	00259	00395	00538	00689	00848	01016	01194	01383	01584	01798	02027	02271	02531	02808	03103	03417	03755	04119	04511	04927	05367	05832	06323	06840	07385	07959
2870	00127	00258	00393	00534	00683	00841	01007	01184	01371	01571	01783	02011	02253	02511	02786	03078	03390	03726	04087	04476	04889	05326	05788	06275	06789	07330	07900
2880	00127	00257	00391	00531	00678	00834	00999	01174	01360	01558	01769	01995	02236	02492	02764	03054	03364	03697	04055	04441	04851	05285	05744	06228	06738	07275	07841
2890	00126	00255	00388	00526	00672	00827	00991	01165	01349	01545	01754	01978	02217	02471	02741	03029	03337	03667	04023	04406	04813	05244	05700	06181	06687	07221	07784
2900	00125	00253	00385	00522	00667	00821	00984	01156	01338	01532	01740	01962	02199	02451	02719	03005	03310	03638	03991	04371	04775	05203	05656	06134	06637	07167	07727

Distance yds.	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
v. ft.																							
2500	1106	1185	1267	1351	1438	1528	1621	1717	1816	1919	2027	2139	2255	2374	2497	2623	2753	2886	3022	3161	3303	3448	3597
2510	1098	1177	1258	1342	1428	1518	1610	1706	1804	1907	2015	2126	2242	2360	2482	2608	2737	2870	3005	3144	3286	3430	3579
2520	1091	1169	1250	1333	1419	1508	1600	1695	1793	1896	2003	2114	2229	2346	2468	2593	2722	2854	2989	3128	3269	3413	3561
2530	1083	1161	1241	1324	1409	1498	1589	1684	1782	1884	1991	2101	2216	2332	2454	2578	2707	2838	2973	3112	3252	3396	3543
2540	1076	1153	1233	1315	1400	1488	1579	1673	1771	1873	1979	2089	2203	2319	2440	2564	2692	2823	2958	3096	3236	3379	3525
2550	1068	1145	1224	1306	1390	1478	1569	1662	1760	1861	1967	2076	2190	2305	2426	2549	2677	2807	2942	3079	3219	3361	3507
2560	1061	1137	1216	1297	1381	1468	1559	1652	1749	1850	1955	2064	2177	2292	2412	2535	2662	2792	2926	3063	3202	3344	3489
2570	1053	1129	1207	1288	1371	1458	1548	1641	1738	1838	1943	2051	2164	2278	2398	2520	2647	2776	2910	3046	3185	3327	3471
2580	1045	1121	1199	1279	1362	1448	1538	1631	1727	1827	1931	2039	2151	2265	2384	2506	2632	2761	2894	3030	3169	3310	3454
2590	1037	1113	1191	1270	1353	1439	1528	1621	1716	1816	1919	2027	2138	2252	2370	2492	2617	2746	2878	2014	3152	3293	3436
2600	1030	1105	1183	1262	1344	1430	1519	1611	1706	1805	1908	2015	2126	2239	2357	2478	2603	2731	2863	2998	3136	3276	3419
2610	1023	1097	1175	1254	1336	1421	1510	1601	1696	1794	1897	2003	2114	2227	2344	2465	2590	2717	2848	2982	3119	3259	3402
2620	1016	1090	1167	1246	1328	1413	1501	1592	1686	1784	1886	1992	2102	2215	2332	2453	2577	2704	2834	2967	3103	3242	3385
2630	1009	1082	1159	1237	1319	1404	1492	1582	1676	1773	1875	1980	2090	2202	2319	2440	2563	2690	2819	2952	3087	3226	3368
2640	1002	1075	1151	1229	1310	1395	1483	1573	1666	1763	1864	1969	2078	2190	2307	2427	2550	2676	2805	2937	3072	3210	3351
2650	09950	1067	1143	1220	1301	1385	1473	1563	1656	1752	1853	1957	2066	2177	3294	2413	2536	2661	2790	2921	3056	3193	3334
2660	09880	1060	1135	1212	1292	1376	1463	1553	1646	1742	1842	1946	2054	2165	2281	2400	2522	2647	2775	2906	3040	3177	3317
2670	09810	1052	1127	1203	1283	1366	1453	1542	1635	1731	1831	1934	2042	2152	2268	2386	2508	2632	2760	2890	3024	3160	3300
2680	09740	1045	1119	1195	1274	1357	1443	1532	1624	1720	1820	1923	2030	2140	2255	2373	2494	2618	2745	2875	3008	3144	3283
2690	09670	1037	1111	1187	1265	1348	1433	1522	1613	1709	1808	1911	2018	2128	2242	2360	2480	2604	2730	2860	2992	3128	3266
2700	09600	1030	1104	1179	1257	1339	1424	1512	1603	1698	1797	1900	2007	2116	2230	2347	2467	2590	2716	2845	2977	3112	3250
2710	09530	1023	1096	1171	1248	1330	1415	1503	1593	1688	1786	1889	1995	2104	2217	2333	2454	2576	2702	2830	2962	3096	3234
2720	09470	1016	1089	1163	1240	1321	1406	1494	1584	1678	1776	1878	1984	2092	2204	2320	2439	2562	2688	2816	2947	3081	3218
2730	09403	1009	1081	1155	1231	1312	1396	1483	1573	1667	1765	1867	1972	2080	2191	2307	2426	2548	2674	2800	2932	3065	3202
2740	09336	1002	1074	1147	1223	1303	1386	1473	1563	1657	1755	1856	1961	2068	2179	2294	2413	2535	2660	2787	2917	3050	3186
2750	09270	09950	1067	1139	1215	1295	1377	1464	1553	1647	1744	1845	1949	2056	2166	2281	2399	2521	2645	2772	2902	3035	3170
2760	09205	09881	1060	1132	1208	1287	1369	1455	1544	1637	1734	1834	1938	2044	2154	2268	2386	2507	2631	2758	2888	3020	3155
2770	09139	09811	1052	1124	1200	1279	1361	1446	1534	1627	1723	1823	1926	2032	2142	2256	2373	2494	2617	2744	2873	3005	3139
2780	09075	09742	1045	1117	1193	1272	1353	1437	1525	1617	1713	1812	1915	2021	2131	2244	2361	2481	2604	2730	2859	2990	3124
2790	09013	09675	1037	1109	1185	1263	1344	1428	1516	1607	1703	1801	1904	2009	2119	2232	2348	2468	2590	2716	2844	2975	3108
2800	08952	09610	1030	1102	1177	1255	1336	1420	1507	1598	1693	1791	1893	1998	2108	2220	2336	2455	2577	2702	2830	2960	3093
2810	08884	09538	1023	1094	1169	1247	1327	1411	1498	1588	1683	1780	1882	1987	2096	2208	2324	2442	2564	2688	2815	2945	3077
2820	08816	09467	1015	1087	1162	1239	1319	1402	1489	1579	1673	1770	1871	1976	2085	2197	2312	2430	2551	2675	2801	2930	3062
2830	08753	09400	1008	1080	1154	1231	1310	1393	1479	1569	1663	1759	1860	1965	2073	2185	2299	2417	2537	2661	2786	2915	3046
2840	08691	09334	1001	1073	1147	1223	1302	1384	1470	1560	1653	1749	1849	1954	2062	2173	2287	2404	2524	2647	2772	2900	3031
2850	08627	09264	09942	1065	1139	1214	1293	1375	1461	1550	1643	1738	1838	1942	2050	2161	2274	2391	2510	2633	2757	2885	3015
2860	08564	09201	09872	1058	1131	1206	1285	1367	1452	1541	1633	1728	1827	1931	2039	2149	2262	2378	2497	2619	2743	2870	3000
2870	08501	09137	09803	1050	1123	1198	1277	1358	1443	1531	1623	1717	1816	1919	2027	2137	2249	2365	2483	2605	2728	2855	2984
2880	08438	09068	09734	1043	1116	1191	1269	1350	1434	1522	1613	1707	1805	1908	2015	2125	2237	2352	2470	2591	2714	2840	2969
2890	08378	09005	09676	1036	1108	1183	1260	1341	1424	1512	1603	1696	1794	1896	2003	2112	2224	2339	2456	2577	2699	2825	2953
2900	08319	08943	09600	1029	1101	1175	1252	1332	1415	1502	1593	1686	1783	1885	1991	2100	2212	2326	2443	2563	2685	2810	2938

Table XI.

Tenuity correction τ for Temperature and Pressure of Atmosphere two-thirds saturated with Moisture.

(From the Rev. F. Bashforth's paper, *Proc. R.A.I.*, Vol. XIII, No. 10.)

F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +
0°	·983	1·021	1·059	1·097	1·134	1·172	38
1	·981	1·019	1·056	1·094	1·132	1·170	38
2	·979	1·017	1·054	1·092	1·130	1·167	38
3	·977	1·015	1·052	1·090	1·127	1·165	38
4	·975	1·012	1·050	1·087	1·125	1·162	38
5	·973	1·010	1·047	1·085	1·122	1·160	37
6	·971	1·008	1·045	1·083	1·120	1·157	37
7	·969	1·006	1·043	1·080	1·118	1·155	37
8	·966	1·004	1·041	1·078	1·115	1·152	37
9	·964	1·001	1·039	1·076	1·113	1·150	37
10	·962	·999	1·036	1·073	1·110	1·147	37
11	·960	·997	1·034	1·071	1·108	1·145	37
12	·958	·995	1·032	1·069	1·105	1·142	37
13	·956	·993	1·029	1·066	1·103	1·140	37
14	·954	·991	1·027	1·064	1·101	1·137	37
15	·952	·989	1·025	1·062	1·098	1·135	37
16	·950	·986	1·023	1·060	1·096	1·133	37
17	·948	·984	1·021	1·057	1·094	1·130	37
18	·946	·982	1·019	1·055	1·091	1·128	36
19	·944	·980	1·017	1·053	1·089	1·125	36
20	·942	·978	1·014	1·051	1·087	1·123	36
21	·940	·976	1·012	1·048	1·084	1·121	36
22	·938	·974	1·010	1·046	1·082	1·118	36
23	·936	·972	1·008	1·044	1·080	1·116	36
24	·934	·970	1·006	1·042	1·078	1·114	36
25	·932	·968	1·004	1·039	1·075	1·111	36
26	·930	·966	1·001	1·037	1·073	1·109	36
27	·928	·964	·999	1·035	1·071	1·106	36
28	·926	·962	·997	1·033	1·069	1·104	36
29	·924	·960	·995	1·031	1·066	1·102	36
30	·922	·958	·993	1·028	1·064	1·099	36
31	·920	·956	·991	1·026	1·062	1·097	35
32	·918	·954	·989	1·024	1·059	1·095	35
33	·916	·952	·987	1·022	1·057	1·093	35
34	·914	·950	·985	1·020	1·055	1·090	35
35	·913	·948	·983	1·018	1·053	1·088	35
36	·911	·946	·981	1·016	1·051	1·086	35
37	·909	·944	·979	1·013	1·048	1·083	35
38	·907	·942	·977	1·011	1·046	1·081	35
39	·905	·940	·974	1·009	1·044	1·079	35
40	·903	·938	·973	1·007	1·042	1·077	35
41	·901	·936	·971	1·005	1·040	1·075	35
42	·899	·934	·968	1·003	1·038	1·072	35
43	·898	·932	·967	1·001	1·036	1·070	35
44	·896	·930	·964	0·999	1·033	1·068	34
45	·894	·928	·963	0·997	1·031	1·066	34
46	·892	·926	·960	0·995	1·029	1·063	34
47	·890	·924	·958	0·993	1·027	1·061	34
48	·888	·923	·957	·991	1·025	1·059	34
49	·886	·920	·955	·989	1·023	1·057	34
50	·884	·919	·953	·987	1·021	1·055	34

F.	26 in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +
50°	·884	·919	·953	·987	1·021	1·055	34
51	·883	·917	·951	·985	1·019	1·053	34
52	·881	·915	·949	·983	1·017	1·051	34
53	·879	·913	·947	·981	1·015	1·048	34
54	·877	·911	·945	·978	1·012	1·046	34
55	·875	·909	·943	·976	1·010	1·044	34
56	·874	·907	·941	·974	1·008	1·042	34
57	·872	·905	·939	·972	1·006	1·039	34
58	·870	·904	·937	·970	1·004	1·037	34
59	·868	·902	·935	·968	1·002	1·035	33
60	·866	·900	·933	·966	1·000	1·033	33
61	·865	·898	·931	·964	·998	1·031	33
62	·863	·896	·929	·962	·996	1·029	33
63	·861	·894	·927	·960	·993	1·027	33
64	·859	·892	·925	·958	·991	1·024	33
65	·857	·890	·923	·956	·989	1·022	33
66	·856	·889	·921	·954	·987	1·020	33
67	·854	·887	·919	·952	·985	1·018	33
68	·852	·885	·918	·950	·983	1·016	33
69	·850	·883	·916	·949	·981	1·014	33
70	·849	·881	·914	·946	·979	1·012	33
71	·847	·879	·912	·944	·977	1·010	33
72	·845	·878	·910	·943	·975	1·008	33
73	·843	·876	·908	·941	·973	1·006	32
74	·842	·874	·906	·939	·971	1·004	32
75	·840	·872	·904	·937	·969	1·001	32
76	·838	·870	·902	·935	·967	·999	32
77	·836	·868	·901	·933	·965	·997	32
78	·834	·867	·899	·931	·963	·995	32
79	·833	·865	·897	·929	·961	·993	32
80	·831	·863	·895	·927	·959	·991	32
81	·829	·861	·893	·925	·957	·989	32
82	·827	·859	·891	·923	·955	·987	32
83	·826	·858	·889	·921	·953	·985	32
84	·824	·856	·887	·919	·951	·983	32
85	·822	·854	·885	·917	·949	·980	32
86	·821	·852	·884	·915	·947	·978	32
87	·819	·850	·882	·913	·945	·976	32
88	·817	·848	·880	·911	·943	·974	31
89	·815	·847	·878	·909	·941	·972	31
90	·814	·845	·876	·908	·939	·970	31
91	·812	·843	·874	·905	·937	·968	31
92	·810	·841	·872	·903	·935	·966	31
93	·808	·839	·870	·902	·933	·964	31
94	·806	·837	·868	·900	·931	·962	31
95	·805	·836	·867	·898	·929	·960	31
96	·803	·834	·865	·896	·926	·957	31
97	·801	·832	·863	·893	·924	·955	31
98	·799	·830	·861	·891	·922	·953	31
99	·797	·828	·859	·889	·920	·951	31
100	·796	·826	·857	·888	·918	·949	31

Table XII.

Particulars of Rifled Guns and Howitzers.

Calibre.	Weight.	Length of bore in calibres.	Maximum charge. Powder.	Maximum charge. Cordite.		Projectile.	Penetration of wrought iron at 1,000 yds.	Muzzle velocity.	Maximum range.
			lb.	lb. oz.	size.	lb.	inches.	f. s.	yards.
B.L. Guns.									
inch.									
16·25	111 tons	30	960 S.B.C.	—	—	1800	32	2087	12,000
13·5 (I to IV)	69 & 67 tons	30	630 ,,	187 0	44	1250	28·2	2016	12,000
12 (I, IA) 12 (III to VII)	47 tons 45 & 46 tons	25·14 25·25	295 Pm.[1] br.	88 8	30	714	20·4	1914	8,000
12 (wire, VIII)	46 tons	35·43	—	174 0	50 & $3\frac{3}{4}$	850	28·6	2367	10,000
12 (wire, IX)	50 ,,	40·0	—	211 0	50 & $3\frac{3}{4}$	850	34·16	2481	12,000
10 (I) 10 (II to IV)	32 ,, 29 ,,	31·75 32	252 Pm.[1] br.	76 0	30	500	20·7	2040	10,000
9·2 (I, II)	22 & 21 tons	25·56	140 ,,	42 0	30	380	15·9	1781	10,000
9·2 (III, IV) 9·2 (V to VII)	24 & 23 ,, 22 tons	31·5	164 ,,	53 8	30	380	18·8	2065	10,000
9·2 (wire, VIII)	25 ,,	40·08	—	63 0	40	380	21·3	2347	12,400
9·2 (wire, IX)	27 ,,	46·71	—	100 0§§	44	380	27·5	2601	13,800
9·2 (wire, X)	28 ,,	46·66	—	103 0	44 & $3\frac{3}{4}$	380	—	—	—
8 (III)	14 ,,	25·1	104 Pm.[1] br.	28 12	20	210	13·5	1953	8,000
8 (IV, VI)	15 & 14 tons	29·61	118 ,,	32 10	20	210	15·0	2150	8,000
8 (VII, VIIA)	12 & 13 ,,	25·5	90 Pm.[1] bl.	22 0	20	180	12·8	2000	8,000
6 (III chase hooped) 6 (IV, VI, & VIA)	5 tons 5 ,,	25·53 26	48 E.X.E.*	— 14 12	— 20	100	10·5	1960	10,000
6 (V)	5 ,,	30·58	45 Pm.[1] bl.	—	—	100	10·2	1890	8,000
6 (wire, VII, VIII)	7 ,,	44·9	—	20 0	20	100	14·8	2460	12,000
6 (IX, X)	7 ,,	50	—	20 0	30	100	—	2610	—
80-pr. (I)	82 cwt.	25·53	25 S.P.	—	—	80	—	1575	5,000
5 (II) 5 (III to V)	38 ,, 40 ,,	25·07 25	$15\frac{1}{2}$,,	4 $7\frac{1}{4}$	$7\frac{1}{3}$	50	6·25	1750	8,700
4 (III to VI)	23 & 26 cwt.	27	12 ,,	3 1	5	25	5·4	1900	7,700
4 (jointed, I)	25 cwt.	25·45	$11\frac{1}{4}$,,	—	—	25	—	—	—
30-pr. (I)	20 ,,	27	—	2 6	10	30	—	1621	6,300
15 ,, (I)	7 ,,	28	—	0 $15\frac{3}{4}$	5	$14\frac{1}{16}$	—	1574†	5,500
15 ,, (IV)	7 ,,	28	—	0 $15\frac{3}{4}$	5	14	—	1581	6,000
12 ,, 7 cwt. (I)	7 ,,	28	4 S.P.	0 $15\frac{3}{4}$	5	$12\frac{1}{2}$	—	1710	5,000
12 ,, 6 ,, (I)	6 ,,	19·66	—	0 $12\frac{7}{16}$	5	$12\frac{1}{2}$	—	1553	5,200
12 ,, 6 ,, (IV)	6 ,,	22	—	0 $12\frac{7}{16}$	5	$12\frac{1}{2}$	—	—	—
32 ,, S.B. (I)‡	42 ,,	13·5	3 R.L.G.[2]	—	—	$54\frac{1}{2}$§	—	—	—
B.L. Howitzers.									
8-in.	70 cwt.	13	—	4 2	$7\frac{1}{2}$	$276\frac{1}{2}$	—	781	5,500
6 ,, 30 cwt. (I)	30 ,,	14	—	1 12	5	$122\frac{9}{16}$	—	777	5,200
6 ,, 25 ,, (I)	25 ,,	12	—	2 1	$3\frac{3}{4}$	$122\frac{9}{16}$	—	779	5,200
5·4-inch (I)	13 ,,	10	—	0 $13\frac{1}{2}$	$3\frac{3}{4}$	60	—	781	3,500
5 ,, (I)	9 ,,	8·4	—	0 $11\frac{7}{16}$	$3\frac{5}{8}$	50	—	782	4,900
R.B.L. Guns.									
7-in.	82 cwt.	14·21	11 R.L.G.[2]	—	—	100	5	1100‖	3,500
7 ,,	72 ,,	14·21 13·93	10 ,,	—	—	100	5	1100‖	3,500
40-pr.	32 & 35 cwt.	22·39	5 ,,	—	—	40	—	1160	3,500
20 ,,	16 cwt.	22·36	$2\frac{1}{2}$,,	—	—	22	—	1130	3,500
20 ,,	15 & 13 cwt.	14·43	$2\frac{1}{2}$,,	—	—	22	—	1000	3,000
12 ,,	8 cwt.	20·46	$1\frac{1}{2}$,,	—	—	$11\frac{1}{4}$	—	1239	3,400
9 ,,	6 ,,	17·5	$1\frac{1}{8}$,,	—	—	$8\frac{1}{2}$	—	1055	3,000

* Only used with the 6-inch Marks III, IV, and VI guns when on V.B., V.C.P., or strengthened A.B. mountings. Only 36 E X.E. is used with Marks III, IV, and VI guns on strengthened A.B. mountings.

† On Mark II carriage the muzzle velocity is 1,569 feet-seconds.

‡ For defence of ditches of forts, &c. § Case shot.

‖ Difference in size of chamber gives same muzzle velocity with different charges.

Table XII—*continued.*

Particulars of Rifled Guns and Howitzers.

Calibre.	Weight.	Length of bore in calibres.	Maximum charge. Powder.	Maximum charge. Cordite.		Projectile.	Penetration of wrought iron at 1,000 yds.	Muzzle velocity.	Maximum range.
			lb.	lb. oz.	size.	lb.	inches.	f/s.	yards.
Q.F. Guns.									
6-in. (I to III)	7 tons	40	27¾ E.X.E., and R.L.G.[4]	13 4	30	100	10 11·6	1882¶ 2154**	— 10,000
4·7 (I to III)	41 cwt.	40	12 S.P.	5 7	20	45	6·7	1786¶	—
4·7 (IV)	42 ,,	40	12 S.P.	5 7	20	45	8	2150**	8,500
4·7 (V)	53 ,,	43·9	—	7 8	20	45	—	2450	—
4 (I to III)	26 ,,	40	—	3 9	15	25	6·6	2300	9,000
12-pr., 12 cwt. (I)	12 ,,	40	3¾ S P.††	1 15	15	12½	5	2210	8,000
12 ,, 8 ,, (I)	8 ,,	28	—	0 13¾	10	12½	3·5	1585	5,100
6-pr. Hotch. (I, II)	8 ,,	40	$1\frac{15}{16}$ Q.F.[1]	0 7¾	5	6	2·5	1818	4,000
6 ,, Nord. (I, II, III)	6 ,,	42·33	$1\frac{15}{16}$ Q.F.[1]	0 7¾	5	6	2·5	1818	4,500
3 ,, Hotch. (I, II)	5 ,,	40	1½ ,,	0 6⅜	5	$3\frac{5}{16}$	1·8	1873	3,400
3 ,, Nord. (I)	4 ,,	45·4	1½ ,,	0 6⅜	5	$3\frac{5}{16}$	1·9	1920	4,000
Q.F.C. Guns.									
6-in. I / IV, VI	5 tons	26·64	27¾ E.X E., and R.L.G.[4]	13 4	30	100	10·3	1913	10,000
II / IV, VI	5 tons	26·64	27¾ E.X E., and R.L.G.[4]	13 4	30	100	10·3	1913	10,000
III / IV, VI	5 tons	26·64	27¾ E.X E., and R.L.G.[4]	13 4	30	100	10·3	1913	10,000
II & III / III, III	5 ,,	26·2	27¾ E.X E., and R.L.G.[4]	13 4	30	100	10·3	1913	10,000
4-in. I / IIIA, IV, V, VI	26 cwt.	27·85	—	3 9	15	25	—	2177	8,800
R.M.L. Guns.									
inch 17·72 (I)	100 tons	20·48	450 Pm.[1] bl. or Pm.[2]	—	—	2000	23	1548	6,400
16 (I)	80 ,,	18	450 Pm.[1] br.	—	—	1700	23	1540	8,000
12·5 (I)	38 ,,	15·84	165 Pm.[1] bl. or Pm.[2]	48 0(*b*)	10	818	16	1442	6,000
12·5 (II)	38 ,,	15·84	200 E.X.E. (*k*)	59 0(*b*)	10	818	17·7	1575	6,500
12 (I)	35 ,,	13·54	110 P.	—	—	714	14	1340	5,600
12 (I, II)	25 ,,	12·09	85 P.	—	—	614	12	1292	6,000
11 (I, II)	25 ,,	13·18	85 P.	25 4	10	548	13	1360	6,000
10·4 (I)	28 ,,	26·00	190 Pm.[1] bl.	—	—	462	17	1810	9,500
10 (I, II)	18 ,,	14·55	70 P.	20 6(*b*)	10	410	12	1379	6,000
10 (III, IV) (*a*)	12 ,,	12·5	48 S.P. (*i*)	—	—	410	—	1048	8,700 (*l*)
9 (I–V)	12 ,,	13·89	50 P.	14 0(*b*)	7½	256	10	1440	6,000
9 (VI) (*c*)	12 ,,	13·89	50 P.	—	—	256	—	1398	5,000
9 (VIABC)	12 ,,	13·89	50 P. (*i*)	—	—	360	—	1194	10,500
8 (I, III)	9 ,,	14·75	35 P.	—	—	180	8	1384	5,500
7 (I—IV)	7 ,,	18	30 P.	—	—	115	8	1561	5,500
7 (I, III)	6½ ,,	15·86	30 P.	—	—	115	8	1525	5,500
7 (I)	90 cwt.	15·86	22 P. (*d*)	—	—	115	7	1325	5,500
6·6 (I)	70 ,,	14·78	25 P.	—	—	100	—	1416	5,000
80-pr. 80 cwt. (I) (*e*)	80 ,,	—	20 P.	—	—	90	—	1153	5,175
80-pr. (I) convtd	5 tons	18·004	12 P.	—	—	90	—	1160	4,600

¶ When fired with powder. ** When fired with cordite at 60° F. †† For paper shot.
§§ In future this gun will use the same charge as the Mark X gun.
(*a*) Bored up from 9-inch calibre—for H.A. fire.
(*b*) For case shot only.
(*c*) These guns are now mounted in India only.
(*d*) 17 P when slide is not fitted with hydraulic buffer.
(*e*) Colonial guns.
(*l*) For gun on same level as target.
(*k*) 210 Pm.[2] for N.S.

TABLE XII—*continued.*

Particulars of Rifled Guns and Howitzers.

Calibre.	Weight.	Length of bore in calibres.	Maximum charge. Powder.	Maximum charge. Cordite.		Projectile.	Penetration of wrought iron at 1,000 yds.	Muzzle velocity.	Maximum range.
			lb.	lb. oz.	size.	lb.	inches.	f/s.	yards.
64-pr. (I, II)	64 cwt.	15·47	$6\frac{1}{2}$ R.L.G.[4]	—	—	65	—	1125	4,000
64 ,, (III)	64 ,,	15·47	10 R.L.G.[4] (*f*)	—	—	65	—	1390	4,000
64 ,, (I) convtd	71 ,,	16·42 }	$8\frac{1}{4}$ R.L.G.[4]	—	—	65	—	1260	4,000
64 ,, ,,	58 ,,	17·24 }							
40 ,, (II)	35 ,, }	22 }	$6\frac{3}{4}$ R.L.G.[4]	—	—	40	— {	1425	4,500
40 ,, (I)	34 ,, }	18 }						1340	4,500
25 ,, (I)	18 ,,	22	$4\frac{1}{2}$ R.L G.[4]	—	—	25	—	1350	4,500
16 ,, (I)	12 ,,	19	$3\frac{3}{16}$ R.L.G[4]	—	—	18	—	1310	4,200
13 ,, (I, II)	8 ,,	28	$3\frac{1}{4}$ R.L.G.[4]	—	—	13	—	1595	6,100
9 ,, (I, II)	8 ,,	21·17	$1\frac{3}{4}$ R.L.G.[2]	—	—	9	—	1330	4,000
9 ,, (I)	6 ,,	17·67	$1\frac{1}{2}$ R.L.G.[2] (*g*)	—	—	9	—	1250	3,000
9 ,, (II, III, IV)	6 ,,	22	$1\frac{3}{4}$ & $1\frac{1}{2}$ R.L.G.[2] (*g*) }	—	—	9	—(*j*)	1390	3,500
2·5-in. (I, II) jointed	} 400 lb.	26·6 {	$1\frac{10}{16}$ R.L.G.[4] or $1\frac{1}{2}$ R.L.G.[2] }	—	—	$7\frac{5}{8}$	—	1440	4,000
7-pr. bronze (II)	200 ,,	10·7	$\frac{1}{2}$ } R.F.G.[2] {	—	—	$7\frac{1}{4}$	—	700	2,800
7 ,, (IV)	200 ,,	12	$\frac{3}{4}$ } R.F.G., or {	—	—	$8\frac{9}{16}$	—	934	2,500
7 ,, (III)	150 ,,	8	$\frac{5}{8}$ } F.G. {	—	—	$7\frac{5}{8}$	—	688	2,000

R.M.L. Howitzers.

Calibre.	Weight.	Length of bore in calibres.	Powder.	lb. oz.	size.	Projectile.	Penetration.	Muzzle velocity.	Maximum range.
8-in. (I, II)	70 cwt.	12	$11\frac{1}{2}$ R.L.G.[2] (*i*)	—	—	180	—	956	6,300
8 ,, (I)	46 ,,	6	10 ,, (*i*)	—	—	180	—	697	3,800
6·6-in. (I, II)	36 ,,	12	5 ,, (*i*)	—	—	100	—	839	5,400
6·3 ,, (I)	18 ,,	7·14	4 (*h*) ,, (*i*)	—	—	70	—	751	4,000
4 ,, (I) jointed	600 lbs.	13	$1\frac{1}{8}$ R.F.G.[2] (*i*)	—	—	20	—	835	4,000

(*f*) 5 lb. R.L.G.[4] full for R.N.R. practice, except Poole.

(*g*) Marks I and IV are for N.S., and use the $1\frac{1}{2}$ lb. charge. Marks II and III are both L.S. and N.S.; for L.S. the $1\frac{3}{4}$ lb. charge is used, and for N.S. the $1\frac{1}{2}$ lb. charge.

(*h*) Will be replaced by $4\frac{1}{4}$ lb. R.L.G.[4].

(*i*) There are also several reduced charges.

(*j*) L.S. with $1\frac{3}{4}$-lb. charge.

TABLE XIII.

Conversion of Measures.

(Chiefly based on data contained in Col. Noble's Useful Tables.)

Length.

Metric to British.

Mètres.	Yards.	Feet	Inches.
1	1·0936	3·2809	39·37
2	2·1873	6·5618	78·74
3	3·2809	9·8427	118·11
4	4·3745	13·1236	157·48
5	5·4682	16·4045	196·85
6	6·5618	19·6854	236·22
7	7·6554	22·9663	275·60
8	8·7491	26·2472	314·97
9	9·8427	29·5281	354·34

British to Metric.

Yds.	Mètres	Ft.	Mètres.	Ins.	Centi-mètres.
1	0·91438	1	0·30479	1	2·5400
2	1·82877	2	0·60959	2	5·0799
3	2·74315	3	0·91438	3	7·6199
4	3·65753	4	1·21918	4	10·1598
5	4·57192	5	1·52397	5	12·6998
6	5·48630	6	1·82877	6	15·2397
7	6·40068	7	2·13356	7	17·7797
8	7·31507	8	2·43836	8	20·3196
9	8·22945	9	2·74315	9	22·8596

Metric Table of Length.

Milli-mètres.
10 = 1 centimètre.
100 = 1 décimètre,
1000 = 1 mètre.
Mètres.
10 = 1 décamètre.
100 = 1 hectomètre.
1000 = 1 kilomètre.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of yards in 2354 mètres (see cols. 1 and 2)	of feet in 12·4 mètres (see cols. 1 and 3).	of inches in 30·5 centimètres (see cols. 1 and 4). Note, 1 m. = 100 cm.	of mètres in 1026 yards (see cols. 5 and 6).	of mètres in 1742 feet (see cols. 7 and 8).	of centimètres in 17·72 ins. (see cols. 9 and 10).
mètres. yards.	mètres. feet.	cms. inches.	yards. mètres.	feet. mètres.	inches. cms.
2000 = 2187·3	10 = 32·809	30·0 = 11·811	1000 = 914·38	1000 = 304·79	10 = 25·400
300 = 328·09	2 = 6·562	·5 = 0·197	20 = 18·29	700 = 213·36	7 = 17·780
50 = 54·68	0·4 = 1·312		6 = 5·49	40 = 12·19	0·7 = 1·778
4 = 4·37				2 = 0·61	0·02 = 0·051
∴ 2354 = 2574·44	∴ 12·4 = 40·683	∴ 30·5 = 12·008	∴ 1026 = 938·16	∴ 1742 = 530·95	∴ 17·72 = 45·009

NOTE.—If a table of conversion is not at hand, an approximation to the equivalent in inches of a distance measured in centimètres may be obtained by multiplying by 0·4: thus, 30·5 cm. multiply by 0·4, and we have 12·2 inches; the real equivalent as shown above is 12·008 inches.

Weight.

Metric to British.

Kilo-grammes.	Tons.	Pounds Avoirdupois.	Grains Troy.
1	·000984	2·2046	15432·3
2	·001968	4·4092	30864·7
3	·002953	6·6139	46297·0
4	003937	8·8185	61729·4
5	·004921	11·0231	77161·7
6	005905	13·2277	92594·1
7	·006889	15·4323	108026·4
8	·007874	17·6370	123458·8
9	·008858	19·8416	138891·1

British to Metric.

Tons.	Metric tons or milliers	Pounds Avoirdupois.	Kilo-grammes	Grains Troy.	Grammes.
1	1·016	1	0·4536	1	·0648
2	2·032	2	0·9072	2	·1296
3	3·048	3	1·3608	3	·1944
4	4·064	4	1·8144	4	·2592
5	5·080	5	2·2680	5	·3240
6	6·096	6	2·7216	6	·3888
7	7·112	7	3·1751	7	·4536
8	8·128	8	3·6287	8	·5184
9	9·144	9	4·0823	9	·5832

Metric Table of Weight.

Milli-grammes.
10 = 1 centigramme.
100 = 1 décigramme.
1000 = 1 gramme.
Grammes.
10 = 1 décagramme.
100 = 1 hectogramme.
1000 = 1 kilogramme.
Kilo-grammes.
10 = 1 myriagramme.
100 = 1 quintal.
1000 = 1 millier, or tonne, or metric ton.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of tons in 35 tonnes. (see cols. 1 and 2).	of pounds in 56·3 kgms. (see cols. 1 and 3).	of grains in 120 grammes (see cols. 1 and 4).	of tonnes in 38 tons. (see cols. 5 and 6).	of kilogrammes in 68 pounds. (see cols. 7 and 8).	of grammes in 85 grains. (see cols. 9 and 10)
tonnes. tons.	kgms. lbs.	grammes. grains.	tons. tonnes.	lbs. kgs.	grains. grammes
30 = 29·53	50 = 110·231	100 = 1543·23	30 = 30·48	60 = 27·216	80 = 5·184
5 = 4·92	6 = 13·228	20 = 308·65	8 = 8·13	8 = 3·629	5 = 0·324
	0·3 = 0·661				
∴ 35 = 34·45	∴ 56·3 = 124·120	∴ 120 = 1851·88	∴ 38 = 38·61	∴ 68 = 30·845	85 = 5·508

NOTE.—7000 grains troy = 1 pound avoirdupois.

TABLE XIII—*continued.*

Pressure.

Metric and Atmospheric to British.						*British to Metric and Atmospheric.*					
Kilogrammes per sq. cm.	Pounds per square inch.	Tons per square inch.	Atmospheres.	Pounds per square inch.	Tons per square inch.	Pounds per square inch.	Kilogrammes per sq. cm.	Atmospheres.	Tons per square inch.	Kilogrammes per sq. cm.	Atmospheres.
1	14·223	·00635	1	14·7	·00656	1	·07031	·068	1	157·49	152·38
2	28·446	·01270	2	29·4	·01313	2	·14062	·136	2	314·99	304·76
3	42·668	·01905	3	44·1	·01969	3	·21093	·204	3	472·48	457·14
4	56·891	·02540	4	58·8	·02625	4	·28124	·272	4	629·97	609·52
5	71·114	·03175	5	73·5	·03281	5	·35155	·340	5	787·47	761·91
6	85·337	·03810	6	88·2	·03938	6	·42186	·408	6	944·96	914·29
7	99·560	·04445	7	102·9	·04594	7	·49217	·476	7	1102·45	1066·67
8	113·783	·05080	8	117·6	·05250	8	·56248	·544	8	1259·95	1219·05
	128·005	·05715	9	132·3	·05906	9	·63279	·612	9	1417·44	1371·43

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of pounds per square inch in 32·1 kilogrammes per square centimètre (see cols. 1 and 2).

kgs. per sq. cm.		lbs. per sq in.
30	=	426·63
2	=	28·45
0·1	=	1·42
32·1	=	456·55

o tons per square inch in 3210 kilogrammes per square centimètre (see cols. 1 and 3).

kgs. per sq. cm.		tons per sq. in.
3000	=	19·05
200	=	1·27
10	=	0·06
∴ 3210	=	20·38

of tons per square inch in 3254 atmospheres (see cols. 4 and 6).

atmospheres.		tons per sq. in.
3000	=	19·69
200	=	1·31
50	=	0·33
4	=	0·03
∴ 3254	=	21·36

of kilogrammes per square centimètre in 15 lbs. on the square inch (see cols. 7 and 8).

lbs. per sq. in.		kgs. per sq. cm.
10	=	0·7031
5	=	0·3516
∴ 15	=	1·0547

of kilogrammes per square centimètre in 18·3 tons per square inch (see cols 10 and 11).

tons per sq. in.		kgs. per sq. cm.
10	=	1574·9
8	=	1259·95
0·3	=	47·25
∴ 18·3	=	2882·1

of atmospheres in 14·6 tons per square inch (see cols 10 and 12).

tons per sq. in.		atmospheres.
10	=	1523·8
4	=	609·5
0·6	=	91·4
∴ 14·6	=	2224·7

Energy.

Metric to British.		*British to Metric.*	
Mètre-tons.	Foot-tons	Foot-tons.	Mètre tons.
1	3·2291	1	0·3097
2	6·4581	2	0·6194
3	9·6872	3	0·9291
4	12·9162	4	1·2388
5	16·1453	5	1·5484
6	19·3743	6	1·8581
7	22·6034	7	2·1678
8	25·8324	8	2·4775
9	29·0615	9	2·7872

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of foot-tons in 4367 mètre-tonnes (see cols. 1 and 2)

mètre-tonnes.		foot-tons.
4000	=	12916·2
300	=	968·72
60	=	193·74
7	=	22·60
∴ 4367	=	14101·26

of mètre-tonnes in 3592 foot-tons (see cols. 3 and 4).

foot-tons.		mètre-tonnes.
3000	=	929·1
500	=	154·84
90	=	27·87
2	=	0·62
∴ 3592	=	1112·43

NOTE.—1000 mètre-tons is called a dinamode in Italy.

Table XIV.

Work capable of being done by One Pound of Exploding Gunpowder, in Expanding from Volume Unity with Unit Gravimetric Density.

(See Noble and Abel, "Researches on Explosives," *Phil. Trans. Roy. Soc.*, 29th May, 1870.)

Gravimetric volume.	Work in foot-tons.	Gravimetric volume.	Work in foot-tons.	Gravimetric volume.	Work in foot-tons.	Gravimetric volume.	Work in foot-tons.	Gravimetric volume.	Work in foot-tons.
1·00	0 000	1·54	33·681	2·95	68·568	6·10	99·282	13·00	127·036
1·01	0 980	1·56	34·500	3·00	69·347	6·20	99·915	14·00	129·602
1·02	1 936	1·58	35·301	3·05	70·109	6·30	100·536	15·00	131·970
1·03	2·870	1·60	36·086	3·10	70·854	6·40	101·145	16·00	134·168
1·04	3·782	1·62	36·855	3·15	71·585	6·50	101·744	17·00	136·218
1·05	4·674	1·64	37·608	3·20	72·301	6·60	102·333	18 00	138·133
1·06	5·547	1·66	38·346	3·25	73·002	6·70	102·912	19·00	139 914
1·07	6·399	1·68	39·069	3·30	73·690	6·80	103·480	20·00	141·647
1·08	7·234	1·70	39·778	3·35	74·365	6·90	104·038	21·00	143·258
1·09	8·051	1·72	40·474	3·40	75·027	7·00	104·586	22·00	144·788
1·10	8·852	1·74	41·156	3·45	75·677	7·10	105·125	23·00	146·242
1·11	9·637	1·76	41·827	3·50	76·315	7·20	105·655	24·00	147·629
1·12	10·406	1·78	42·486	3·55	76·940	7·30	106·176	25·00	148·960
1·13	11·160	1·80	43·133	3·60	77·553	7·40	106·688	26·00	150·232
1·14	11·899	1·82	43·769	3·65	78·156	7·50	107·192	27·00	151·452
1·15	12·625	1·84	44·394	3·70	78·749	7·60	107·688	28 00	152·622
1·16	13·338	1·86	45·009	3·75	79·332	7·70	108·177	29·00	153·743
1·17	14 038	1·88	45·614	3·80	79·905	7·80	108·659	30·00	154·819
1·18	14 725	1·90	46·209	3·85	80·469	7·90	109·133	31·00	155·857
1·19	15 400	1·92	46·795	3·90	81·024	8·00	109·600	32·00	156·856
1·20	16 063	1·94	47·372	3·95	81·570	8·10	110·060	33·00	157·824
1·21	16·716	1·96	47·940	4·00	82·107	8·20	110·514	34·00	158·771
1·22	17·359	1·98	48·499	4·10	83·157	8·30	110·962	35 00	159·678
1·23	17·992	2·00	49·050	4·20	84·176	8·40	111·404	36·00	160·556
1·24	18·614	2·05	50·383	4·30	85·166	8·50	111·840	37·00	161·411
1·25	19·226	2·10	51·673	4·40	86·128	8·60	112·270	38·00	162·241
1·26	19·828	2·15	52·922	4·50	87·061	8·70	112·695	39·00	163·046
1·27	20·420	2·20	54·132	4·60	87·975	8·80	113·114	40·00	163·828
1·28	21·001	2·25	55·304	4·70	88·861	8·90	113·528	41·00	
1·29	21·572	2 30	56·439	4·80	89·724	9·00	113·937	42·00	
1·30	22·133	2·35	57·539	4·90	90·565	9·10	114·341	43·00	
1·32	23·246	2·40	58·605	5·00	91·385	9·20	114·739	44·00	
1·34	24 324	2 45	59·639	5·10	92·186	9·30	115·133	45·00	
1·36	25·371	2·50	60·642	5·20	92·968	9·40	115·521	46·00	
1·38	26·389	2·55	61·616	5·30	93·732	9·50	115·905	47·00	
1·40	27·380	2·60	62·563	5·40	94·479	9·60	116·284	48·00	
1·42	28·348	2·65	63·486	5·50	95·210	9·70	116·659	49·00	
1·44	29·291	2·70	64·385	5·60	95·925	9·80	117·029	50·00	171
1·46	30·211	2·75	65·262	5·70	96·625	9·90	117·395		
1·48	31·109	2·80	66·119	5·80	97·310	10·00	117·757		
1·50	31·986	2·85	66·955	5·90	97·981	11·00	121·165		
1·52	32·843	2·90	67·771	6·00	98·638	12·00	124·239		

Table XV.

Four Figure Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	Fourth Figure. 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	6	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5723	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	7	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	5	6	7

TABLE XV—*continued.*

Four Figure Logarithms.

No.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											Fourth Figure.								
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	4	4

TABLE XVI.

Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1235	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2223	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	5	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	5	6

TABLE XVI.—*continued.*

Numbers to Logarithms.

logs.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5961	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	13
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7601	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Table XVII.

Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff.	Cosec.	Tan.	Diff.	Cotan.	Secant.	Diff.	Cosine.	
0° 0′	Infin. neg.	Infin.	Infinite.	Infin. neg.	Infin.	Infinite.	10·0000	0	10·0000	90° 0′
0 1	6·4637	3010	13·5363	6·4637	3010	13·5363	10·0000	0	10·0000	89 59
0 2	6·7648	1761	13·2352	6·7648	1761	13·2352	10·0000	0	10·0000	89 58
0 3	6·9408	1249	13·0592	6·9408	1249	13·0592	10·0000	0	10·0000	89 57
0 4	7·0658	969	12·9342	7·0658	969	12·9342	10·0000	0	10·0000	89 56
0 5	7·1627	792	12·8373	7·1627	792	12·8373	10·0000	0	10·0000	89 55
0 6	7·2419	669	12·7581	7·2419	669	12·7581	10·0000	0	10·0000	99 54
0 8	7·3668	512	12·6332	7·3668	512	12·6332	10·0000	0	10·0000	89 52
0 10	7·4637	414	12·5363	7·4637	414	12·5363	10·0000	0	10·0000	89 50
0 12	7·5429	348	12·4571	7·5429	348	12·4571	10·0000	0	10·0000	89 48
0 14	7·6099	300	12·3901	7·6099	300	12·3901	10·0000	0	10·0000	89 46
0 16	7·6678	263	12·3322	7·6678	263	12·3322	10·0000	0	10·0000	89 44
0 18	7·7190	235	12·2810	7·7190	235	12·2810	10·0000	0	10·0000	89 42
0 20	7·7648	212	12·2352	7·7648	212	12·2352	10·0000	0	10·0000	89 40
0 22	7·8061	193	12·1939	7·8062	193	12·1938	10 0000	0	10·0000	89 38
0 24	7·8439	177	12·1561	7·8439	177	12·1561	10·0000	0	10·0000	89 36
0 26	7·8787	164	12·1213	7·8787	164	12·1213	10·0000	0	10·0000	89 34
0 28	7·9109	152	12·0891	7·9109	152	12·0891	10·0000	0	10·0000	89 32
0 30	7·9408	137	12·0592	7·9409	137	12·0591	10·0000	0	10·0000	89 30
0 35	8·0078	118	11·9922	8·0078	118	11·9922	10·0000	0	10·0000	89 25
0 40	8·0658	104	11·9342	8·0658	104	11·9342	10·0000	0	10·0000	89 20
0 45	8·1169	93	11·8831	8·1170	93	11·8830	10·0000	0	10·0000	89 15
0 50	8·1627	84	11·8373	8·1627	84	11·8373	10·0000	0	10·0000	89 10
0 55	8·2041	77	11·7959	8·2041	77	11·7959	10·0001	0	9·9999	89 5
1 0	8·2419	70	11·7581	8·2419	70	11·7581	10·0001	0	9·9999	89 0
1 5	8·2766	65	11·7234	8·2767	65	11·7233	10·0001	0	9·9999	88 55
1 10	8·3088	60	11·6912	8·3089	60	11·6911	10·0001	0	9·9999	88 50
1 15	8·3388	56	11·6612	8·3389	56	11·6611	10·0001	0	9·9999	88 45
1 20	8·3668	53	11·6332	8·3669	53	11·6331	10·0001	0	9·9999	88 40
1 25	8·3931	50	11·6069	8·3932	50	11·6068	10·0001	0	9·9999	88 35
1 30	8·4179	46	11·5821	8·4181	46	11·5819	10·0001	0	9·9999	88 30
1 40	8·4637	42	11·5363	8·4638	42	11·5362	10·0002	0	9·9998	88 20
1 50	8·5050	38	11·4950	8·5053	38	11·4947	10·0002	0	9·9998	88 10
2 0	8·5428	35	11·4572	8·5431	35	11·4569	10·0003	0	9·9997	88 0
2 10	8·5776	32	11·4224	8·5779	32	11·4221	10·0003	0	9·9997	87 50
2 20	8·6097	30	11·3903	8·6101	30	11·3899	10·0004	0	9·9996	87 40
2 30	8·6397	28	11·3603	8·6401	28	11·3599	10·0004	0	9·9996	87 30
2 40	8·6677	26	11·3323	8·6682	26	11·3318	10·0005	0	9·9995	87 20
2 50	8·6940	25	11·3060	8·6945	25	11·3055	10·0005	0	9·9995	87 10
3 0	8·7188	24	11·2812	8·7194	24	11 2806	10·0006	0	9·9994	87 0
3 10	8·7423	22	11·2577	8·7429	22	11·2571	10·0007	0	9·9993	86 50
3 20	8·7645	21	11·2355	8·7652	21	11·2348	10·0007	0	9·9993	86 40
3 30	8·7857	20	11·2143	8·7865	20	11·2135	10·0008	0	9·9992	86 30
3 40	8·8059	19	11·1941	8·8067	19	11·1933	10·0009	0	9·9991	86 20
3 50	8·8251	18	11·1749	8·8261	18	11·1739	10·0010	0	9·9990	86 10
4 0	8·8436	18	11·1564	8·8446	18	11·1554	10·0011	0	9·9989	86 0
4 10	8·8613	17	11·1387	8·8624	17	11·1376	10·0012	0	9·9989	85 50
4 20	8·8783	16	11·1217	8·8795	16	11·1205	10·0012	0	9·9988	85 40
4 30	8·8946	16	11·1054	8·8960	16	11·1040	10·0013	0	9·9987	85 30
40	8·9104	15	11·0896	8·9118	15	11·0882	10·0014	0	9·9986	85 20
4 50	8·9256	15	11·0744	8·9272	15	11·0728	10·0015	0	9·9985	85 10
5 0	8·9403	14	11·0597	8·9420	14	11·0580	10·0017	0	9·9983	85 0
5 10	8·9545	14	11·0455	8·9563	14	11·0437	10·0018	0	9·9982	84 50
5 20	8·9682	13	11·0318	8·9701	13	11·0299	10·0019	0	9·9981	84 40
5 30	8·9816	13	11·0184	8·9836	13	11·0164	10·0020	0	9·9980	84 30
5 40	8·9945	13	11·0055	8·9966	13	11·0034	10·0021	0	9·9979	84 20
5 50	9·0070	12	10·9930	9·0093	12	10·9907	10·0023	0	9·9977	84 10
	Cosine.	Diff. for 1′.	Secan	Cotan.	Diff. for 1′.	Tan.	Cosec.	Diff. for 1′.	Sine.	Angle.

Table XVII—*continued.*

Angle.	Sine.	Diff.	Cosec.	Tan.	Diff.	Cotan.	Secant.	Diff.	Cosine.	...
6° 0′	9·0192	12	10·9808	9·0216	12	10·9784	10·0024	0	9·9976	84° 0′
6 10	9·0311	12	10·9689	9·0336	12	10·9664	10·0025	0	9·9975	83 50
6 20	9·0426	11	10·9574	9·0453	11	10·9547	10·0027	0	9·9973	83 40
6 30	9·0539	11	10·9461	9·0567	11	10·9433	10·0028	0	9·9972	83 30
6 40	9·0648	11	10·9352	9·0678	11	10·9322	10·0029	0	9·9971	83 20
6 50	9·0755	10	10·9245	9·0786	11	10·9214	10 0031	0	9·9969	83 10
7 0	9·0859	10	10·9141	9·0891	10	10·9109	10·0032	0	9·9968	83 0
7 10	9·0961	10	10·9039	9·0995	10	10·9005	10·0034	0	9·9966	82 50
7 20	9·1060	10	10·8940	9·1096	10	10·8904	10·0036	0	9·9964	82 40
7 30	9·1157	10	10·8843	9·1194	10	10·8806	10·0037	0	9·9963	82 30
7 40	9·1252	9	10·8748	9·1291	9	10·8709	10·0039	0	9·9961	82 20
7 50	9·1345	9	10·8655	9·1385	9	10·8615	10·0041	0	9·9959	82 10
8 0	9·1436	9	10·8564	9·1478	9	10·8522	10·0042	0	9·9958	82 6
8 10	9·1525	9	10·8475	9·1569	9	10·8431	10·0044	0	9·9956	81 50
8 20	9·1612	9	10·8388	9·1658	9	10·8342	10·0046	0	9·9954	81 40
8 30	9·1697	83	10·8303	9·1745	85	10·8255	10·0048	2	9·9952	81 30
9 0	9·1943	78	10·8057	9·1997	80	10·8003	10·0054	2	9·9946	81 0
9 30	9·2176	74	10·7824	9·2236	76	10·7764	10·0060	2	9·9940	80 30
10 0	9·2397	70	10·7603	9·2463	72	10·7537	10·0066	2	9·9934	80 0
10 30	9·2606	67	10·7394	9·2680	69	10·7320	10·0073	2	9·9927	79 30
11 0	9·2806	64	10·7194	9·2887	66	10·7113	10·0081	2	9·9919	79 0
11 30	9·2997	61	10·7003	9·3085	63	10·6915	10·0088	3	9·9912	78 30
12 0	9·3179	58	10·6821	9·3275	61	10·6725	10·0096	3	9·9904	78 0
12 30	9·3353	56	10·6647	9·3458	59	10·6542	10·0104	3	9·9896	77 30
13 0	9·3521	54	10·6479	9·3634	57	10. 6366	10·0113	3	9·9887	77 0
13 30	9·3682	52	10·6318	9·3804	55	10·6196	10·0122	3	9·9878	76 30
14 0	9·3837	50	10·6163	9·3968	53	10·6032	10·0131	3	9·9869	76 0
14 30	9·3986	48	10·6014	9·4127	51	10·5873	10·0141	3	9·9859	75 30
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